THERMAL CONDUCTIVITY - BASIS OF A POTENTIAL METHOD FOR DETERMINING IN SITU SNOW DENSITY

580-74

By

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Density is a common term in the description of the physical properties of snow. As a value derived from snow course measurements or as a time series describing snowpack metamorphosis, it forms a basis for assessing our snow resource and evaluating our management efforts. Both direct and indirect methods of determining snow density are available. Measuring density directly as a weight per unit volume by vertical or horizontal sample is quite familiar. Methods which can be used to determine snow density indirectly require:

1. A relationship between snow density and another measurable parameter; for example:
   (a) air temperature at time of snowfall
   (b) optical properties of snow crystals
   (c) attenuation of ionizing radiation
   (d) measurement of dielectric strength

and

2. A procedure for measuring the indexing parameter.

Indirect methods are of operational value only if they offer some advantage over the weighed sample. Most procedures, therefore, are directed toward a fast, repeatable, and non-destructive measurement. To the listing above can be added the measurement of thermal conductivity and the following procedure.

Thermal Conductivity-Density Relationship

Thermal conductivity is the proportionality constant in the equation describing heat flow through a slab of uniform material. In equation form

$$H = -Ak \frac{dt}{dz}$$

where the flow of heat H (cal/sec) across the area A (cm^2) is in response to the temperature gradient dt/dz. The dimensions of the constant k are cal sec^{-1} cm^{-1} $^oC^{-1}$. Materials with lowest k's are the best insulators. Air has a k of 0.057×10^{-3} compared with copper at 920×10^{-3}.

In the conduction process, addition of heat increases the thermal motion of the atoms in the material. Materials with more atoms/unit volume, i.e., of greater density, are better conductors of heat; also materials with relatively more free electrons (metals) are the best conductors of heat. The effect of density on k is easily noted by comparing a material in a solid, maximum density state with an expanded product of the same chemical makeup. Our acquaintance with glass products in cooking utensils of high k to fiberglass batts of low k is commonplace.

In snow, the density versus k relationship should be similar to that of other solids. Fig. 1 shows values of k for snow at different densities. This is a composite curve from List (2). Yen (3) shows more variable results from a number of studies. Differences in technique by the various authors amplifies the natural variability. Although the relationship may lack precision, the first criterion of a causal relationship between snow density and thermal conductivity is satisfied.

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Instrumental Techniques

Several techniques appear in the literature for the determination of snow thermal conductivity. Laboratory techniques which require removal of an undisturbed sample or use equipment too cumbersome for field sampling do not fill our operational needs. Several "transient state" probes for measurement of soil thermal conductivity and adaptable to snow have been reported. Use of these probes is fairly well standardized and has found a place in the soils laboratory.

The customary measurement technique for these probes notes the temperature rise upon application of a known and constant heat input. Through a clever solution of the equation relating heat input to the temperature/time relationship, a graphical solution for thermal conductivity of the medium is made. Unfortunately, the time required for each sample is usually several minutes in addition to the inconvenience of recording several timed temperature measurements and making a subsequent graphical analysis.

In the soils laboratory, measurement time is not critical but with snow, changes may be occurring within the snow, reducing accuracy. Using a large probe (0.6 cm x 76 cm), Jaafar and Picot (1) measured k in snow over a limited range of densities. About 10 minutes were required for each measurement. Probes of this type have also been used for determination of thermal conductivity in permafrost and glacial ice. Although this type of probe appears suitable for use in snow, an excessively long probe and the time required for each measurement are problems. A smaller, modified probe allowing a single value, density-related measurements (as the design objective here) is examined in Fig. 2.

Application of heat through the winding embedded in the outer surface warms the entire probe as in customary designs. Heat flow to the inside of the probe is delayed by the path composed of air and the nylon spacers. A temperature differential is measured between the inner and outer tubes with the thermocouple shown. The temperature rise internally is a complex function of the k of the internal path and the k of the external medium. A theoretical analysis of this differential heating between inside and outside can be made but is not considered here. Intuition suggests that the outside should heat more rapidly as the external medium becomes less conductive. This is the same effect as with customary probe designs.

