AN ICE MELT FORMULA 1/

583-74

By

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At the 1969 Western Snow Conference in Salt Lake City, I presented a paper entitled "A Snow Pack Evapo-Sublimation Formula."

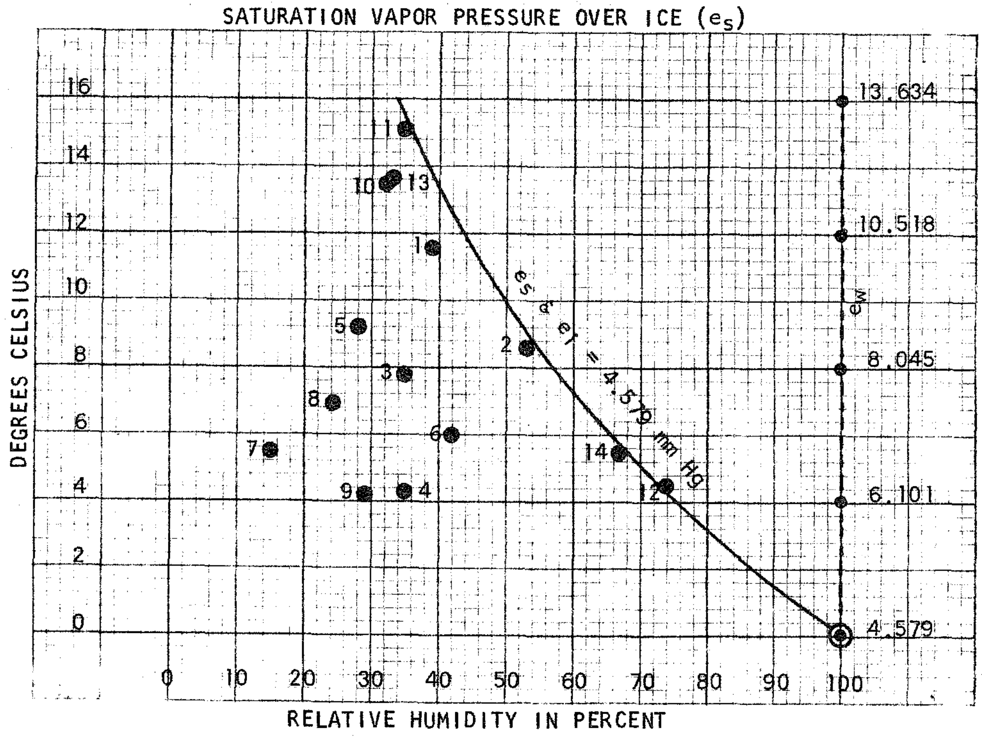
During the development of the evaporation formula, it was necessary to keep accurate records of runoff from the ice. Drainage from the sample was collected in a container beneath the tray from which there was no additional evaporation. The runoff-melt was carefully weighed and used in computing the evaporation from each of the samples.

Consequently, the temperature and humidity values obtained with each sample in the evaporation analysis are identical to the data in the runoff-melt analysis. These data are given in a pamphlet entitled "Ice Ablation Formulae."

A month or two after the 1969 meeting, these data were used to develop the runoff-melt formula for atmospheric vapor pressures ranging from zero to saturation over ice.

RUNOFF-MELT AT VAPOR PRESSURES
LESS THAN THE SATURATION VAPOR PRESSURE OVER ICE

PLATE 1.



The curve for saturation vapor pressure over ice in terms of degrees Celsius and relative humidity in percent is shown in Plate 1, where

$$\%RH_s = (e_s/e_w)100$$

1/ Presented at Western Snow Conference, April 16-20, 1974, Anchorage, Alaska.

