This paper is published with the approval of the Director of the Division of Building Research, National Research Council, Ottawa.

References Cited

(1) Halliday, I. G., 1950: The liquid water content of snow measurement in the field. Journal of Glaciology, 7:357-361.

(2) Bader, H., 1948: Theory of non-calorimetric methods for the determination of the liquid water content of wet snow, Schweizerische Mineralogische and Petrographische Mitteilungen, 28:355-361.

(3) Gerdel, R. W., 1954: The transmission of water through snow, Trans. American Geophysical Union, 35:3:475-485.

(4) Aura, H. and S. Kinesita, 1954: Measurement of free water content in snow by centrifugal separator, Low Temperature Science, Series A12 - Ser. CI L91.

(5) Proctor, R. R., 1933: Design and construction of rolled earth dams, Engineering News-Record, 111:245-248, 286-289.

(6) Burmister, D. M., 1948: The importance and practical use of relative density in soil mechanics, Reprint from Procs., American Society for Testing Materials. Vol. 48. 20p.

(7) SIPRE, 1952: Instructions for the Measurement of the Free Water Content in Snow, Snow, Ice and Permafrost Research Establishment Operation Manual No. 2, Corps of Engineers, U.S. Army, Wilmette, Illinois.

(8) Krumbein, W. C., 1955: Experimental design in earth sciences, Trans. American Geophysical Union, 36:1:1-11.

(9) Gerdel, R. W., Discussion of the transmission of water through snow, Trans. American Geophysical Union, 35:475-485, 1954; 36:2:347, 1955.

(10) Church, J. E., 1948: The evolution of snow-melt by dyes and drip-pan, Assoc. hydrologie sci. Assemblée Générale d'Oslo, 19-20 août, 1948, 2:115-117.

THERMODYNAMICS OF TRANSPIRATION IN HEAVY FOREST DURING ACTIVE SNOW MELT

by

George Mondrillo[1/]

1. Introduction. - Hydrologists have long recognized the importance of transpiration in their computations of loss in the basin water balance. With the various methods for estimating transpiration currently in use, it is possible to obtain a satisfactory degree of accuracy with seasonal totals. Thornthwaite's method (1), which uses a temperature index of heat supply, has been used for monthly estimates for the Willamette Basin Snow Laboratory (WBSL) and the North Santiam River, Oregon McClain (2), Miller (3). Since the source of transpiration heat supply is primarily solar radiation incident upon the forest, a temperature index alone cannot be expected to define potential transpiration completely. The method is popular because it is relatively simple and makes use of ordinarily available temperature data. However, Thornthwaite (4) recognizes the limitations of his method and suggests an energy balance approach for non-advective conditions.

2. Detailed analysis of the non-advective energy balance is particularly convenient for densely forested snow-covered areas during active spring melt. During the melting season, there is an abundant supply of water to plant roots, and transpiration can proceed at potential rate. If incident radiant energy is known, transpiration energy may be evaluated as a residual difference between input energy and the melt energy equivalent of measured runoff from the basin. This is possible because snow and forest canopy act almost as ideal black bodies for radiant energy in the long-wave spectrum and because transpiration is the principal operating loss mechanism during active spring melt. It is the purpose of this appendix to present an illustration of the thermodynamic relationships involved in forest transpiration during non-advective weather.

3. For illustrative purposes, a detailed energy balance has been worked out for WBSL with data from a non-advective period in the 1949 spring melt season. Starting with incident radiant energy at tree-top level,

---

1/ Hydraulic Engineer, Snow Investigations Unit, Corps of Engineers, Portland, Oregon.

magnitudes of heat flux are computed for the various thermodynamic processes (radiative exchange between forest and atmosphere, radiative exchange between forest and snow, condensation, convection, and transpiration).

