SIMULATION OF DAILY SNOW WATER EQUIVALENT AND MELT[1/]

528-71

By

Donald W. Willen[2/], Clyde A. Shumway and James E. Reid[3/]

Introduction

If the growing demands for natural resources are to be met, resource managers must improve their techniques of predicting outputs from alternative treatments of these resources. Predictions of water yield depend on the most reliable estimates of the various components in the "hydrologic cycle". One of the most difficult estimates is the quantity and timing of on-site delivery of snowmelt water from management units with persistent seasonal snowpacks.

Fundamental studies of the U. S. Army Corps of Engineers (1956) have described the processes affecting snow accumulation and melt. Several empirical and analytical models have estimated snowmelt with varying degrees of sophistication (Anderson and Crawford, 1964; U. S. Army Corps of Engineers, 1966; Crawford and Linsley, 1966; Rantz, 1964; and Riley, et al., 1969). The output of these models usually involved elaborate estimates of depth, density, and water equivalent for periods exceeding one day.

The objective of the work reported here was to provide the hydrologist with a reasonably accurate simulation model that predicts daily snow deposition, depletion, and net on-site delivery of snowmelt water.

This work was done as part of a larger effort to develop a water-yield prediction model. The concept of that portion of the water yield model, reported here, is that of net snow accumulation. For convenience of discussion only, this concept is divided into three main parts: (I) precipitation form and interception; (II) the snowmelt process; and (III) snowpack condition.

I. Precipitation form and interception

Precipitation form. Precipitation may occur as rain, snow, or a combination of both. Discrimination among the forms of precipitation requires an assumption about the concept of base temperature. As used in our model, base temperature is assumed to be the ambient surface air temperature at which a precipitating cloud cover overhead would be at 32° F. Thus it involves the average elevation from which precipitation falls on the given site, the elevation of the site, and the average lapse rate for that location.

We define the precipitation form as:
Rain, when the daily minimum temperature > "base" temperature.
Snow, when the daily maximum temperature ≤ "base" temperature.
Rain + Snow, when the daily maximum temperature > "base" temperature and the daily minimum temperature ≤ "base" temperature.

In the combination event, the proportion of total precipitation falling as snow and as rain must be determined. To do this we used two simplifying assumptions. First, the ambient air temperature follows the general form of a sine function passing through maximum and minimum temperatures. Second, without loss of generality, we can represent one-half the diurnal temperature fluctuation as a right triangle (Fig. 1).

1/ Presented at the Western Snow Conference, Billings, Montana, April 20-22, 1971.

2/ Current Address: George Washington National Forest, Harrisonburg, Virginia.

3/ Pacific Southwest Forest and Range Experiment Station, Forest Service, U. S. Dept. of Agriculture, Berkeley, California 94701.

Let:

A = Maximum-minimum temperature
B = Maximum-base temperature for snowfall
C = Time base (Any length of time)
D = Time that temperature is above base temperature
P = Total daily precipitation (rain + snow)
S = Proportion of precipitation occurring as snow (inches, water equivalent)
(Note: rain equals P-S)

The proportion of precipitation falling as rain is equivalent to the proportion of time that the temperature is above the base temperature, or D/C i.e., proportion falling as rain = D/C, but by similar triangles, D/C = B/A.

Then,

$$S = P[1.0 - (B/A)]$$

Interception. Precipitation was adjusted for interception by vegetation. Based on data from Rowe and Hendrix (1951) and the U. S. Army Corps of Engineers (1956) empirical equations were developed to define the parameters by cover density classes for the equation:

$$I = \alpha + \beta p$$

In which:

I = total interception (inches)
α = interception storage of the vegetation
β = interception rate with respect to cover density
p = total precipitation (inches)

We made four basic assumptions: (1) there is no difference between rain and snow interception (Rowe and Hendrix, 1951); (2) interception is less than total precipitation; (3) interception storage increases as cover density increases to a "saturation" point, from which interception continues at a lesser rate; (4) the interception rate increases as cover density increases, suggesting that evaporation potential increases are due to the greater surface area of the canopy.

Rowe and Hendrix (1951) reported a relationship for a stand of 40 percent cover density:

$$I \text{ (inches)} = 0.11 + 0.06\, P \text{ (inches)}$$

This function agrees closely with many of the interception relationships reported by Zinke (1967).