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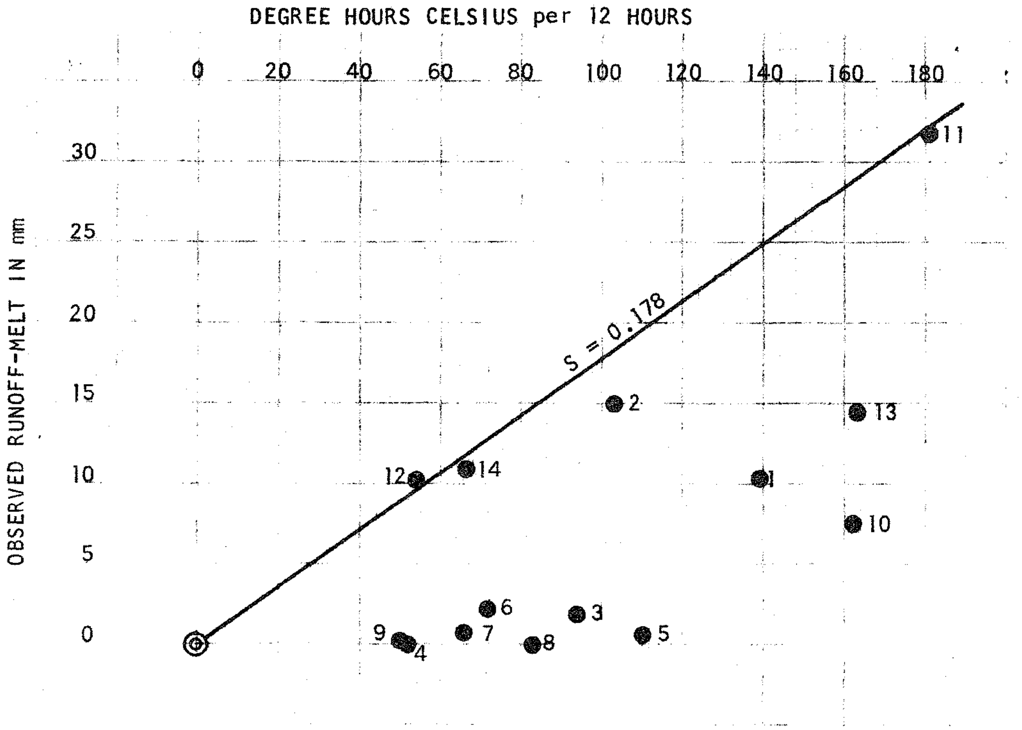
RH_s is the relative humidity at the saturation vapor pressure over ice. e_s is the saturation vapor pressure over ice and remains constant for 0° and above freezing temperatures. e_w is the vapor pressure of water for varying temperatures. The location of the points is determined by measured temperatures and relative humidity values for each of the samples.

CHART 1
HOGADON BASIN

Sample No.	Obs. °C	DHC	M_m = 0.178DHC	M_o Obs. Melt	M_o/M_m	Obs. RH	e_w	e_a
1	11.6	139	24.74	10.29	42%	39%	10.244	3.995
2	8.6	103	18.33	14.99	82%	53%	8.380	4.441
3	7.8	94	16.73	1.91	11%	35%	7.936	2.778
4	4.3	52	9.26	0.00	0%	35%	6.230	2.180
5	9.2	110	19.58	0.64	3%	28%	8.727	2.444
6	6.0	72	12.82	2.16	17%	42%	7.013	2.945
7	5.5	66	11.75	0.76	6%	15%	6.775	1.016
8	6.9	83	14.77	0.00	0%	24%	7.462	1.791
9	4.2	50	8.90	0.13	1%	29%	6.187	1.794
10	13.5	162	28.84	7.49	26%	32%	11.604	3.713
11	15.1	181	32.22	31.75	99%	35%	12.870	4.505
12	4.5	54	9.61	10.16	106%	74%	6.318	4.675
13	13.6	163	29.01	14.48	50%	33%	11.680	3.854
14	5.5	66	11.75	10.92	93%	67%	6.775	4.539

PLATE 2.



