Blue Canyon Studies

To provide wind direction and velocity measurements at another "mountain top" station, recorders were installed at Blue Canyon where they will be operated with the cooperation of the Weather Bureau. The Blue Canyon Station is on a flat-topped ridge some 15 miles to the west of the laboratory headquarters. Wind records from this station will be correlated with and supplement those taken on Mt. Lincoln.

Conclusion

The new basic measurements of the meteorological elements and snow at five stations in the Central Sierra, together with the Corps of Engineers-Weather Bureau records, will provide sound physical base for studies of snow accumulation, snow melt, and evaporative losses from snow and soil in the Central Sierra.

THE USE OF FOURIER SERIES IN STREAMFLOW FORECASTING

By

Cleve H. Milligan[1/]

Scope

Brevity and clarity require that only the basic idea involved in the Use of Fourier Series in streamflow forecasting be presented in this paper. Several variants on the procedures herein described are currently under investigation. These additional studies are in varying stages of completion. As far as they have been completed they show considerable promise.

The method herein described has been used only on the Logan River in Utah. As soon as possible it will be tried on other streams. Thus far, however, the objective has been to experiment with several forecasting methods for one stream before advancing to other streams.

Basic Data

Basic data should be consistent. That is, all data including precipitation, streamflow, temperature, snow survey, etc. should be corrected for any changes in location of the station or for any other changed conditions or procedures under which the data were collected.

Data available for these studies included:

1. Daily and average monthly temperatures at valley stations.
2. Daily and monthly precipitation at valley stations.
3. Daily and monthly streamflow, Logan River near Logan, Utah.
4. April 1 snow survey data at 7 mountain stations in the Logan River drainage. At some stations snow data are available for other times of the year, but in this analysis only the April 1 data were used. The method described herein, however, can be adapted to snow data collected on a monthly basis.

Soil moisture data are now being collected at 4 stations in the Logan River watershed. Data are insufficient yet to be included in this analysis. The method can be adapted to include soil moisture data as soon as sufficient data are available.

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Example

Table 1. VALUES OF TRIG. FUNCTIONS FOR USE IN FOURIER SERIES.

One water year = 2π radians or 360°

MONTH	COS θ	SIN θ	COS 2 θ	SIN 2 θ
Oct.	1.000	.000	1.000	.000
Nov.	.866	.500	.500	.866
Dec.	.500	.866	- .500	.866
Jan.	.000	1.000	-1.000	.000
Feb.	- .500	.866	- .500	- .866
Mar.	- .866	.500	.500	- .866
Apr.	-1.000	.000	1.000	.000
May	- .866	- .500	.500	.866
June	- .500	- .866	- .500	.866
July	.000	-1.000	-1.000	.000
Aug.	.500	- .866	- .500	- .866
Sept.	.866	- .500	+ .500	- .866

FORMULAS FOR COMPUTATION OF FOURIER COEFFICIENTS

$$\text{Means} = \frac{\Sigma x}{n} = \frac{\Sigma x}{12}$$

$$\text{Coeff. of } \cos\theta = \frac{\Sigma(x_i \cos\theta)}{6}$$

$$\text{Coeff. of } \sin\theta = \frac{\Sigma(x_i \sin\theta)}{6}$$

$$\text{Coeff. of } \cos 2\theta = \frac{\Sigma(x_i \cos 2\theta)}{6}$$

$$\text{Coeff. of } \sin 2\theta = \frac{\Sigma(x_i \sin 2\theta)}{6}$$

See pp. 337-338 Harmonic Analysis "Mathematical Methods in Engineering" by Von Karman

x = month by month physical data such as temperature, precipitation, run-off, etc.

Table 2 COMPUTATION OF MEANS FOR A TYPICAL YEAR (1950)

Month	$(\bar{T})$ Logan Temp.		$(\bar{P})$ Richmond Precip.		$(\bar{F})$ Logan R. run-off	
	Year	°F.	Year	Inches	Year	A.F.
Oct.	1949	44.9	1949	3.62	1948	9,860
Nov.	49	43.4	49	1.45	48	8,480
Dec.	49	26.6	49	2.35	48	7,700
Jan.	1950	24.1	1950	3.21	949	6,990
Feb.	50	30.3	50	0.80	49	6,020
Mar.	50	36.5	50	2.24	49	7,670
Apr.	50	48.2	50	2.04	49	18,610
May	50	56.5	50	1.93	49	42,900
June	50	64.0	50	1.36	49	33,130
July	50	73.3	50	0.69	49	18,350
Aug.	50	71.4	50	0.84	49	13,020
Sept.	1950	62.4	50	1.04	1949	10,840
Σx_i		581.6		21.57		183,570
* Means		48.47		1.798		15,298

* These means are the $\bar{T}$, $\bar{P}$ and $\bar{F}$ in equations (4), (5), and (6).

