MAXIMUM SNOW DEPTHS AND SNOW LOADS ON ROOFS IN CANADA

By

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The annual maximum depths of snow on the ground at over 200 stations in Canada for the available winters from 1941 to 1959 were tabulated. For each station an extreme value distribution of the form $y = -\log_e(-\log_e P)$ was fitted to the observed maxima and then used to obtain the maximum snow depth for a 30-year return period. The maximum snow depths were then used to estimate the design snow loads on roofs. The distribution of snow depths and loads are shown on maps of Canada.

INTRODUCTION

In many parts of Canada the weight of snow on the roof of a building exceeds, at times, the maximum loads which the roof would otherwise have to carry. The load of snow then becomes the determining factor in specifying the required strength of the roof. The maximum load of snow that will probably occur during the life of the roof is necessary for design purposes but with short snow cover records its determination is difficult. In the1940's a rule of thumb based on average precipitation records was used. In 1953 a more rational method was introduced and now, with longer snow cover records available, the estimates can be revised. These revised estimates of maximum snow loads are presented as shown on charts coverning most of Canada, but actual figures for particular locations can be obtained from the Division of Building Research in Ottawa.

HISTORY

The National BuildingCode of Canada published in 1941 by the National Research Council of Canada (1) contained a paragraph requiring roofs having a slope of 20 degrees or less to be capable of supporting a live load of from 20 to 40 pounds per square foot of roof surface. For steeper roofs the loading could be reduced. As a guide to the selection of the live load the following formula was to be used:

$$L = S + R$$

where S is the sum of the average snowfalls in January, February and March, in inches, over a number of years; R the sum of the average rainfall in January, February and March, in inches, over a number of years; and L is related to live load by the following table:

L	Live Load, lb/sq ft.
Less than 20	20
20-30	30
More than 30	40

This method was based on weather records that were readily available for many stations and involved no calculation other than a simple addition. The approach was completely empirical. The change of units from inches to pounds per square foot was not meant to imply a particular snow density but merely a fortuitous equivalence of the numerical values of the average snowfall and the extreme snow load. The addition of rainfall had little effect in most cases but did indicate that the authors of the code were aware of the prevalence of roof failures during early spring rains. The method did not provide for any difference in loads between stations where winter thaws were common and those where snow ordinarily accumulated for a few months. Nor is it known why the design load was limited to 40 pounds per square foot.

Several years later, the Associate Committee on the National Building Code appointed a Technical Committee on Climate. This technical committee recommended that the design snow load be based on the maximum measured depth of snow on the ground instead of on average snowfalls. M. K. Thomas assembled the snow depth records that were available at that time, selected the extremes, converted these to loads by assuming that the density of the snow on the ground was about 0.2 times that of water, added the maximum weight of rain that might be expected to fall in one day in the late winter or early spring, and plotted the resulting snow loads on a map of Canada. It was assumed that snow loads on flat unheated

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roofs would be the same as on the ground. This map was published in the National Building Code of Canada 1953 (2). The details of the computations of maximum snow loads on horizontal surfaces are given by Thomas (3).

Thomas' method is a more rational approach to the problem, and a considerable improvement over the older method. Basing the computed loan on a single observation (the maximum observed snow depth in the period from 1941 to 1950) instead of on a statistic based on a larger sample is a procedure that can be questioned. Using a smaple of ten or less to compute extreme value statistics is also questionable. These were the only alternatives and Thomas chose the former. His choice of 0.2 for the density of the snow on the ground and of one day as the rainfall period which might occur without runoff were arbitrary but probably reasonable. The present work is primarily a report on a revision of Thomas' chart, based on the additional snow cover records obtained since 1950, and using a somewhat different method of computing the extreme load.

ASSEMBLY AND SELECTION OF SNOW COVER RECORDS

The reports of depths of snow on the ground in Canada prior to 1941 were very scattered and incomplete. In that year a number of stations started taking daily measurements of snow cover and the number and reliability has gradually increased since then. Up to eighteen years of records are now available for some stations. It was decided to use a statistical analysis of the annual maximum depths of snow instead of merely selecting the one extreme value in the whole period of record as Thomas had done. Accordingly the annual maximum depths of snow on the ground were tabulated for each station and an extreme value distribution equation was fitted to these values using the method explained by Gumbel (4).