In Plate 2, the observed runoff-melt for each sample in the evaporation analysis is plotted against the same degree hours Celsius that were recorded for each of the evaporation samples. Three of these samples are close to the point of the saturation vapor pressure over ice. Samples 11, 12 and 14 recorded zero evaporation and zero vapor deficits in the evaporation formula. Consequently, these samples should provide close to maximum rates of melt for their corresponding temperature values.

The change of state from ice to water was complete for those three samples. There was no vaporization as this would require a vapor deficit, and a vapor deficit would reduce the vapor pressure to less than maximum with a corresponding reduction in a major melt factor--the latent heat of condensation.

The temperature equation for the maximum rate of melt originates at 0.000 mm of melt and zero degree hours Celsius and is placed through the three points of maximum melt. The slope of the curve is 0.178 and the equation is:

$$M_m = 0.178DHC$$

where M_m is the maximum rate of melt in millimeters for 12 hours and DHC is the degree hours Celsius for 12 hours.

The derivation of the balance of the runoff-melt formula is as follows:

The ratio of the observed melt (M_o) and the maximum rate of melt for each sample in Plate 2 becomes the ordinate in Plate 3. See Chart 1.

One gram of condensation on ice at 0^o will melt 7½ grams of ice for a total of 8½ grams of water. This is termed "condensation-melt," and since condensation requires water vapor, it is related exclusively to the vapor pressure of the overriding air. This relationship is given by an exponential curve that is inversely proportional to the curve for the vapor deficit versus evaporation.

Excepting that some air movement is necessary to maintain constant values of temperature and humidity in the area adjacent to the surface of the ice, it appears that variation in wind velocity does not provide a corresponding variation in runoff-melt, so the equation for runoff-melt contains no wind factor. The 1800 to 0600 hour formula is:

$$M = (0.178DHC)\%M_m{:}e_a$$

where $\%M_m{:}e_a$ is the percent of maximum runoff-melt indicated by the vapor pressure.

The curve for the Vapor Deficit versus the Rate of Evaporation is taken from "Ice Ablation Formulae" and shown in Plate 4.

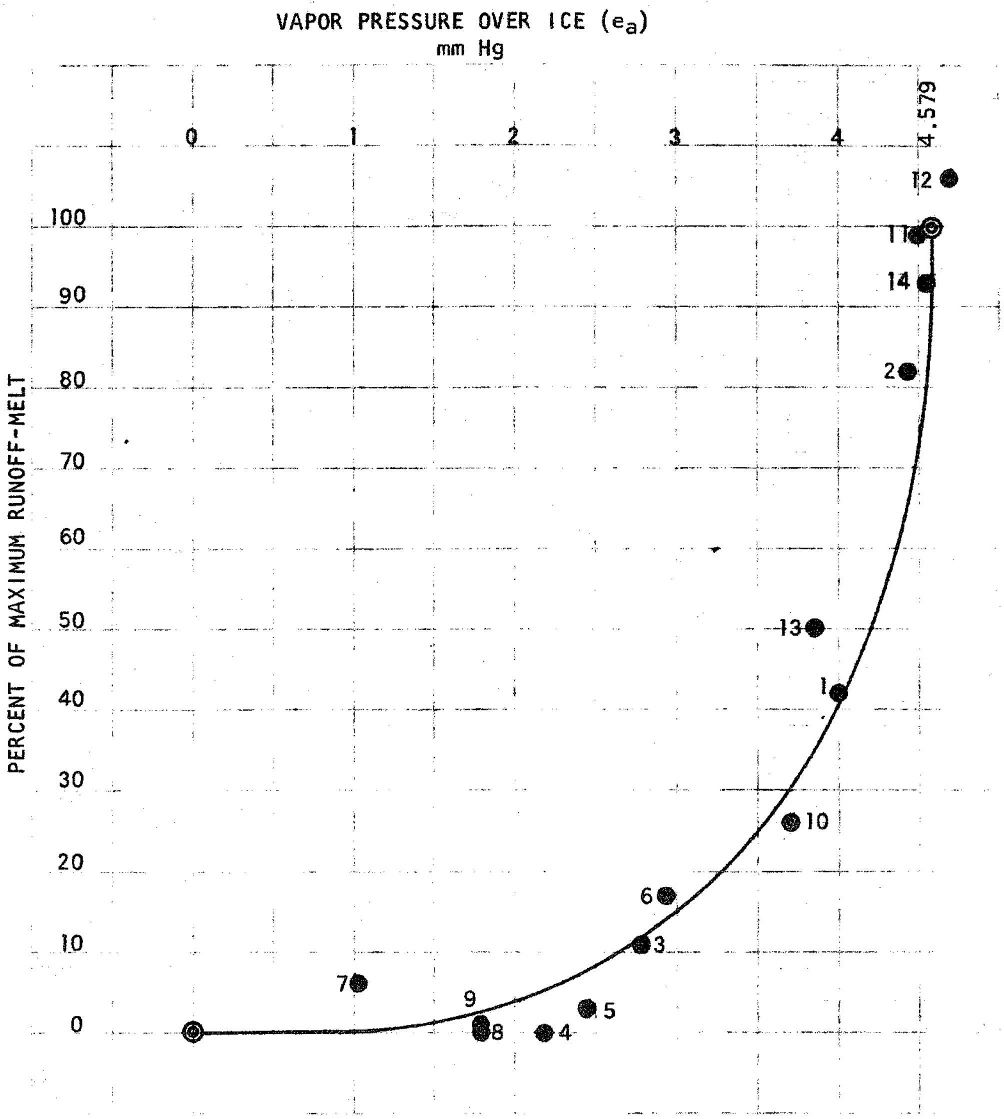
The equation for runoff-melt and the evaporation equation were derived from the same group of ice samples. Consequently, the wind, temperature and humidity data for each equation were identical. However, the curve for the rate of evaporation is inversely proportional to the curve for the rate of runoff-melt. The reason being that evaporation is related to the vapor pressure of the air.