From snow hydrology data, we developed a relationship that expresses percent water equivalent intercepted as a function of canopy density in percent. The resultant linear equation was: Avg. Storm Interception (%) = 0.3684 (cover density (%)). Solving this equation for 40 percent cover density yields a value of 14.736 percent interception. Using the Rowe and Hendrix function as a base equation and adjusting the coefficients by a scaling factor of the ratio of percent interception for a given cover density to 14.7360, we developed a set of equations describing interception rates for various cover densities.

The interception-storage coefficient was derived from plotting Rowe's and Hendrix's storage values with respect to storm size. This linear relationship was combined with the interception rates for various cover densities. These combined relationships gave reasonable results (Fig. 2). They also satisfied our assumptions. The interception rate relationships could be estimated by two equations for the (α) and (β) coefficients with respect to a cover density:

$$\alpha_{CD} = 0.2711 \text{ (cover density, decimal fraction)}$$

$$\beta_{CD} = 0.1478 \text{ (cover density, decimal fraction)}$$

in which CD = cover density.

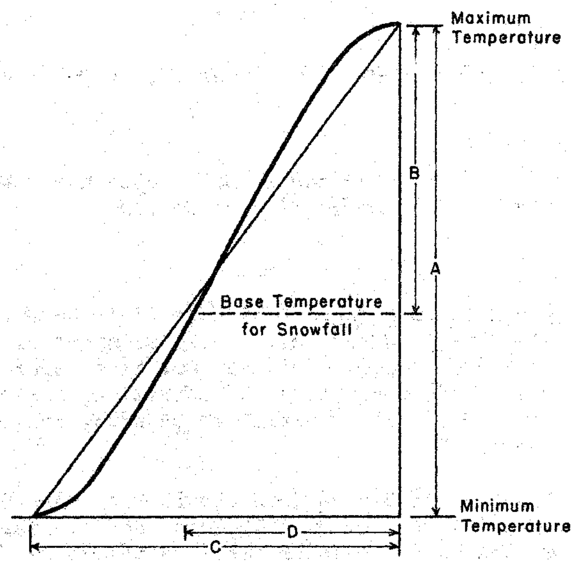

Figure 1--The right triangular relationship of one-half of the diurnal temperature wave. A is the temperature range; B is the temperature range between maximum and base temperature for snowfall; C is the time base; and D is the time that temperature is above the base temperature.

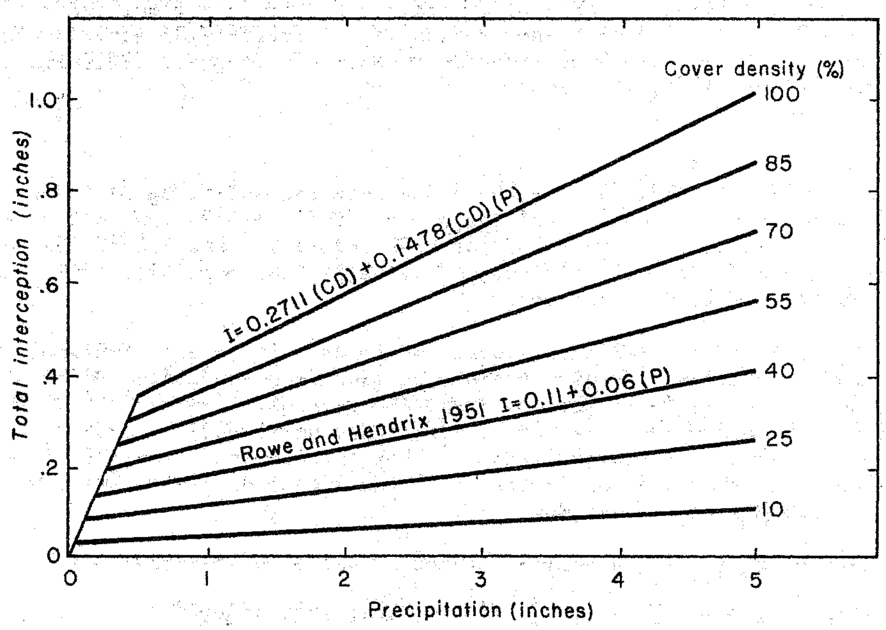

Figure 2--Total interception (I) as a function of precipitation (P) and cover density (CD).