The tabulation of annual maximum snow depths was not as straighforward as the above statement might imply. The depths of snow on the ground at the <u>end</u> of each month are published in the "Monthly Record" (5) but neither the monthly nor the annual maxima are published. Abstracts for annual maximum depths were prepared by the Climatology Division under the direction of J. G. Potter for the fifteen winteres from 1941 to 1956, and were used by him to draw a map of the mean annual maximum depths of snow cover (6).

Most of the records used inthis paper were copied from Potter's manuscript abstracts or from microfilm copies of his abstracts. Where Potter had entered a value of the annual maximum depth of snow cover this was accepted as a "usable value" unless there was some fairly obvious reason for questioning it. If no annual maximum was entered the largest of the depths at the ends of the months were recorded and used as explained below. Even if some of the month-end values were missing the largest of the otheres was recorded and the numberof missing months noted. Table II is a fictitious example of reports from one station. The maximum monthend values and the number of missing months are shown in Columns 6 and10.

TABLE II

EXAMPLE OF SELECTION OF REPORTS AND THEIR ANALYSIS

Winter beginning in	Month end Snow Cover Dec.	Jan.	Feb.	Mar.	Month End Max	Adj. Mo-End Max.	Rept. Ann. Max.	No.Yrs. Smaller n	Months Missing m	4n/12m	Accepted Values h
1941	–	–	–	–			–				–
1942	1	6	9	7	9	11	–				11
1943							26				26
1944							18				18
1945	–	–	7	3	7	9	–	2	2	8/24	0
1946	7	12	–	10	12	15	–	6	1	24/12	15
1947	7	12	–	–	12	15	–	6	2	24/24	–
1948	–	–	24	–	24	30	–	11	3	44/36	30
1949							10				10
1950		–					25				25
1951	4	9	12	8	12		11				12
1952							22				22
1953							32				32
1954							7				7
1955							11				11
1956							6				6
1957							20				20
1958							14				14
Column 1	2	3	4	5	6	7	8	9	10	11	12

N = 15 $\Sigma h^2 = 5425$ $S_h = 7.97$

$\Sigma h - 259$ $\bar{h} = 17.27$ A dash (–)represents missing report

A blank represents a report that was available but not needed.

In most cases, and certainly on the average, the largest of the month-end snow depths will be less than the maximum for the winter. If the month-end values are to be used they should be multiplied by some factor greater than unity to make them comparable with annual maxima. To obtain a value for this factor both the annual maxima and the maxima of the month-end snow depths were tabulated for 76 stations in Eastern Canada whenever both values were available for the winters beginning in 1951 to 1955. Similar values were not readily available for more stations in the East nor for any Western stations but this sample was probably sufficient. The sums of these values are shown in Table I.

TABLE I

RATIO OF THE ANNUAL MAXIMUM DEPTH TO THE MAXIMUM OF THE MONTH-END DEPTHS OF SNOW COVER

	No. of Stations	Sums of Annual Maxima	Sums of Max. Month-end Depths	Ratio
Ontario	28	3389	2823	1.200
Quebec	19	2791	2316	1.205
Atlantic Provinces	29	3338	2560	1.304
Totals	76	9518	7699	1.236

The ratios in the last column of the table indicate that in Ontario and Quebec month-end values should be increased by 20 per cent and in the Atlantic Provinces by 30 per cent. The difference in these percentages may be significant but the effect on the final snow depth estimates would be small. Since similar ratios were not available for the rest of the country the ratio (1.236) based on all 76 values was used everywhere.

A table was drawn up showing the first one hundred multiples of 1.236 each rounded off to the nearest integer. The table was used to adjust all of the useful annual maximum month-end depths to values comparable with the annual maximum depths. These were then considered "usable annual maximum depths of snow cover". The example given in Table II for the winter beginning in 1942 illustrates this.

A criterion for accepting or rejecting adjusted values of month-end maxima for years in which one or a few month-end depths were missing had to be devised. The simplest solution would be to reject them all, but if this had been done some of the record maximum snow depths would have been discarded.

In almost all cases the maximum month-end snow depths are reported in one of the four months, December, January, February, or March. If reports are missing for m of these months (where m can be 1, 2 or 3) and if it is assumed that the maximum depth is equally likely to occur in each of these four months, then $m/4$ is an estimate of the probability that the annual maximum is missing.