With the fixed internal path with constant k, the temperature differential between inside and outside rapidly increases to a maximum value dependent on probe design and thermal constants of the external medium. Overall temperature rise of the probe, the differential temperature developed between inside and outside, and the time to maximum differential temperature are all related. Materials with high k show lower overall temperature rise, lower differential temperatures, and faster response. In snow with density of 0.2 g cm^{-3}, about 15 seconds are needed to reach maximum temperature differential.

Materials for construction and several complete probes with epoxy coatings are as shown in Fig. 3. The unit mounted on the fiberglass whip was selected for calibration and field use.

The probe in Fig. 3 was calibrated using several reference mediums (Fig. 4). Fluids were at room temperature. The calibration equation has an r^2 of 0.95. This curve was produced with an energy input of 0.55 watt. Lower energy input rotates the curve clockwise reducing the resolution, especially at the higher conductivities. Heating obviously must be restricted in snow to avoid melting around the probe. Some knowledge of the snow's temperature profile is therefore desirable. Operation at temperatures warmer than -0.5^{o} to -1.0^{o} is not recommended with the present probe configuration. The thermistor in the probe tip determines temperatures in the profile and aids in evaluating probe operation.

Field Calibration and Use

Meter readings (reflecting differential temperatures inside and outside the probe) and samples of snow at various densities were taken at five locations over a 2-month period during 1973-74 (Fig. 5). In the majority of cases, the 250-ml core (20.4 cm long x 1.975 cm radius) enclosed the volume measured by the probe. If the core was lost, the measurement was repeated, or in very uniform appearing layers, a second horizontal core was taken adjacent to the original. All samples were taken in plastic bags to the laboratory for weighing.

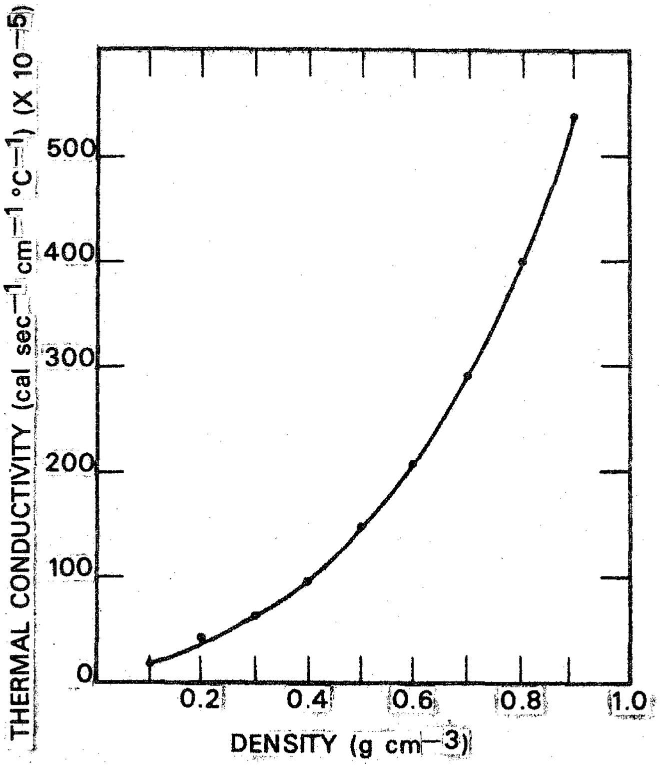

Figure 1. Relationship between snow density and thermal conductivity (data from list, 1966)