Evaporation and runoff-melt occurred simultaneously, but at varying degrees. The wind was, of course, a major factor on the evaporation side, but could not be related to runoff-melt.

At below freezing temperatures, the vapor pressure of ice is not a constant as it is in above freezing temperatures, so the Vapor Deficit versus the Rate of Sublimation is a family of curves. If the expression, $e_s - e_a$, is changed to $1 - RH_o/RH_s$, the curves for evaporation and sublimation are identical.

The dotted line in Plate 4 represents the vapor deficit versus evaporation as a linear function and as it is commonly used, for example: $E = C(e_s - e_a)$. At vapor deficits in the area of 1 mm, the difference in the rate of evaporation is 51 percent.

PLATE 3.



RUNOFF-MELT AT VAPOR PRESSURES ABOVE
THE SATURATION VAPOR PRESSURE OVER ICE

During the spring of 1968, the vapor pressures at Hogadon Basin seldom exceeded the saturation vapor pressures over ice, and then to only a slight degree, so there was no information as to the extent of the rate of runoff-melt in the area between the vapor pressure of ice and that of water.

In June 1972, research was continued on Mount Hood, Oregon where average humidity values for a given temperature are considerably greater than those in central Wyoming.

In Plate 5, again, the curve for the saturation vapor pressure over ice is given. Samples were obtained during the night to eliminate radiation. Temperatures were above freezing and the relative humidity values were determined once an hour with a sling psychrometer.

PLATE 4.