If N is the total number of unadjusted annual maximum snow depths for a particular station, and if n is the number of unadjusted values which are smaller than the adjusted value under consideration, then n/N is approximately the probability that the annual maximum is not larger than the value under consideration, or that this value is the maximum.

If n/N is greater than $m/4$ then it is more probable that the value under consideration is the annual maximum than that the annual maximum is missing. The value was therefore accepted if n/N was greater than $m/4$, that is if $4n/Nm$ was greater than unity. The winters beginning in 1945, 46, 47 and 48 in Table II illustrate this rule.

Annual maxima reported as such were accepted even if one or more of the month-end values were missing, as in the winter beginning in 1950 in the example. If an annual maximum was reported but was less than one of the month-end values then the month-end value was accepted as the annual maximum without adjustment as in the winter beginning in 1951 in the example.

The annual maxima for the winters beginning in 1956, 1957 and 1958 were supplied by the Climatology Division of the Meteorological Branch, Department of Transport, and no adjustments were made. The final list of "usable" annual maximum depths of snow cover is shown in the last column in the example.

The accuracy of the observations, or more correctly the degree to which the observations represent conditions in the surrounding area, depends on the exposure of the observing site, and, perhaps even more, on the attitude of the observer, who may unconsciously tend to overemphasize or underemphasize drifted areas. Exposure, as used here, includes elevation, slope, and shelter from the wind or sun by buildings or trees. It is doubtful if the representativeness of the observed values could be improved by adjusting them to correct for differences in exposure.

ANALYSIS

Since each of the annual values is the maximum of all the snow depth measurements during a winter it was assumed that the values would fit an extreme value distribution of the formula $y = -\log_e(-\log_e P)$ which was used by Gumbel (4). In this equation P is the probability that a value of the reduced variate (i. e. the reduced maximum snow depth) will exceed y. Once an equation has been fitted to the annual maxima for a particular station, this equation can be used to compute the snow cover depth which has any given probability of occurrence, or to compute the snow cover depth which will be exceeded once, on the average, in a given number of years.

The method of fitting requires the calculation of the mean snow depth ($\bar{h}$) and the standard deviation of the snow depths (s_h). Two dimensionless constants which depend only on the number of annual values used (N) are also required; the expected mean of the reduced extremes ($\bar{y}N$) and the expected standard deviation of the reduced extremes (σ_N). These constants are tabulated by Gumbel (4) for values of N from 20 to 100. The values of N in this study range up to a maximum of 18 winters and hence a table of values of $y_{\bar{n}}$ and σ_N had to be computed for values of N from 7 to 18 using equations given by Gumbel. These values are given in Table III.

TABLE III

EXPECTED MEAN, $\bar{y}_N$, AND STANDARD DEVIATION, σ_N, OF REDUCED EXTREMES AS FUNCTIONS OF THE NUMBER OF EXTREMES, N.

N	$\bar{y}_N$	σN
7	0.47735	0.87493
8	0.48428	0.90432
9	0.49015	0.92882
10	0.49521	0.94963
11	0.49962	0.96758
12	0.50350	0.98327
13	0.50695	0.99713
14	0.51005	1.00948
15	0.51284	1.02057
16	0.51537	1.03060
17	0.51768	1.03973
18	0.51980	1.04809
19	0.52175	1.05575
20	0.52355	1.06283

In the example, N - 15 and the mean annual maximum snow cover and standard deviation work out to:

$$\bar{h} = 17.27$$
$$s_h - 7.97$$

From Table III for N = 15 the following values can be read:

$$\bar{y}_N = 0.51284$$
$$\sigma N = 1.02057$$

These four values are used to calculate 1α and μ as follows:

$$\frac{1}{\alpha} = \frac{s_h}{\sigma_N} = 7.81$$

$$\mu = \bar{h} - \bar{y}_N . 1/\alpha = 13.26$$

These two values are the constants in the final equation:

$$h = \mu + y . 1/\alpha$$

$$\text{or } h = 13.26 + 7.81\ y$$

where y is the reduced extreme and is related to the probability by the equation:

$$y = -\log_e\ (-\log_e P).$$

In this equation P is the probability that a given value will exceed h. The return period (T) or the average length of time between annual maxima exceeding h is related to the probability by the equation:

$$\frac{1}{T} = 1 - P$$

Some representative values of T, P and y are shown in Table IV. The values of y were obtained from (7).