4. Data. - Magnitudes of the hydrometeorological variables used in the analysis are averages computed from data of the five-day period 9-13 May 1949 from the WBSL clear-weather snow-melt runoff study, Research Note No. 19 (5). During this period mean daily temperatures and vapor pressures remain essentially constant, no precipitation occurred, and each day's snow-melt contribution to the hydrograph was approximately constant. These data are as follows:

Air temperature	60°F	(15.6°C)
Vapor pressure	0.310 in. Hg	(10.52 mb)
Generated runoff	1.24 in.	(3.15 cm)

With the above data and the assumptions of the following paragraph the construction of the mean daily energy balance may be accomplished.

5. Assumptions. - The following basic assumptions are fulfilled by the conditions of the period under study and are responsible for the relatively simple structure of the energy balance:

a. Net advection = 0
b. Net basin energy change = 0
c. Net moisture-content change
of air = 0
of soil = 0

6. Analysis. - Snow under dense forest canopy has little opportunity to lose heat by radiative cooling at night and remains essentially at a constant temperature of 32°F during the melting season. For the equilibrium conditions of paragraph 5, the average forest temperature and the average air temperature are equal. The forest canopy acts as a reservoir for the input energy and releases heat by long-wave radiation exchange with the atmosphere and the snow, by transpiration, and by convection. Since both snow and soil have already been primed by early season melt, losses (melt minus runoff) are accounted for by transpiration. The energy used for transpiration is computed as the difference between the net all-wave energy exchange with space and the energy required to produce the melt water for runoff from the snow.

7. Constants and incidental computations. - The following constants and relationships are used in pertinent computations:

a. Incident short-wave radiation = 800 ly (extrapolated from Medford, Oregon).

b. Downcoming long-wave radiation = $0.75\sigma T^4$ (= 617 ly/day for air at 60°F). Emissivity value from Research Note No. 25 (6).

c. Basin albedo = 0.13 (for pine forest). Reflected short-wave radiation = 0.13 x 800 = 104 ly

d. Long-wave radiation from trees = σT^4 = 823 ly/day for trees at 60°F (forest emissivity = 1.00).

e. Long-wave radiation from snow = σT^4 = 661 ly/day for snow at 32°F (snow emissivity = 1.00).

f. Net long-wave exchange, forest-snow = forest-snow = 823 - 661 = 162 ly to snow.

g. Net energy input at tree-top level:

Down:	Short-wave plus long-wave, 800 + 617 =	1417 ly
Up :	Short-wave plus long-wave, -104 - 823 =	-927 ly
Net input		490 ly

h. Heat of fusion of snow (with 3% free water)

= 0.97 x 80 x 2.54 = 197 ly/in.

i. Heat of vaporization at 60°F = 587 x 2.54 = 1491 ly/in
Heat of vaporization at 32°F = 595 x 2.54 = 1511 ly/in

j. Specific heat of liquid water = (1.0/1.8) x 2.54 = 1.41 ly/in./°F

Specific heat of water vapor = (0.5/1.8) x 2.54 = 0.71 ly/in./°F

8. Basin energy balance and transpiration loss. - With the assumption that there is no net gain or loss of energy by the basin, total input energy and total output energy are equal. Net all-wave radiant energy constitutes the net heat input to the basin at tree-top level. The combined energy equivalents of transpiration and runoff constitute the output energy. Since heat input and melt energy equivalent of runoff are known, transpiration energy may be found by equating input and output energy:

$$H_r = H_t + H_q$$

where H_r = all-wave radiant energy input = 490 ly (par 7g)
H_t = energy to transpiration loss, ly
H_q = energy equivalent of runoff = 197 x 1.24 = 244 ly

Substituting known values in the above expression:

$$490 = H_t + 244$$
$$H_t = 246 \text{ ly}$$

9. Water equivalent of transpiration energy. The water equivalent of the net transpiration energy (246 ly) is found by analysis of the heat-consuming processes involved in the production of transpiration water loss. (Q_t = transpiration loss in inches).