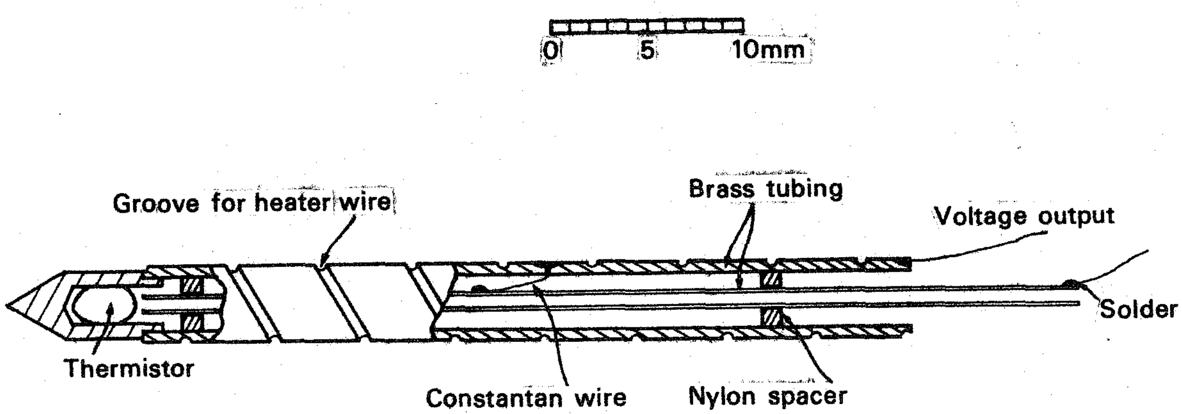

Figure 2. Cross section of snow density probe. Thermistor leads not shown.