TABLE IV

REPRESENTATIVE VALUES OF THE RETURN PERIOD, T,
THE PROBABILITY, P, AND THE REDUCED EXTREME, y

T years	P %	y from (7)
2	50	0.36651
5	80	1.49994
10	90	2.25037
15	93.3	2.67376
20	95	2.97020
30	96.6	3.38430
50	98	3.90194
100	99	4.60015

In the present study the return period of 30 years was chosen as the standard. This is the same as the standard normal period for climatological records but otherwise is quite arbitrary. From Table IV the value 3.3843 can be obtained for y. If this is substituted in the equation:

$$h = 13.26 + 7.81\ y$$

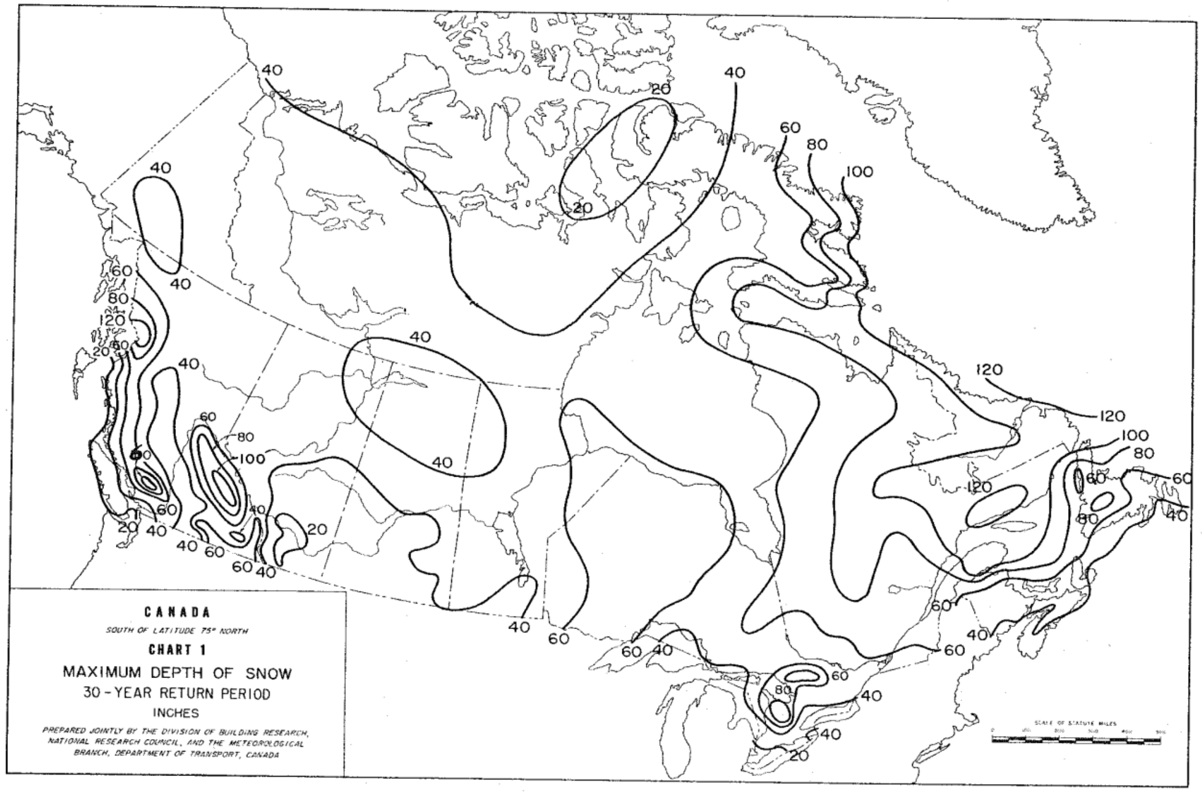
it yields the value:

$$h = 39.69.$$

This is the depth of snow cover which has one chance in 30 of being exceeded in a particular year, or which will be exceeded once in 30 years, on the average, at the station used in the example.

For the eighteen winteres from 1941 to 1958 there were 24 stations with complete records, that is, 18 "usable" annual maxima. An additional 91 stations had records ranging from 14 to 17 winteres and a further 100 stations, from 10 to 13 winters. Some stations with still shorter records were used, particularly in areas where there were very few stations; there were 67 stations with records for only 7 to 9 winteres. At a few stations in British Columbia there were some winteres with no snow. For these the analysis had to be modified but the details will not be given here.