Processes involved in transpiration loss[1/]	Required energy ly
a. Melting of transpiration water from snow	197 x Q_t
b. Heating of liquid transpiration water from 32 to 60°F	40 x Q_t
c. Vaporization of transpiration water at 60°F	1491 x Q_t
d. H_t, energy to transpiration loss	1728 x Q_t = 246
Transpiration loss	Q_t = 0.14"

10. Snow melt provides the water for transpiration loss and for runoff:

	Melt water - in.	Melt energy - ly
Transpiration loss	0.14	28
Runoff	1.24	244
Total melt	1.38	272

11. Melt components may be evaluated as follows:

	Energy - ly	Melt - in.
Short-wave melt (transmission through forest = 0.10, A = 0.50)	40	0.20
Long-wave melt (net exchange, forest - snow = 823 - 661)	162	0.82
Remainder (condensation and convection)	70	0.36
Total melt	272	1.38

The remainder of 0.36" above may be separated into respective condensation and convection melt components by writing expressions for them with the coefficients of Research Note No. 25 (6) (adjusted for use with mean daily temperatures in °F and mean daily vapor pressures in in. Hg):

Condensation melt $1.0671 \, (e_a - e_s) \, v$
Convection melt $0.00278 \, (T_a - T_s) \, v$

where v = mean daily wind speed in miles per hour at the one foot level. Although conditions are calm, a wind speed

1/ Transpiration loss is distinguished from total transpiration by the amount of condensation. See paragraph 12.

may be found which is equivalent to turbulent exchange in the forest, by substituting known values in the above expressions, adding them together, and equating their sum to the remainder of 0.36 inch. Thus:

Condensation melt	1.0671 (0.31 - 0.18) v	= 0.139 v
Convection melt	0.00278 (60 - 32) v	= 0.078 v
Sum		= 0.217 v = 0.36 in.
		v = 1.66 mph

All the components of melt have been determined:

Component	Energy - ly	Melt - in.	Percent
Short-wave radiation	40	0.20	15
Long-wave radiation	162	0.82	60
Condensation	45	0.23	16
Convection	25	0.13	9
Total	272	1.38	100

12. Total transpiration and condensation. - The net amount of transpiration loss (0.14 in., see par. 9) does not represent the total amount of water involved in the transpiration process. For the assumed average conditions, condensation on the snow is occurring continuously and water for condensation is supplied by total transpiration. Therefore, the total quantity of water transpired must be greater than the transpiration loss by an amount which is equal to the condensate. The condensate releases energy to the snow in cooling to 32°F and in condensing at 32°F. The computation for finding the condensate Q_e, is similar to that for finding the water equivalent of transpiration energy (par. 9), except that for condensation the specific heat of water vapor and the latent heat of vaporization at 32°F (par. 7, i, j) are used:

Process	Energy - ly
Cooling of condensate vapor from 60°F to 32°F	20 x Q_e
Condensation at 32°F	1511 x Q_e
Total energy released to snow by condensation	1531 x Q_e = 45 ly
Condensate	Q_e = 0.03 in.

Total transpiration may be computed by adding this quantity of condensate to the net transpiration:

	Energy - ly	Water - in.
Condensation	45	0.03
Net transpiration	246	0.14
Total transpiration	291	0.17

The energy for total transpiration may also be represented as the combined energies of the heat-consuming processes in the production of total transpiration:

Component	Process	Water in.	Energy ly
Net transpiration	Melting at 32°F, heating of liquid water to 60°F, and vaporization at 60°F	0.14	246
Condensate	Heating of liquid water from 32°F to 60°F and vaporization at 60°F. (no melt energy for condensate)	0.03	45
Total transpiration = potential transpiration		0.17	291

13. Comparison with similar work. - Halstead (4) has indicated a type of computation to be used for non-advective conditions with the assumption that 85 percent of net radiation is consumed for transpiration. A comparison is made with his computation for pine forest in the following table.