Note: The method described herein is not limited to the particular months used in this table. For purposes of this example these were used.

Table 3 COMPUTATION OF TEMPERATURE COEFFICIENTS (1950)

Mo.	Coeff. for Cos θ, (A_{T1})			Coeff. for Sin θ, (A_{T2})			Coeff. for Cos 2 θ, (A_{T3})			Coeff. for Sin 2 θ, (A_{T4})		
	x	Cos θ	x Cos θ	x	Sin θ	x Sin θ	x	Cos 2 θ	x Cos2θ	x	Sin 2θ	x Sin 2θ
Oct.	44.9	1.000	44.90	44.9	.000	.000	44.9	1.000	44.90	44.9	.000	.000
Nov.	43.4	.866	37.58	43.4	.500	21.70	43.4	.500	21.70	43.4	.866	37.58
Dec.	26.6	.500	13.30	26.6	.866	23.04	26.6	-.500	-13.30	26.6	.866	23.04
Jan.	24.1	.000	.00	24.1	1.000	24.10	24.1	-1.000	-24.10	24.1	.000	.000
Feb.	30.3	-.500	-15.15	30.3	.866	26.24	30.3	-.500	-15.15	30.3	-.866	-26.24
Mar.	36.5	-.866	-31.61	36.5	.500	18.25	36.5	.500	18.25	36.5	-.866	-31.61
Apr.	48.2	-1.000	-48.20	48.2	.000	.000	48.2	1.000	48.20	48.2	.000	.000
May	56.5	-.866	-48.93	56.5	-.500	-28.25	56.5	.500	28.25	56.5	.866	48.93
June	64.0	-.500	-32.00	64.0	-.866	-55.42	64.0	-.500	-32.00	64.0	.866	55.42
July	73.3	.000	.00	73.3	-1.000	-73.30	73.3	-1.000	-73.30	73.3	.000	.000
Aug.	71.4	.500	35.70	71.4	-.866	-61.83	71.4	-.500	-35.70	71.4	-.866	-61.83
Sept.	62.4	.866	54.04	62.4	-.500	-31.20	62.4	+.500	+31.20	62.4	-.866	-54.04
Σ			9.63			-136.67			-1.05			-8.75
$\frac{\Sigma}{6}$		A_{T1} = 1.61			A_{T2} = -22.78			A_{T3} = -0.18			A_{T4} = -1.46	

Table 4 COMPUTATION OF PRECIPITATION COEFFICIENTS

(Precipitation at Richmond, Utah) 1950

Mo.	x	(A_{P1}) Coeff. of Cos θ		(A_{P2}) Coeff. of Sin θ		(A_{P3}) Coeff. of Sin 2 θ		(A_{P4}) Coeff. of Sin 2 θ	
		Cos θ	x Cos θ	Sin θ	x Sin θ	Cos 2θ	x Cos 2θ	Sin 2θ	x Sin θ
Oct.	3.62	1.000	3.6200	.000	.000	1.000	3.6200	.000	.000
Nov.	1.45	.866	1.2557	.500	.7250	.500	.7250	.866	1.2557
Dec.	2.35	.500	1.1750	.866	2.0351	-.500	-1.1750	.866	2.0351
Jan.	3.21	.000	.0000	1.000	3.2100	-1.000	-3.2100	.000	.0000
Feb.	0.80	-.500	-.4000	.866	.6928	-.500	-.4000	-.866	-.6928
Mar.	2.24	-.866	-1.9398	.500	1.1200	.500	1.1200	-.866	-1.9398
Apr.	2.04	-1.000	-2.0400	.000	.0000	1.000	2.0400	.000	.0000
May	1.93	-.866	-1.6714	-.500	-.9650	.500	.9650	.866	1.6714
June	1.36	-.500	-.6800	-.866	-1.1778	-.500	-.6800	.866	1.1778
July	0.69	.000	.0000	-1.000	-.6900	-1.000	-.6900	.000	.0000
Aug.	0.84	.500	.4200	-.866	-.7274	-.500	-.4200	-.866	-.7274
Sept.	1.04	.866	.9006	-.500	-.5200	.500	.5200	-.866	-.9006
Σ			+.6401		+3.7027		2.415		1.8794
A_p			.107		.617		.402		.313