The records from each of these 282 stations were analyzed as explained above and in each case the depth of snow cover for a thirty-year return period was calculated. These values were plotted on a map of Canada and the Chart 1 was drawn.


CANADA
SOUTH OF LATITUDE 75° NORTH
CHART 1
MAXIMUM DEPTH OF SNOW
30-YEAR RETURN PERIOD
INCHES
PREPARED JOINTLY BY THE DIVISION OF BUILDING RESEARCH, NATIONAL RESEARCH COUNCIL, AND THE METEOROLOGICAL BRANCH, DEPARTMENT OF TRANSPORT, CANADA

SNOW LOADS

The analysis of maximum snow cover depths was originally undertaken as a basis for computing maximum snow loads on roofs for design purposes for any municipality in Canada. Using snow depths on the ground as a basis for computing maximum loads on roofs is a questionable procedure but there is at present no alternative since snow loads on roofs are seldom measured.

To convert snow depths to pressures or loads it was necessary to assume a density for the snow. In Canada it is usually assumed that, on the average, fresh snow has a density of one tenth the density of water. The density of old snow ranges from 0.2 to 0.4 or even more. The average density of the snow cover may vary across the country, but the available reports were not thought to justify the use of different densities in different regions. In any case, it is the average density at the time of maximum snow cover in a 30-year period that concerns us here. This maximum would almost certainly occur immediately after an unusually heavy snowfall and hence a large proportion of the snow cover would have a very low density. If the density of the old snow was about 0.3, then it would seem reasonable to assume a mean density of about 0.2 for the whole cover. In practice it is more convenient to assume that one inch of snow corresponds to a pressure of exactly one pound per square foot. This corresponds to a density of 0.192. Chart 1 can now be considered as a chart of maximum loads on the ground (in pounds per square foot) due to snow alone.

Since roof failures in Canada are often connected with early spring rains, it is considered advisable to add to the load due to snow alone the load of rain water that might be retained in the snow. Since rainfall is measured daily at all precipitation stations, it is most convenient to use the maximum one-day rainfall that might occur at the time of year when snow depths are greatest.

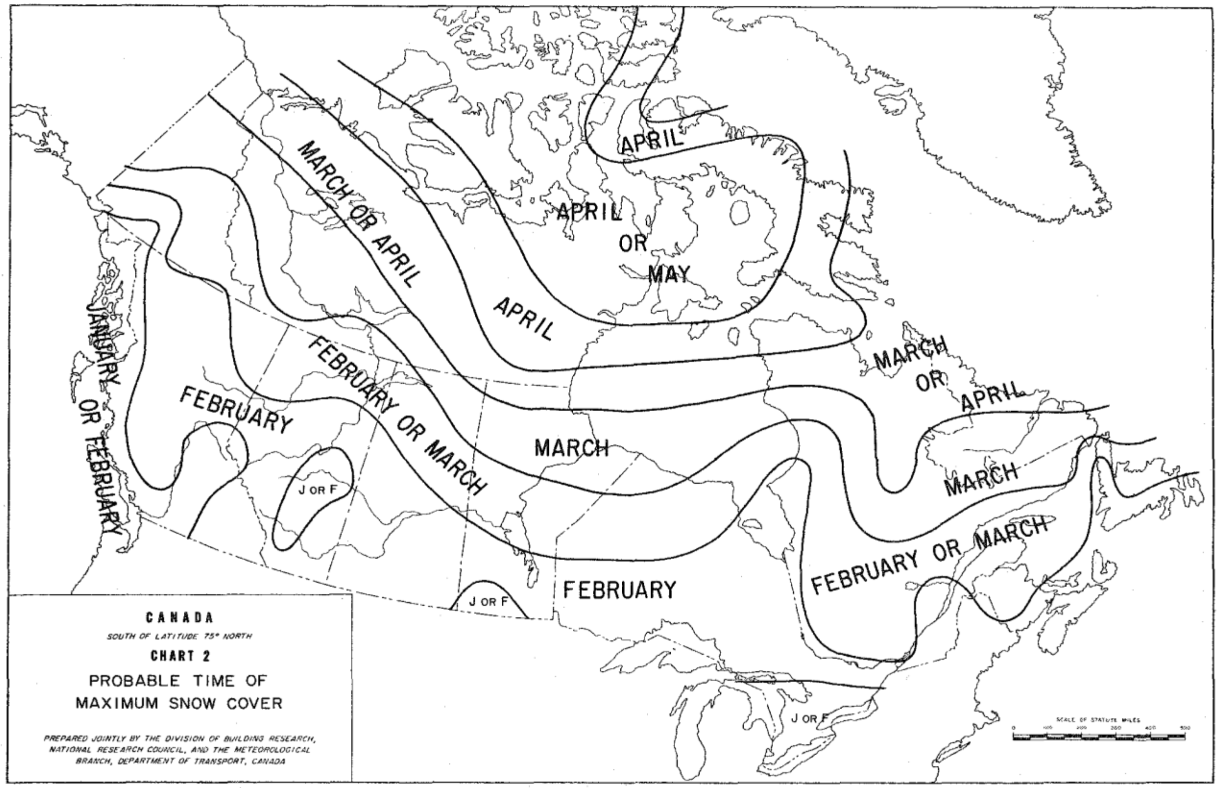
To determine the period for which maximum rainfalls should be considered the manuscript abstracts of month-end snow depths for the decade 1951 to 1960 were studied. The month or months in which the maximum month-end snow depths most frequently occurred were noted for each station and plotted on a map. Chart 2 shows, for example, the region in which the maximum month-end snow depth is equally likely to occur at the end of either January or February; this region is marked "February" to indicate that the annual maximum will most probably occur in this month. Similarly, the region in which the maximum month-end snow depth is most likely to occur at the end of February is marked "February or March" to indicate that the annual maximum will probably occur in one of these months.