THE VAPOR DEFICIT OVER ICE ($e_s - e_a$)
mm Hg

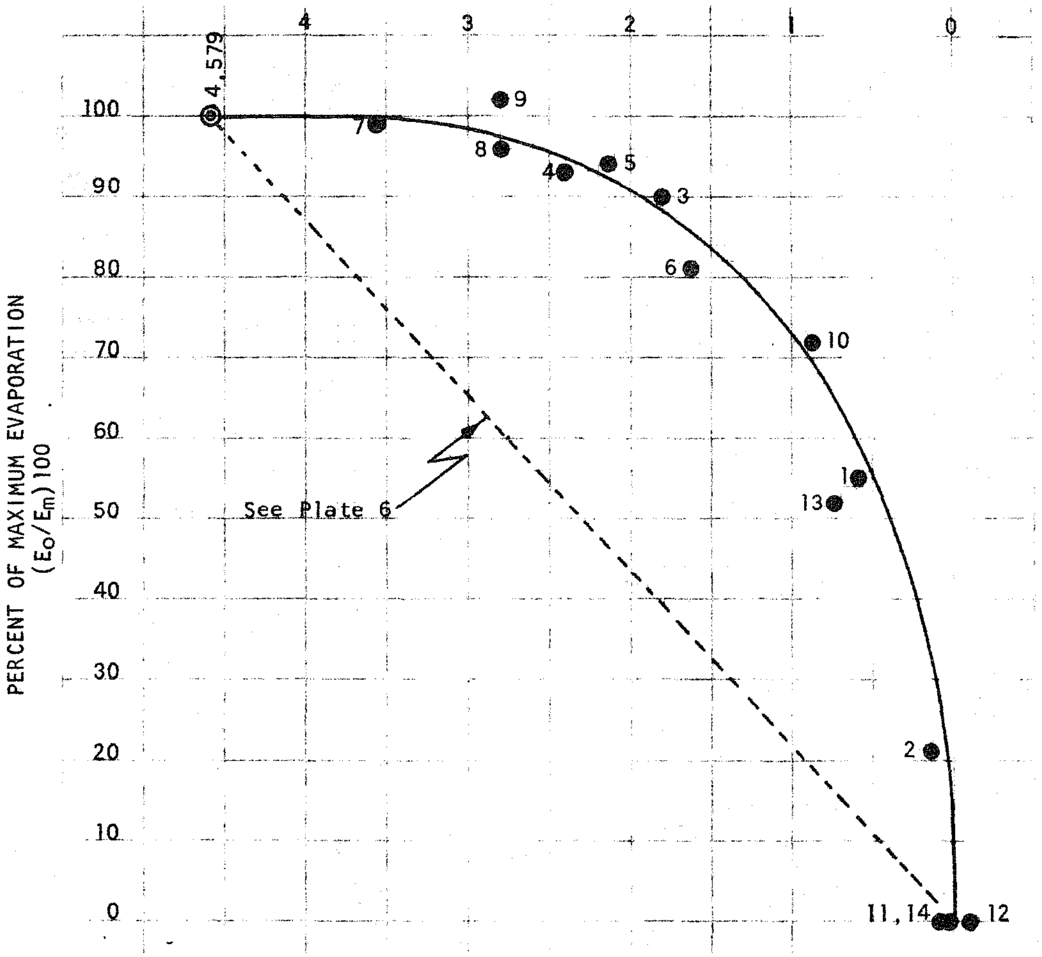
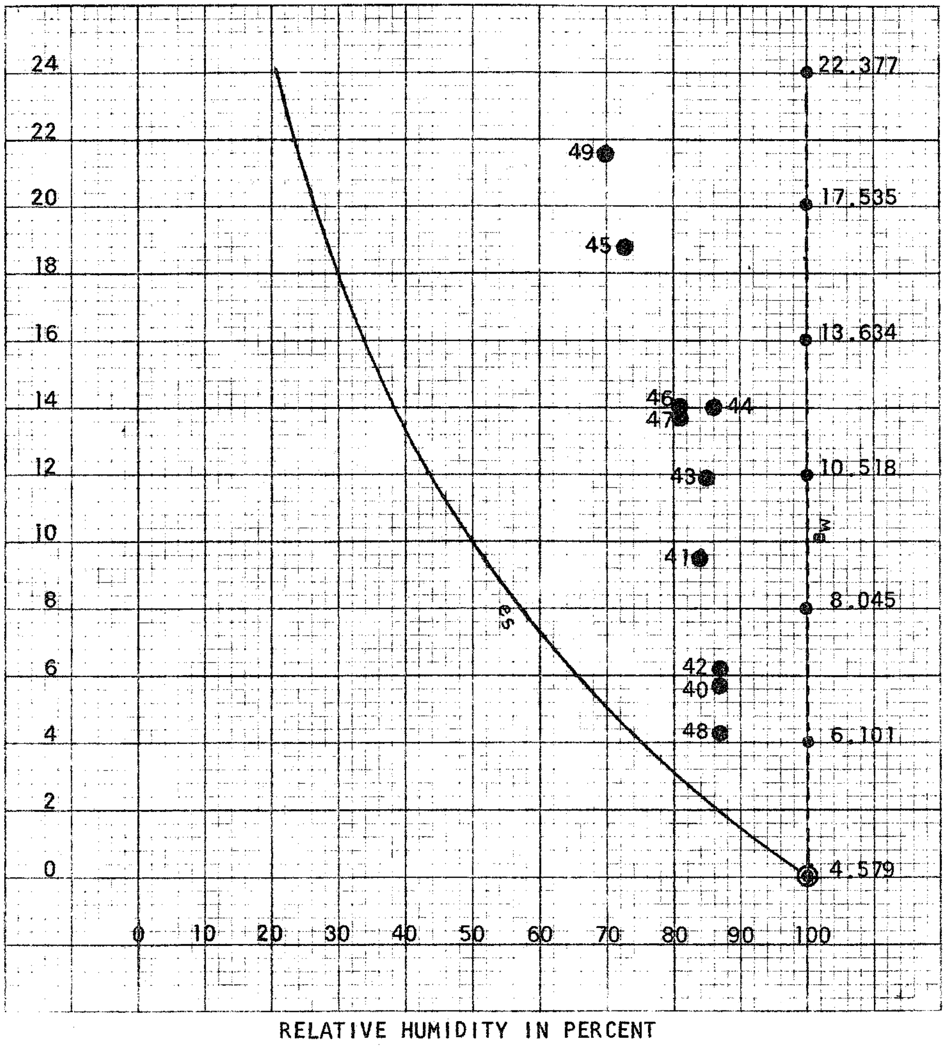