Table 5 COMPUTATION OF RUNOFF COEFFICIENTS (LOGAN RIVER) 1950

Mo.	x Runoff	(A_{F_1}) Cos θ	Coeff. for cos θ: x Cos θ	(A_{F_2}) Sin θ	Coeff. for sin θ: x Sin θ	(A_{F_3}) Cos 2θ	Coeff. for cos 2 θ: x Cos 2θ	(A_{F_4}) Sin 2θ	Coeff. for sin 2 θ: x Sin 2θ
Oct.	9,860	1.000	9,860	.000	000	1.000	9,860	.000	000
Nov.	8,480	.866	7,344	.500	4,240	.500	4,240	.866	7,344
Dec.	7,700	.500	3,850	.866	6,668	- .500	- 3,850	.866	6,668
Jan.	6,990	.000	000	1.000	6,990	-1.000	- 6,990	000	000
Feb.	6,020	- .500	- 3,010	.866	5,213	- .500	- 3,010	-.866	- 5,213
Mar.	7,670	- .866	- 6,642	.500	3,835	.500	3,835	-.866	- 6,642
Apr.	18,610	-1.000	-18,610	.000	000	1.000	18,610	000	000
May	42,900	- .866	-37,151	- .500	-21,450	.500	21,450	.866	37,151
June	33,130	- .500	-16,565	- .866	-28,691	- .500	-16,565	.866	28,691
July	18,350	.000	000	-1.000	-18,350	-1.000	-18,350	000	000
Aug.	13,020	.500	6,510	- .866	-11,275	- .500	- 6,510	-.866	-11,275
Sept.	10,840	.866	9,387	- .500	- 5,420	.500	5,420	-.866	- 9,387
Σ			-45,027		-58,240		+ 8,140		+47,337
A_F			- 7,505		- 9,707		+ 1,357		+ 7,889

Table 6 TABULATION OF COEFFICIENTS (1950)

Loc. or Coefficients	1950 Data	Coding Corrections Added		X'
Franklin Basin	41.3		5×10^{-2}	2.065
Tony Grove Lk.	50.3		2×10^{-2}	1.006
Tony Grove R.S.	14.0		5×10^{-2}	.700
Spring Hollow Lk.	19.8		5×10^{-2}	.990
Spring Hollow U.	34.7		5×10^{-2}	1.735
Mount Logan	41.9		5×10^{-2}	2.095
Garden City S.	28.9		5×10^{-2}	1.445
Logan Temp. Mean	48.47		2×10^{-1}	9.694
A_{T_1}	1.61	+1.54 = 3.14	$\times 2 \times 10^{-1}$	.630
A_{T_2}	-22.78	+27.81 = 5.03	$\times 1 \times 10^{-1}$	.503
A_{T_3}	- 0.18	+2.23 = 205	$\times 2 \times 10^{-1}$	.410
A_{T_4}	- 1.46	+5.34 = 3.88	$\times 2 \times 10^{-1}$	.776
Rich. Precip. M.	1.798		1	1.798
A_{P_1}	.107	+.994 = 1.101	1	1.101
A_{P_2}	.617	+.611 = 1.228	1	1.228
A_{P_3}	.402	+.358 = .760	1	.760
A_{P_4}	.313	+.700 = 1.013	1	1.013
Logan R. Flow	15,298		$\times 1 \times 10^{-4}$	1.530
A_{F_1}	-7507		1×10^{-4}	.683
A_{F_2}	-9707		1×10^{-4}	1.033
A_{F_3}	1357		1×10^{-4}	.580
A_{F_4}	7889		1×10^{-4}	.789

Table 7 MEANS AND FOURIER COEFFICIENTS FOR PREDICTION EQUATION

Coefficients For Prediction Equation	Intercept b_o	$\Sigma b_i x''$	Y''	Code Values	Y'	Correction (decade)	($\bar{Y}$) Values of Coeff. A_o, A_T, A_2, A_3
Mean ⟶	3.4284	-1.4065	2.0219	1×10^4	20,219		20,219
A_0	.4883	- .2182	.2701	1×10^4	2,701	-14,331	-11,630
A_1	-4.9156	+5.3172	.4016	1×10^4	4,016	-20,036	-16,020
A_2	-4.3514	+4.8200	.4694	1×10^4	7,156	- 4,445	+ 249
A_3	1.9743	- .8146	1.1597	1×10^4	12,123		+11,597

b_i = regression coefficients from IBM multiple correlation analysis

$Y'' = b_o + \Sigma b_i x$ $\quad Y' = Y'' \times$ Code Value $\quad Y = Y' +$ Correction

Table 8 PREDICTED AND ACTUAL FLOW, LOGAN RIVER 1950.