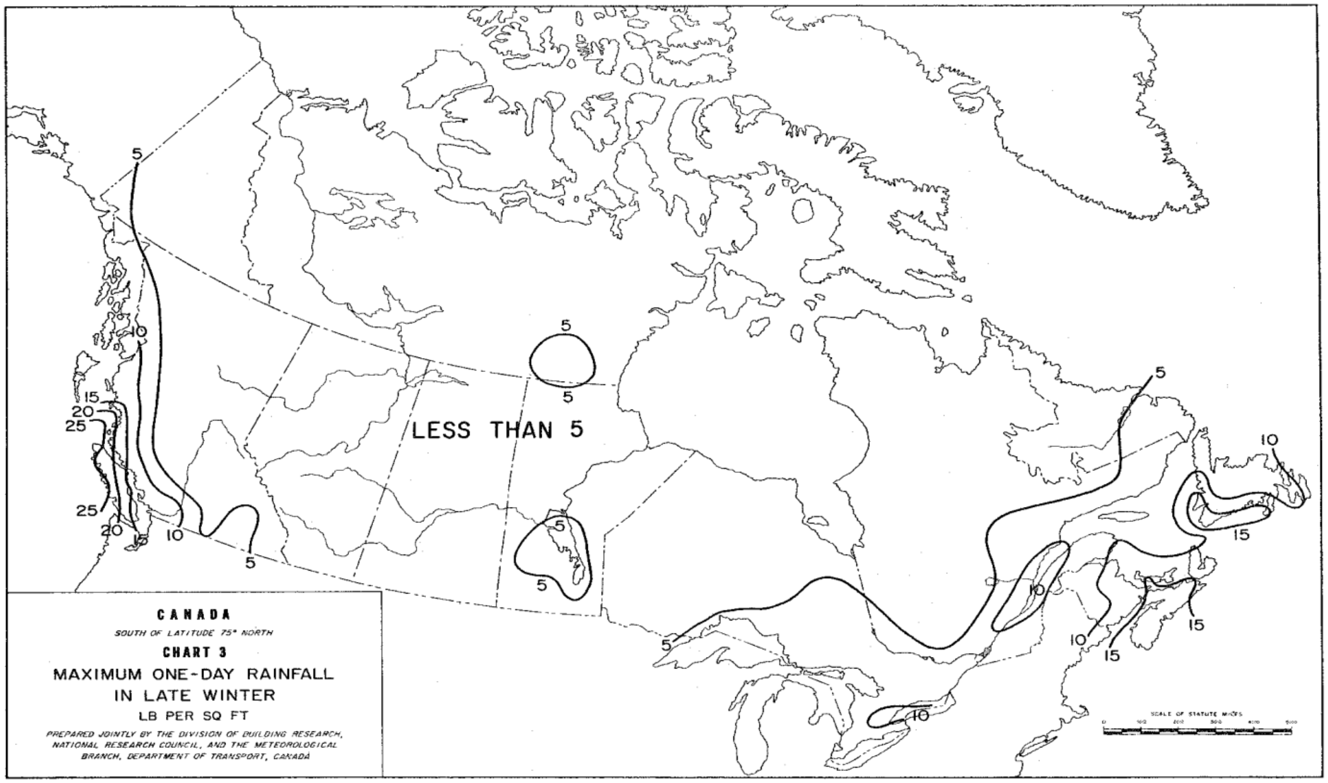
For the stations in the "February" region of Chart 2, the maximum one-day rainfall occurring in January, February or March during the twenty years from 1941 to 1960 was noted. For stations in other regions marked with a single month the corresponding three-month maximum was noted. For stations in regions marked with two months the maximum one-day rainfall in these two months was noted. These rainfalls were plotted on a map and drawn up as a chart showing the maximum reported one-day rainfall for the two or three month period when the maximum snow cover is most likely to occur. The name was abbreviated to "Maximum one-day rainfall in late winter," and is shown in Chart 3.