For all cover densities the initial interception storage rate was:

$$I = 0.6893\ (p).$$

This rate would continue until:

$$P = \frac{\alpha_{CD}}{0.6893-\beta_{CD}}$$

When the precipitation exceeded this value, then interception would continue at the rate:

$$I = \alpha_{CD} + \beta_{CD}\ (p).$$

Thus interception was defined for the total model. But a user has the option of whether or not to account for interception in a snowmelt simulation run.

II. The snowmelt process

Development of the snowmelt function. Available field data restricted the use of an all-inclusive variable model and required that a simpler model be developed which would reasonably describe or simulate the snowmelt process. Our first approach was to develop an all inclusive model from research data, run tests to determine sensitivity of variables, develop coefficients or delete insignificant variables or both, and test a reduced model which meets available field data.

Research data from the Central Sierra Snow Laboratory, near Soda Springs, Calif., for the 1948-49 and 1949-50 water years (U. S. Army Corps of Engineers, 1956) were used and met the data requirements for the energy balance terms described by Sellers (1965). The eight variables describing heat were: solar radiation (H_{SW}), terrestrial longwave radiation (H_{lw}), heat stored in rain (H_r), conductive heat from the earth surface (H_e), convective heat transfer (H_c), latent heat of vaporization (H_V), latent heat of fusion (Hf), and the energy state of the snowpack (H_s). The total heat (H) or energy balance was then:

$$H - (H_{sw} + H_{lw} + H_r + H_e + H_c + H_V + H_f + H_s) = 0.$$

Each variable was evaluated as to variations in value and the resultant effect on snowmelt in sensitivity tests. These test runs showed that solar radiation, albedo, air temperature, forest cover density, and cloud density were the most significant variables in predicting snowmelt. The variables of dewpoint, wind, and elevation which are components of the sensible heat, advection, and convection terms produced less significant (less than 10%) changes in snowmelt. Of the significant variables the terrestrial longwave and cloud density terms were usually not available from field data. The longwave radiation was then estimated by using a Brunt (1944) equation:

$$R = \frac{Q_a \downarrow}{\sigma T_a^4}$$

in which R is a ratio of back radiation to the black-body radiation, e_a is the vapor pressure at air temperature, and (a) and (b) are regression coefficients. The values of 6.11 mb of vapor pressure over ice at 0^oC, and 0.740 for coefficient (a) and 0.0049 for coefficient (b), were used to solve for R = 0.757 (Kobert, 1964; Reifsnyder and Lull, 1965; Sellers, 1965; and the U. S. Army Corps of Engineers, 1956).

We were unable to derive reliable algorithms to estimate cloud cover. Graphical analysis of percent cloud cover disclosed a distinct bimodal distribution which was asymptotic to the two bounds of the distribution, 0 and 100 percent. Five percent of the observations occurred between cloud densities of 15-80%. These observations produced unrealistically large, negative longwave radiation losses to the surrounding atmosphere from the snowpack. For the purposes of the model we assumed that the effect of clouds was implicitly "indexed" by the measured solar radiation.

The simplified model for the snowmelt function was thus expressed as:

$$\text{Snowmelt}(cal/cm^2) = R_{SW}(1.0-A)+C(ST_a^4-ST_s^4)+(1.0-C)(0.757\ ST_a^4-ST_s^4)\ ,$$

in which R_{SW} = total daily shortwave radiation, cal. cm^{-2}, A = albedo, decimal fraction; C = cover density, decimal fraction, S = Stefan-Boltzmann constant, 1.17×10^{-7} cal. cm^{-2}, $^{o}K^{-4}$; T_a = air temperature, absolute (^{o}C + 273.16); T_s = snow surface temperature, absolute (^{o}C + 273.16); and 0.757 = coefficient of longwave back radiation, as described above.

Albedo functions were derived from existing data (U. S. Army Corps of Engineers, 1956) and described by the functions:

"Accumulation period;"
Albedo, decimal fraction = 0.8113 - 0.0677 ln x;
"Melt period;"
Albedo, decimal fraction = 0.7244 - 0.1092 ln x,

in which x = the number of days since last snowfall. Solar radiation input to the snowpack under a forest canopy was adjusted by a transmission coefficient from data by Reifsnyder and Lull, (1965): Transmission coefficient (decimal fraction) = -0.1017- 0.3904 ln(CD), in which CD = cover density (decimal fraction). Mean daily snow surface temperature was assumed to follow daily ambient temperatures below 32° F. The snow surface temperature was held constant at 32° F, when the ambient air temperature was at or exceeded 32° F.