Month	$\bar{F}$	$A_0 \cos \theta$	$A_1 \sin \theta$	$A_2 \cos 2\theta$	$A_3 \cos 2\theta$	Predicted Monthly Flow A.F.	Actual Flow A.F.
Oct.	20,219	-11,630	0	249	0	8,838	10,480
Nov.	20,219	-10,072	- 8,010	125	10,043	12,305	8,060
Dec.	20,219	- 5,815	-13,873	-125	10,043	10,449	7,450
Jan.	20,219	0	-16,020	-249	0	3,950	7,660
Feb.	20,219	+ 5,815	-13,873	-125	-10,043	1,993	6,670
Mar.	20,219	+ 7,158	- 8,010	125	-10,043	9,449	8,610
Apr.	20,219	11,630	0	249	0	32,098	21,820
May	20,219	10,072	8,010	125	10,043	48,469	49,110
June	20,219	5,815	13,873	-125	10,043	49,825	68,780
July	20,219	0	16,020	-249	0	35,990	40,020
Aug.	20,219	- 5,815	13,873	-125	-10,043	18,109	19,790
Sept.	20,219	-10,072	8,010	125	-10,043	8,239	14,430
		Totals for Year				239,714	262,880
	July-Sept. Runoff using 32-Year Mean Temp.					62,338	74,240
		July-Sept. Totals using Actual Temps.				73,250	74,240

Objective

The main objective of this analysis was to develop a procedure or a mathematical model which will utilize the foregoing data available and give an accurate, unbiased prediction of expected streamflow.

The method developed should not be highly arbitrary. Sometimes, for example, we have arbitrarily weighted the snow measurements collected at different stations, because for a particular series of years such a weighting procedure gives a more accurate forecast for most of those years. There is no guarantee, however, that such a weighting scheme will work equally well over another different series of years.

Fourier Series

A Fourier Series may be expressed as,

$$f(x) = Ao + \sum_{1}^{\infty} \left(An \cos \frac{n\pi x}{c} + B_n \sin \frac{n\pi x}{c}\right) \text{------------} (1)$$

Where A_n and B_n are Fourier coefficients defined as follows:

$$A_n = \frac{1}{c} \int_{-c}^{c} f(x) \cos \frac{n\pi x}{c} \, dx \text{------------} (2)$$

$$B_n = \frac{1}{c} \int_{-c}^{c} f(x) \sin \frac{n\pi x}{c} \, dx \text{------------} (3)$$

c is in the interval over which $f(x)$ is expressed, that is $-c \leq x \leq c$. $f(x)$ can have only a finite number of finite discontinuities and maxima and minima over the interval c.

For purposes of this analysis equation (1) simply states that if $f(x)$ represents actual temperature, precipitation, or streamflow data plotted on a time scale, we can fit a trigonometric curve of the form represented by the right hand side of the equation to these data by an appropriate selection of the coefficients $A_1, A_2, A_3,$ ——— A_n and $B_1, B_2, B_3,$ ——— B_n.

Equations (2) and (3) suggest the manner in which these coefficients must be selected.[1]

In this analysis equation (1) was simplified and utilized to represent mean monthly temperature, monthly precipitation, and monthly streamflow in time as follows:

(a) Temperature:

$$T_M = \bar{T} + A_{T_1} \cos \theta + A_{T_2} \sin \theta + A_{T_3} \cos 2\theta = A_{T_4} \sin 2\theta \text{------------} (4)$$

where, T_M = Mean temperature for any month of the year in question.

$\bar{T}$ = Average temperature for 12 months (mean of mean monthly temperatures).

$A_{T_1}, A_{T_2}, A_{T_3}, A_{T_4}$ are Fourier coefficients determined from actual temperature data.

θ = time (2π radians or 360 degrees is one year).

Only 4 terms of the series were considered necessary to represent mean monthly temperature adequately. The accuracy can be increased to any desirable level by adding additional terms to the series. This, however, increases the work of computation.

1/ See "Mathematical Methods in Engineering" by Von Karman and Biot, pp. 337-338.

(b) Precipitation:

$$P_M = \bar{P} = A_{p_1} \cos \theta + A_{p_2} \sin \theta + A_{p_3} \cos 2\theta = A_{p_4} \sin 2\theta \text{ ———— } (5)$$

where,

P_M = Precipitation for any month of the year in question.

$\bar{P}$ = Mean monthly precipitation for a given station for the year in question.

A_{p_1}, A_{p_2} — etc. are Fourier coefficients determined from actual precipitation data.

(c) Streamflow:

$$F_M = \bar{F} + A_{F_1}, \cos \theta + A_{F_2} \sin \theta + A_{F_3} \cos 2\theta = A_{F_4} \sin 2\theta \text{ ———— } (6)$$

where,

F_M = Total streamflow in acre feet for any month for the year in question.