If winter rain is absorbed by the snow cover into which it falls then one inch of rainfall will add 5.2 pounds per square foot to the snow load. For the stations where the maximum depth of snow cover (or maximum load due to snow alone) and the additional load due to the maximum one-day rainfall were both available they were added to obtain the maximum snow load on the ground. If one or the other was missing it was estimated from the appropriate chart and used to compute the maximum snow load on the ground. At a few stations in British Columbia the weight of rain exceeded the load due to snow alone. It is very unlikely that snow could hold more than its own weight of water and in these cases the maximum snow load was assumed to be twice the load due to snow alone. All these maximum snow loads were plotted and Chart 4 drawn up using the gridding method to ensure the consistency of the three charts even in the sparcely populated areas.

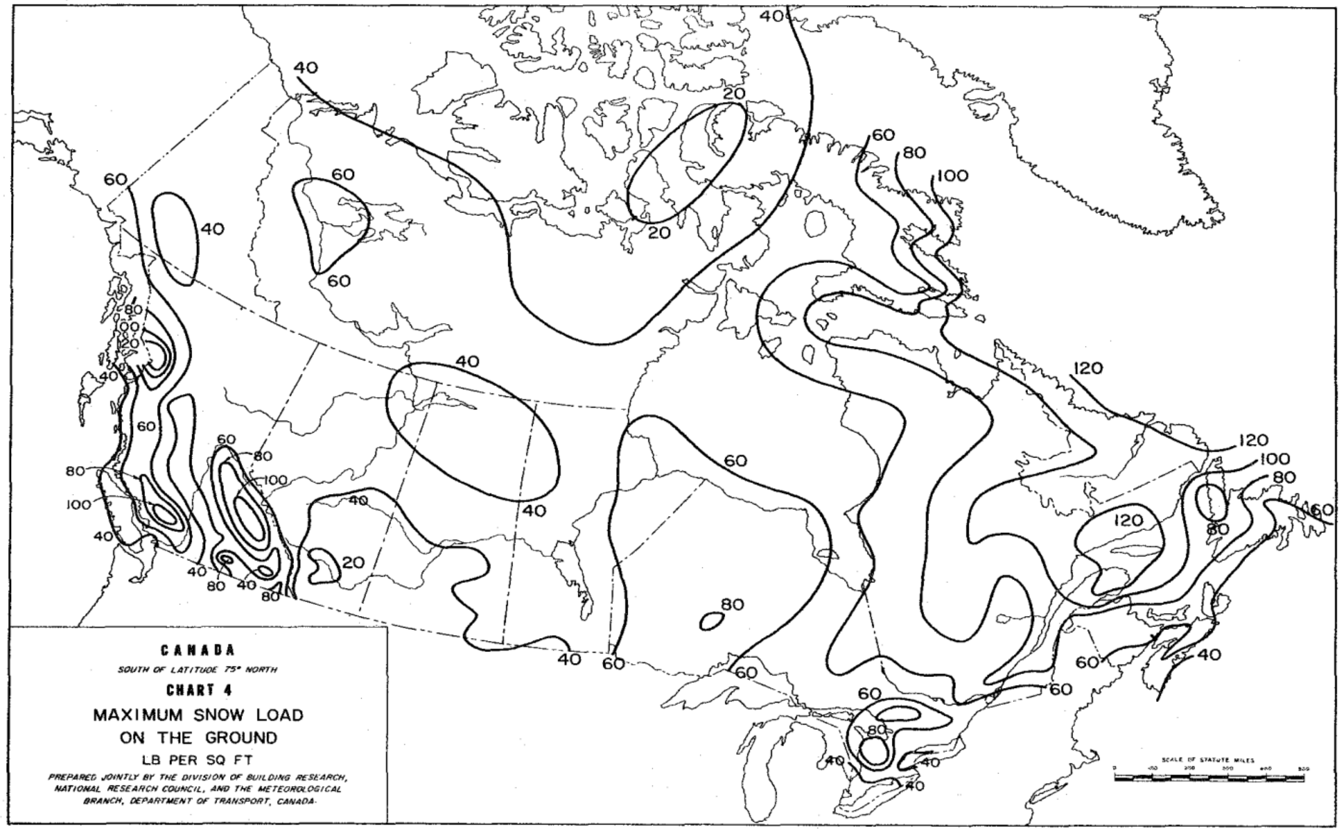
Design Snow Loads on Roofs

During the last five winters, the Building Structures Section of the Division of Building Research, National Research Council, Canada has been conducting a survey of snow loads on roofs, and in particular comparing roof loads with loads on the ground. On the basis of these observations the Associate Committee on the National Building Code has decided that until more information is available, the design snow load for flat or low-slope roofs would be 80 per cent of the maximum snow load on the ground. Further decreases in the design snow load are permitted for more steeple sloping roofs and substantial increases are required for roofs on which snow accumulation may be more rapid due to drifting and for roofs onto which snow may slide from higher roofs. The detailed requirements are given in the National


CANADA
SOUTH OF LATITUDE 75° NORTH
CHART 2