III. The snowpack condition

This portion of the model was the "main" function governing the control of and accounting for gains and losses of heat to the snowpack. All heat inputs or outputs from precipitation, radiative energy, and the net heat capacity of the snowpack were expressed in calories. The snowpack was considered a dynamic heat reservoir.

Each precipitation event was handled as an entity, and its form determined. And if desired, the net precipitation to the site was calculated from the interception function. Once a snowpack was developed the daily heat gains and losses were algebraically added to the heat capacity (pack condition) of the snowpack. When sufficient heat was produced to bring the pack condition to zero (0), the pack was considered to be isothermo at 0°C. Additional heat input converts the pack from ice to water which is delivered to the site on which the pack exists.

Results and Discussion

Data input/output. Two years (1948-49, 1949-50) of research data were used from the U. S. Army Corps of Engineers studies at the Central Sierra Snow Laboratory. The daily data inputs required to run the simulation model were total terrestrial measured insulation (cal. cm^{-2}); albedo (percent); maximum and minimum temperature (°F); vegetative cover density (percent); precipitation (inches); and base temperature for snowfall (°F). When available the measured water equivalent was also included as input, but was not included in the model. This parameter allowed us to compare simulated water equivalent to actual water equivalent. The simulation run for each year commenced at a time when we concluded that a snowpack was potentially forming. This timing always preceded the date when the snowpack was first measured. The first actual observation was always made when the snowpack contained more than 12 inches of water equivalent.

An example of a selected portion of the printed output for 1948-49 is shown in Figure 3 and the terms used are defined in Table 1. A simulation run for 1948-49 is graphically shown in Figure 4.

1948-49 Season. The simulated and the actual snow water equivalent compared very closely (Fig. 4). The graph suggests that precipitation events of less than 1 inch of water equivalent can cause significant changes to the snowpack. However, one unit of precipitation will not necessarily produce an equal amount of water equivalent increase, because of the net effect of the precipitation and melt. The simulation run did not vary as much as the actual observations. This difference is especially evident during the accumulation period when actual increases in water equivalent occurred but without any recorded precipitation. We assumed this discrepancy to be related to sampling error and internal transfer of water that may have occurred in the pack. As much as 3 inches of water equivalent existed between the observed and simulated data for only short periods of time. At maximum pack, however, this difference was negligible, although the simulated run tended to extend the duration of the maximum period for about 6 days. This "lag" effect persisted during the