CHART 2
TIMBERLINE LODGE

Sample No.	°C	DHC	M_m = 0.178DHC	M_o Obs. Melt	M_o/M_m	RH_o	e_w	e_a
40	5.7	68	12.10	9.12	75%	87%	6.870	5.977
41	9.5	114	20.29	13.92	69%	84%	8.905	7.480
42	6.2	74	13.17	9.78	74%	87%	7.111	6.187
43	11.9	143	25.45	18.72	74%	85%	10.449	8.882
44	14.0	168	29.90	29.51	99%	86%	11.987	10.309
45	18.8	226	40.23	39.88	99%	73%	16.272	11.879
46	14.0	168	29.90	29.95	100%	81%	11.987	9.709
47	13.7	164	29.19	24.64	84%	81%	11.757	9.523
48	4.3	52	9.26	7.06	76%	87%	6.231	5.421
49	21.6	259	46.10	35.71	78%	70%	19.349	13.544

PLATE 5.

SATURATION VAPOR PRESSURE OVER ICE



The points in Plate 5 are all in the area between the saturation vapor pressure over ice (e_s) and the vapor pressure of water (e_w). Since e_a is always greater than e_s, there is no evaporation--only runoff-melt.

In Plate 6 the observed melt is plotted against the observed degree hours Celsius. The linear equation is anchored at (0,0) and placed through the points of maximum observed melt. The slope of the equation is again 0.178 as it was in the area of zero to e_s.

At this stage, there was no way to determine what the maximum rate of melt should be. It had to be assumed that:

$$M_m = 0.178DHC$$

Plate 7 graphically illustrates the rate of runoff-melt at vapor pressures greater than the vapor pressure of ice.

The ratio of the observed melt and the maximum melt, in percent, is the ordinate in Plate 7. The abscissa is the vapor pressure in mm Hg.