PROBABLE TIME OF
MAXIMUM SNOW COVER
PREPARED JOINTLY BY THE DIVISION OF BUILDING RESEARCH, NATIONAL RESEARCH COUNCIL, AND THE METEOROLOGICAL BRANCH, DEPARTMENT OF TRANSPORT, CANADA
JANUARY OR FEBRUARY
FEBRUARY
J OR F
MARCH OR APRIL
FEBRUARY OR MARCH
APRIL
APRIL OR MAY
APRIL
MARCH
MARCH OR APRIL
MARCH
FEBRUARY OR MARCH
FEBRUARY
J OR F
J OR F



LESS THAN 5
5
10
15
20
25
CANADA
SOUTH OF LATITUDE 75° NORTH
CHART 3
MAXIMUM ONE-DAY RAINFALL
IN LATE WINTER
LB PER SQ FT
PREPARED JOINTLY BY THE DIVISION OF BUILDING RESEARCH,
NATIONAL RESEARCH COUNCIL, AND THE METEOROLOGICAL
BRANCH, DEPARTMENT OF TRANSPORT, CANADA



CANADA

SOUTH OF LATITUDE 75° NORTH

CHART 4

MAXIMUM SNOW LOAD ON THE GROUND

LB PER SQ FT

PREPARED JOINTLY BY THE DIVISION OF BUILDING RESEARCH, NATIONAL RESEARCH COUNCIL, AND THE METEOROLOGICAL BRANCH, DEPARTMENT OF TRANSPORT, CANADA.

Code of Canada, 1960.

CONCLUSION

The charts which are themselves the real conclusions of this analysis, show only the general distribution in Canada of the following elements:

1. The maximum depth of snow on the ground for a 30 year return period.
2. The approximate time of year when the maximum depth of snow is most likely to occur.
3. The maximum one-day rainfall at that time of year.
4. The maximum snow load on the ground based on charts 1 and 3.

Charts on such a small scale cannot show local differences in the weather elements even where these are known to exist. The weather observations used in the analysis were all taken at places where people were living, and hence the charts apply only to populated areas. This is particularly significant in mountainous areas where the lines of the chart apply only to the populated valleys and not to the mountain slopes where in some cases much greater snow depths are known to accumulate.

ACKNOWLEDGMENTS

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