SNOWMELT SIMULATION TEST CSSL DATA 1948 TO 1949

COVDEN = .10

DATE	RADIN	RADLWN	ALBEDO	TMAX	TMIN	MTEMP	TPPT	PRECIP	WEQUIV	CALHET	SNMELT	NMELT	PACK	PCON	BT
32849	584.0	-133.3	.81	30.9	16.9	23.9	42.49	.06	.00	-45.4	.00	.00	40.13	890.27	35.6
32949	668.0	-139.8	.81	42.3	17.0	29.7	42.49	.00	42.00	-39.3	.00	.00	40.13	890.27	35.6
33049	284.0	-135.2	.81	30.1	21.1	25.6	43.03	.54	40.00	-92.5	.00	.00	40.67	902.55	35.6
33149	640.0	-141.4	.81	39.8	22.3	31.0	43.07	.04	.00	-45.1	.79	.00	40.71	900.84	35.6
40149	639.0	-142.1	.81	46.3	17.0	31.7	43.07	.00	40.20	-45.9	.00	.00	40.71	900.84	36.8
40249	313.0	-137.3	.76	40.5	26.0	33.2	43.07	.00	.00	-78.5	.00	.00	40.71	900.84	36.8
40349	693.0	-118.7	.74	49.7	25.6	37.6	43.07	.00	41.50	26.7	10.50	.00	40.71	874.15	36.8
40449	693.0	-113.1	.57	55.8	22.1	38.9	43.07	.00	.00	122.8	48.36	.00	40.71	751.33	36.8
40549	666.0	-110.7	.55	54.7	24.3	39.5	43.07	.00	40.00	129.0	50.77	.00	40.71	622.37	36.8
40649	438.0	-115.4	.72	51.4	25.4	38.4	43.09	.02	.00	-19.2	.94	.00	40.73	620.05	36.8
40749	234.0	-136.2	.81	41.0	26.0	33.5	43.17	.08	39.70	-101.0	1.81	.00	40.81	615.91	36.8
40849	669.0	-111.1	.81	51.8	27.0	39.4	43.18	.01	.00	-10.5	.51	.00	40.82	614.64	36.8
40949	716.0	-98.0	.81	58.8	26.0	42.4	43.18	.00	38.30	9.7	3.82	.00	40.82	604.95	36.8
41049	708.0	-88.9	.65	61.6	27.3	44.5	43.18	.00	.00	109.4	43.05	.00	40.82	495.59	36.8
41149	626.0	-93.4	.60	59.9	27.0	43.4	43.18	.00	.00	104.0	40.96	.00	40.82	391.55	36.8
41249	729.0	-97.8	.57	56.8	28.1	42.4	43.18	.00	35.00	150.4	59.20	.00	40.82	241.19	36.8
41349	731.0	-82.0	.55	64.0	28.0	46.0	43.18	.00	.00	181.1	71.28	.00	40.82	60.13	36.8
41449	689.0	-95.6	.53	55.5	30.4	42.9	43.18	.00	35.10	163.3	23.67	.51	40.31	.00	36.8
41549	723.0	-88.3	.51	61.0	28.2	44.6	43.18	.00	.00	193.1	.00	.95	39.36	.00	36.8
41649	728.0	-86.9	.50	62.3	27.5	44.9	43.18	.00	.00	204.8	.00	1.01	38.35	.00	36.8
41749	572.0	-99.8	.48	55.3	28.7	42.0	43.18	.00	.00	135.3	.00	.67	37.68	.00	36.8
41849	484.0	-108.7	.47	50.1	29.8	39.9	43.18	.00	.00	94.6	.00	.47	37.22	.00	36.8
41949	600.0	-110.7	.46	50.0	29.0	39.5	43.18	.00	31.30	146.4	.00	.72	36.50	.00	36.8
42049	683.0	-93.2	.45	59.0	28.0	43.5	43.18	.00	.00	204.7	.00	1.01	35.49	.00	36.8
42149	714.0	-84.4	.44	61.0	29.9	45.4	43.18	.00	29.10	231.9	.00	1.14	34.35	.00	36.8

Figure 3--A selected portion of printed output of results in which the simulation model was tested. Data tested were from U. S. Army Corp of Engineer studies at the Central Sierra Snow Laboratory, California. See Table 1 for definition of symbols.

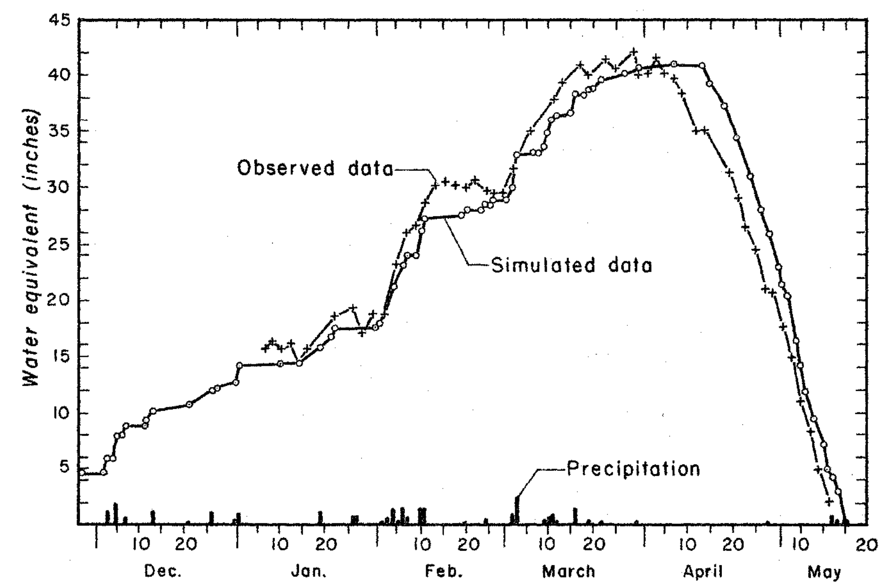

Figure 4--Daily Observed vs. Simulated Snow Water Equivalent for the 1948-49 Snow Season.

depletion period but the difference became less with time. From May 1 until disappearance of the snowpack on May 15 the "lag" was a constant two days. The over-all rate was 1.25 inches of water equivalent per day for the observed depletion and 1.37 inches for the simulated depletion.