At the point of saturation vapor pressure over ice, the rate of runoff-melt reaches a maximum (4.579, 100%). Beginning at this point the curve for vapor pressures above 4.579 mm Hg abruptly drops into a cup that bottoms out at approximately (7.2, 69%). It again indicates a maximum rate of melt at about (9.9, 100%). Thence, it remains constant at 100 percent for vapor pressures up to about 12 mm Hg.

PLATE 6.

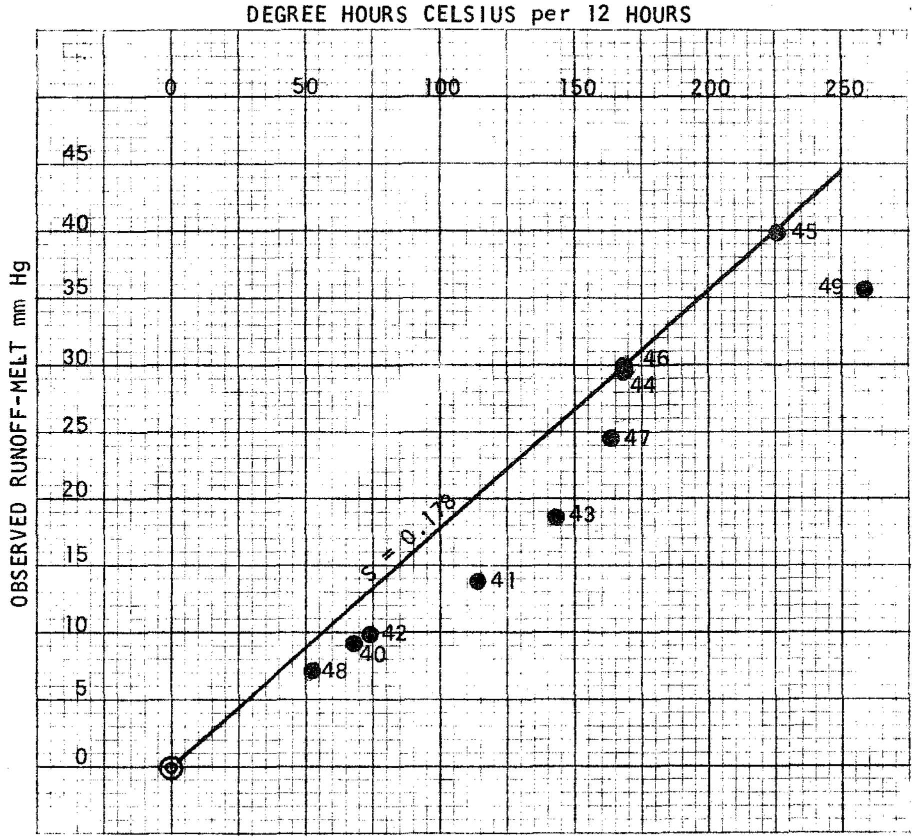
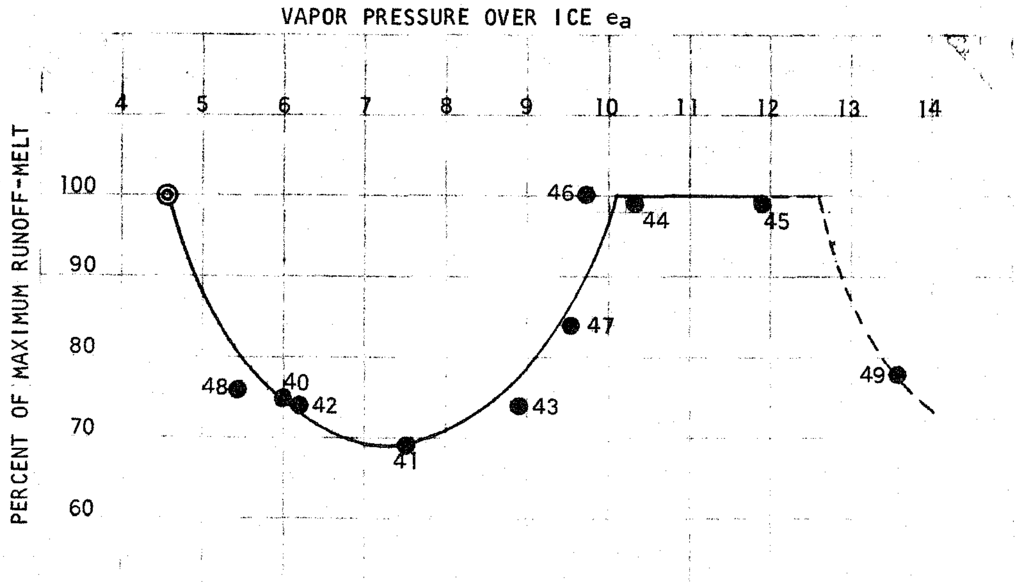