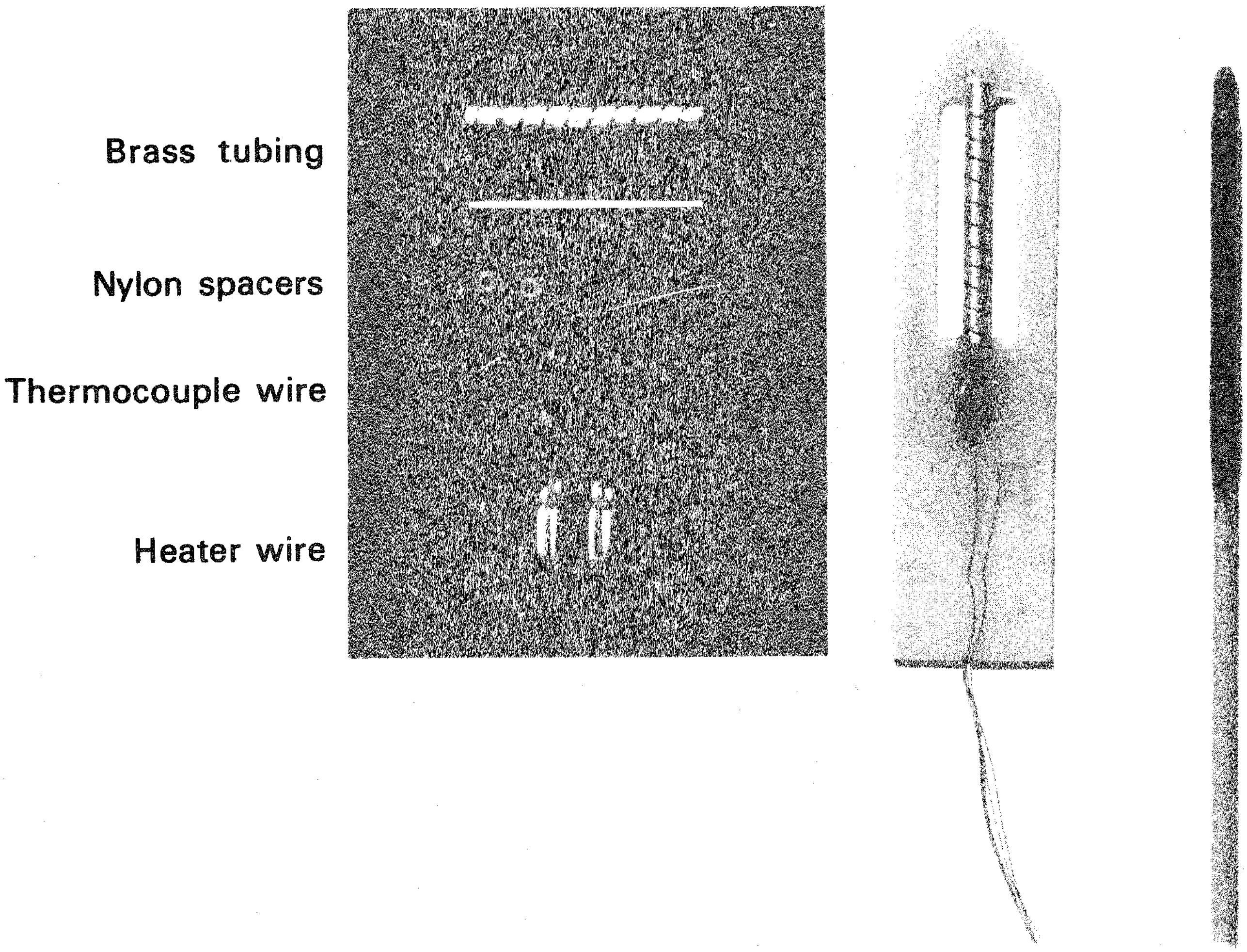

Figure 3. Materials for construction of snow density probes and completed models at right.

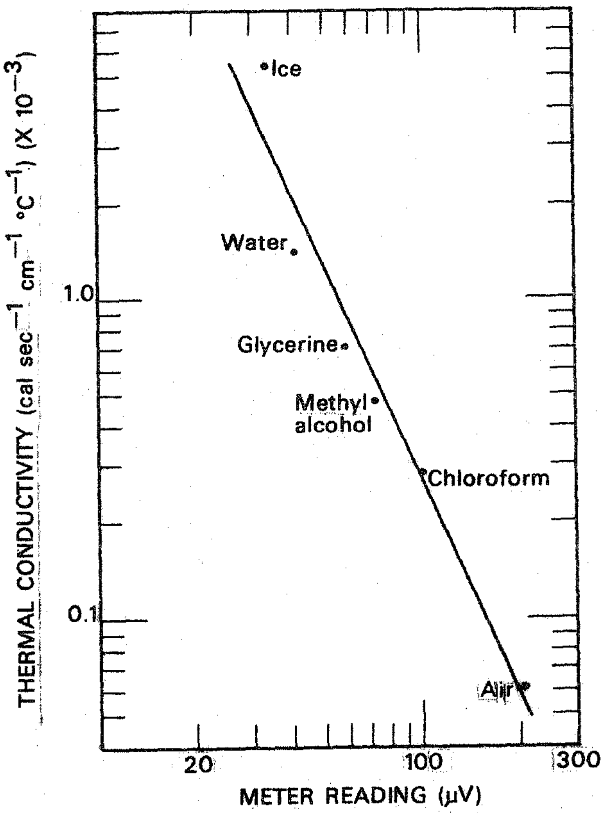

Figure 4. Calibration of probe against materials of known thermal conductivity.

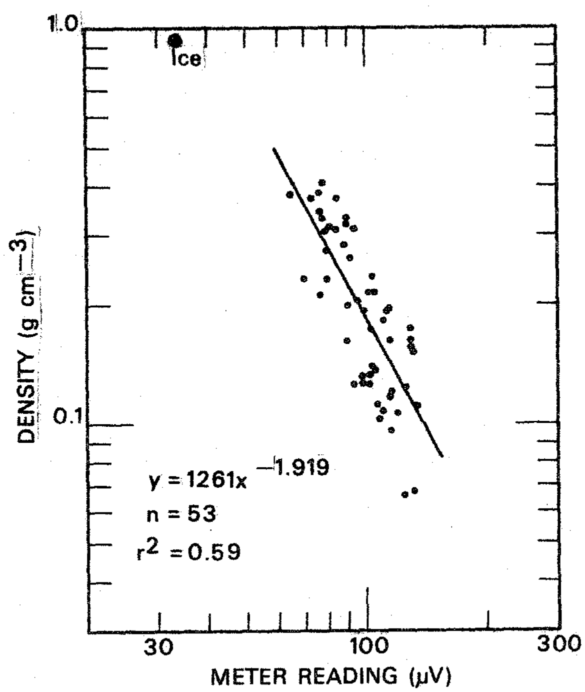

Figure 5. Calibration of snow density probe against core density samples.