Table 1. Comparison of daily amounts of potential evapotranspiration for non-advective conditions

	Riverside, Calif.	Miami, Fla.	WBSL, Ore.
Latitude	34°N	26°N	44°N
Date	25 June	4 June	11 May
Total Short-wave radiation (sun and sky), ly	600	511	800
Albedo (pine forest)	0.14	0.14	0.13
Absorbed short-wave radiation, ly	516	440	696
Long-wave exchange	-118	- 83	-206
Net radiation	398	357	490
Warming of trees (15% net for Riverside, Miami)	-60	-54	0
Snow melt (less condensation), ly	0	0	-227
Net energy available to transpiration, ly	338	303	263
Latent heat of vaporization, ly/cm	585	578	603*
Potential evapotranspiration, cm	0.578	0.524	0.436
in.	0.23	0.21	0.17

*Energy consumed in transpiration = latent heat of vaporization at 60°F plus energy required to heat transpiration water from 32°F to 60°F

Data for Riverside and Miami extracted from Tables 1 and 2, page 206 (4).

Even though the daily potential evapotranspiration amounts computed by Halstead are larger than the WBSL figure for less solar radiation, he has indicated that his values are too high because the soil-warming energy was not taken into account in his computations.

14. The energy flow diagram of figure 1 illustrates the flux of component energies obtained from the above relationships (par. 1-14). In the diagram it can be seen that the sum of heat fluxes directed toward any junction in the network equals the sum of the heat fluxes directed away from the junction. The widths of the paths have been made proportional to the quantities of heat flux which they carry so that their relative magnitudes are emphasized. The heat flux amounts are summarized in the following table:

	Energy, ly	Percent
Heat supply :		
Incident short-wave radiation	800	56
Incident long-wave radiation	617	44
Total	1417	100
Distribution of heat supply:		
Reflected short-wave radiation	104	7
Outgoing long-wave radiation	823	58
Net input to basin	490	35
Total	1417	100
Distribution of net input energy to basin:		
Transpiration loss	246	50
Runoff	244	50
Total	490	100
Distribution of melt energy:		
Condensation Melt	45	17
Other (short-wave, long-wave convection)	227	83
Total	272	100

15. Interpretations from the energy balance. - The following information may be learned from the non-advective energy balance for snow-covered areas in dense forest:

a. Transpiration loss consumes approximately fifty percent of the net input energy to the basin.

b. Average transpiration loss = 0.14"/day (approximately 10% of melt) for the period 9-13 May 1949.

c. The coefficients of condensation and sensible heat transfer for CSSL (Research Note No. 25) are applicable to calm conditions in dense forest with an equivalent wind of 1.6 mph.

d. The role of condensation in the non-advective heat budget is clarified. For the assumed conditions the condensate water does not appear either in the runoff or in the transpiration loss but undergoes a repeated cycle of evaporation and condensation, aiding the transpiration process by partly supplying the water requirements of the vegetation, and aiding melt by returning some of the total transpiration energy to the snow (see chart 1).

16. The energy balance computed in this appendix is limited entirely to the assumptions stated in paragraph 3 and is not intended to apply to other conditions. It is believed that daily amounts of potential evapotranspiration from forested, snow-covered areas computed in this way are more accurate than can be obtained by other methods and should be used where possible as a check on other computations and measurements.

References Cited

(1) Thornthwaite, C. W.: "An approach toward a rational classification of climate," Geographical Review, Vol. 38, No. 1, pp. 55-94, 1948

(2) McClain, M. H.: "Precipitation, evapotranspiration, and runoff, Willamette Basin Snow Laboratory," Research Note No. 20,* 30 July 1954.

(3) Miller, S.: "Forecasting seasonal runoff by the water-balance method," Research Note No. 22,* 30 September 1954.

(4) Thornthwaite, C. W.: "A re-evaluation of the concept and measurement of potential evapotranspiration," The Johns Hopkins University, Publications in Climatology, Vol. VII, No. 1, pp.200-209, 1954

(5) Mondrillo, G. and C. E. Jencks: "Clear-weather snow-melt runoff in a densely forested area, Willamette Basin Snow Laboratory," Research Note No. 19,* 15 May 1954

(6) Hildebrand, C. E. and T. H. Pagenhart: "Lysimeter studies of snow melt," Research Note No. 25,* 1 March 1955.

*Snow Investigations Research Note, U. S. Army, Corps of Engineers, North Pacific Division, Portland, Oregon.