1949-50 Season: The results were similar to the 1948-49 results. The simulated run had less variation than the actual data. In a series of three maximum "peaks", the simulated run generally underestimated each actual peak about 4 inches of water equivalent. The "lag" effect was not evident between simulated and the actual data, and the melt rates were about 1.47 inches of water equivalent per day for simulated data, and 1.53 inches for the actual data. The predicted versus actual water equivalent was virtually the same for the last 23 days, and the snowpack "disappeared" on the same date.

We are quite satisfied with the results for both years of data. This model meets our objectives in terms of both simulating progressive seasonal water equivalent on a daily basis, and also providing the capabilities of predicting on-site delivery of melt-water in a larger water-yield model. We plan to refine the model in the future for the Rocky Mountain and Northeastern snow zones. Such work will require greater attention to nonisothermal snowpack conditions for "dry-cold" snow; to the redistribution of snow by wind; and to vegetative cover effects.

Table 1--Definition of Parameters in the Printed Output of Results from Tests of the Simulation Model.

Parameter	Definition
RADIN	Total daily measured radiation calories per square centimeter.
RADLWN	Outgoing, daily, longwave radiation, calories per square centimeter.
ALBEDO	Daily albedo, decimal fraction.
MTEMP	Mean daily temperature, ^{o}F to ^{o}C.
TPPT	Daily, accumulated precipitation, inches.
PRECIP	Daily; precipitation, inches.
WEQUIV	Measured snow water equivalent, inches.
CALHET	Net daily calories.
SNMELT	Heat available to reduce PCON, inches of water equivalent.
NMELT	Net daily simulated melt, inches.
PACK	Daily simulated snowpack water equivalent, inches.
PCON	Daily pack condition (status of net heat capacity of snowpack), calories.
BT	Base temperature for snowfall, ^{o}F.

REFERENCES

Anderson, E. A., and N. H. Crawford, 1964; The Synthesis of Continuous Snowmelt Runoff Hydrographs on a Digital Computer. Tech. Rept. 36, Dept. Civ. Engr., Stanford University, Stanford, Calif. 103 pp.

Brunt, D., 1944; Physical and Dynamical Meteorology. Cambridge University Press. 119 pp.

Crawford, N. H., and R. K. Linsley, 1966; Digital Simulation in Hydrology: Stanford Watershed Model IV. Tech. Rept. 39, Dept. Civ. Engr., Stanford University, Stanford, Calif., 210 pp.

Koberg, G. E., 1964; Methods to Compute Longwave from the Atmosphere and Reflected Solar Radiation from a Water Surface. U. S. Geological Survey. Prof. Paper 272-F, pp. 107-136.

Rantz, S. E., 1964; Snowmelt Hydrology of a Sierra Nevada Stream. U.S.G.S. Water Supply Paper 1779-R. 35 pp.

Reifsnyder, W. E., and H. W. Lull, 1965; Radiant Energy in Relation to Forests. U. S. Dept. Agr. Tech. Bull. 1344. 111 pp.

Riley, J. P., Chadwick, D. G., and K. O. Eggleston, 1969; Snowmelt Simulation. 37th Annual West. Snow Conf. Proc. Apr. 15-17, 1969. pp. 49-56.

Rowe, P. B. and T. M. Hendrix, 1951; Interception of Rain and Snow by Second Growth Ponderosa Pine. Trans. Amer. Geophysical Union 32(6): pp. 903-908.

Sellers, W. D., 1965; Physical Climatology. University of Chicago Press. 272 pp.

U. S. Army Corps of Engineers, 1956; Snow hydrology Summary Report of the Snow Investigations. North Pacific Division, Portland, Oregon. 437 pp.

U. S. Army Corps of Engineers, 1966; Basin Rainfall and Snowmelt Computation. Hydrologic Engineering Center, General Computer Prog. 23-J2-L226.

Zinke, P. J., 1967; Forest Interception Studies in the United States. IN, International Symposium on Forest Hydrology. Edited by W. E. Sopper and H. W. Lull. pp 137-161.