$\bar{F}$ = Mean monthly streamflow in acre feet for the year in question.

A_{F_1}, A_{F_2} etc. are Fourier coefficients determined from actual streamflow data.

The Fourier coefficients in equations (4), (5) and (6) measure the effect of antecedent temperature, precipitation, and streamflow on the predicted streamflow. They help set the stage for the streamflow forecast to be made by use of another Fourier series as follows:

(d) Prediction formula:

$$F_{M_p} = \bar{F}_M + A_o \cos \theta + A_1 \sin \theta + A_2 \cos 2\theta + A_3 \sin 2\theta \text{ ———— } (7)$$

where,

F_{M_p} = Predicted monthly flow, usually for each month after April 1, or some other predictio date.

$\bar{F}_M$ = Mean monthly flow

A_o, A_1, A_2, A_3 Fourier coefficients related to the Fourier coefficients of equations (4), (5), and (6) and to water content of snow. That is,

$A_o = \Phi_o$ (A_T, A_p, A_F, water content of snow)

$A_1 = \Phi_1$ (A_T, A_p, A_F, water content of snow)

$A_2 = \Phi_2$ (A_T, A_p, A_F, water content of snow)

$A_3 = \Phi_3$ (A_T, A_p, A_F, water content of snow)

These functional relationships are determined by multiple correlation.

The foregoing discussion gives only the germ of the idea for using Fourier Series in streamflow forecasting. Many variants on the procedure as here outlined are possible. The method is highly flexible. Accuracy can be improved by adding more terms to the Fourier series. If data are available, a series can be fitted to snow curves rather than using single measurements. The time interval can be decreased to 2-week periods rather than to use periods of one month. In this connection, the method might be useful for flood forecasting. However, we need to be able to predict temperatures for the streamflow prediction period before great accuracy can be obtained in prediction of flood flows.

The entire procedure can be mechanized by programming the computations for IBM machines. An example, showing how the forecast would have been made for Logan River near Logan, Utah, for the year 1950, is included to clarify the procedures.

The comparison between the computed July-Sept. runoff using 32-Year mean temperatures for the months April-September and the actual temperatures for these same months indicates that if summer-time temperatures could be predicted more accurately in advance the forecasts could be materially improved.

Temperatures during the summer prediction period influence runoff a good deal. The consumptive use of water by the vegetation on the watershed is an important factor in determining the volume of runoff. If temperatures are higher than normal, consumptive use will be high and runoff will be low. On the other hand, if summer temperatures are lower than normal, the runoff will be higher. So far the temperatures used in the formula for the prediction period have always been the 32-year normals for the summer months.

Studies are now underway to develop a method for predicting in advance more accurately what these expected summer temperatures will be. If these studies are fruitful then the method can still be further improved.

Comparison of the last two columns of table 8 indicate considerable error in the predictions of volume of runoff for some of the month. A modification in the procedure of computing the fourier coefficients has now been developed which gives greater accuracy in computing the monthly flows. The results of these studies were not available soon enough to be included in this paper.

DISCUSSION

By

George R. Hamburg and H. Boyd Phillips 1/

Mr. Milligan's paper presents a very interesting method of forecasting stream flow. The Fourier system, to our knowledge, has not previously been used in forecasting. The author introduces here a new tool which in time could be very useful in forecasting inflow. The accuracy of the example given is of a relatively high order. The results of the predicted monthly flow in comparison with the actual flow at Logan are very significant, as the total difference for the year is within 9 percent of the total actual flow. The author should be encouraged into further investigation and research in this matter.

It would be interesting to note what results would be obtained with the use of more recent data. For example, we would like to see how the forecast for 1956 would have turned out using data through 1955. If available, we should appreciate receiving from Mr. Milligan copies of the 1957 forecast for the Logan River so that it can be compared with the actual flow that occurs this year.

Any technique which recognizes evapotranspiration losses as the melt season progresses is a much needed contribution to the science and art of forecasting. We are looking with anticipation to achievement of the objective set forth in Paragraph 4, page 52, in which Mr. Milligan states that studies are underway to develop a method for predicting summer temperatures more accurately.

We should like to suggest that if he has not done so, Mr. Milligan's forecast might be improved with the use of Logan, Utah, precipitation data rather than Richmond, which we believe would be more indicative of basin conditions. We would also like to ask Mr. Milligan if the precipitation data at Tony Grove Ranger Station would be useful in improving the forecast.

We should also like to know to what degree the results were improved by using more than four terms, as stated in the paragraph on page 50. Mr. Milligan also states, in the paragraph on page 52,

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