Several factors are involved in the scatter around the calculated curve (r^2 = 0.59); some are related to the apparently simple task of taking core samples. The probe also examines material nearest the probe itself, and the density of the larger core sample is assumed to be uniform throughout. A core sampler of much smaller volume would not be practical; small errors in length or loss of material in removal would magnify problems. Addition of the ice point in Fig. 4 improves r^2 to 0.66. If time and snow conditions had permitted, additional samples would have been included, especially at the limits of the density range. With more familiarity with the probe's operation and earlier recognition of the need for extremely precise density measurements, this relationship would have improved.

As an example of our use of the probe, a sample site was selected to follow density changes with time at two depths (20 and 40 cm) under a canopy and in the open. As with all the measurements for calibration, a battery-powered microvoltmeter served as the readout. High capacity mercury batteries were used for probe heat. An additional circuit enabled readout of temperature on the same meter.

The results of the first sample (January 1974) are shown in Table 1. Density variability increases with depth and is greater under the canopy. The newly fallen snow appears of relatively constant density, having undergone little settling or consolidation. As an interesting statistic, the number of samples to provide a sample within 10% of the mean at P = 0.05 for the two upper levels are calculated (column 8, Table 1).

TABLE 1. DENSITY VARIABILITY AT TWO DEPTHS AND PROSPECTIVE SAMPLING REQUIREMENTS

Location (1)	Samples (2)	Depth in centimeters (3)	Density in grams per cubic centimeter: Mean (4)	High (5)	Low (6)	Standard deviation (7)	Estimate of sample size[a] (8)
Open	22	20	0.105	0.118	0.082	0.00865	3
	22	40	0.170	0.194	0.138	0.01583	4
	6	80	0.438	0.749	0.114	0.22177	
Forest	22	20	0.116	0.190	0.083	0.02966	27
	22	40	0.183	0.344	0.119	0.05628	41

[a]0.05 probability level. Error 10 percent of mean.

Discussion and Recommendations

The criteria for a potential method of determining in situ snow density appear to be satisfied. First, thermal conductivity is related to the density of the medium, and second, the instrumental procedure is practical. Some instrumental problems of this prototype should be discussed:

1. The snow pack and air are rarely at the same temperature. As the probe is moved between samples, it heats or cools to approach air temperature. Time to equilibrate in the snow at the next station depends on the air to snow temperature difference and requires up to 90 seconds for a 15° C differential. Reduction of probe mass, a more thermally conductive interior, removal of the extra thermistor and its wires, and a thinner epoxy coating would decrease equilibration time.

2. If-large temperature gradients exist between air and snow, axial flow of heat affects the probe's operation, The wire size used for the probe amplifies this effect. Although a single #32 AWG wire does not conduct much heat, the problem of heat flow increases substantially with the seven lead wires in the prototype. With a 46° C temperature gradient, air at 20° to ice at -26°, heat transferred to the probe's interior created a 28-microvolt offset (about 0.5° C) after an equilibrium state was reached. As a correction procedure, any offset voltage times a constant (1.5) is added algebraically to the final measurement. A factor of 1.5 was found to produce the best fit for the calibration curve.

3. Cost of the probe is low, suggesting that probes could be left in place throughout the season. Equilibration time would disappear and heat loss minimized. Probes could be rigidly fixed at several heights or allowed to settle following selected layers.

4. Interface problems occurring with insertion or around implanted probes are not definable at present.

Improvements to correct minor design problems and further study on the interface questions are underway.

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