PLATE 7.



The last observation, No. 49, plots at (13.5, 78%). Assuming that a second cup is forming and that it would be identical to the first, the rate of melt would drop away from 100 percent at 12 to 13 mm Hg.

Summary

1. The annual variation in alpine snowpack evapo-sublimation is the major correction factor in Rocky Mountain snowmelt-runoff forecast equations.

2. The conventional evaporation equation is $E = C(e_s - e_a)$, where the rate of evaporation is proportional to the vapor pressure gradient.

The results at Hogadon Basin indicate that evaporation rates for ice are not linearly related to the difference in vapor pressures. They are, instead, related to the vapor deficit by an exponential curve (Plate 4).

3. The rate of sublimation for ice is related to the vapor deficit, $e_s - e_a$, by the identical curve developed for evaporation.

4. The rate of runoff-melt for ice is related to the overriding vapor pressure by an exponential curve that is inversely proportional to the vapor deficit curve for evapo-sublimation (Plate 3).

5. Sublimation is determined by four major parameters: wind velocity, below freezing air temperatures, net radiation and the vapor deficit which controls the rate of vaporization. With respect to snowpack ablation, loss from other parameters is considered negligible.

Evaporation is also determined by the four major parameters: wind velocity, above freezing air temperatures, net radiation and the vapor deficit.

Runoff-melt is determined by three major parameters: air temperatures, net radiation and the vapor deficit.

6. The samples in the nighttime evaporation and runoff-melt series are in the area where the vapor pressure of the atmosphere is less than the vapor pressure of melting ice (Plate 1). The samples were placed in a shallow tray about 4 m above bare ground, on a treeless ridge.

Both evaporation and runoff-melt data were obtained from each sample. These rates are governed by the vapor deficits and the vapor pressures, respectively. When the atmospheric vapor pressure becomes equal to the vapor pressure of melting ice, the rate of runoff-melt reaches a maximum and evaporation ceases.

The maximum rate of nighttime melt, over open bare ground appears to be: $M_m = 0.178DHC$ (Plate 2).

7. The samples in the nighttime runoff-melt series are in the area where the atmospheric vapor pressure is greater than the vapor pressure of melting ice (Plate 5). The ablation is from runoff-melt alone. There is no evaporation.

The samples were placed in a shallow tray about 50 cm above bare ground and in a mature spruce-fir-cedar forest. The maximum rate of runoff-melt proved to be the same as that on the open, bare ridge, so that $M_m = 178DHC$ (Plate 6).

The nighttime runoff-melt equation for ice samples depends on three major factors:

A. Convection-melt. The heat transfer from the air to the ice.

B. Condensation-melt. The condensation of vapor from the air to the ice and the release of the latent heat of condensation.

C. Radiation-melt. Under labatory conditions, the maximum rate of melt was about 71 percent of the maximum rate obtained at Mount Hood.

This would indicate that long wave radiation from the surrounding terrain is a major melt factor in the runoff from the ice samples, but is not a major melt factor in snowpack runoff.

The forest and cloud cover are the sources of long wave radiation to the snowpack, so long wave radiation is probably a minor melt factor in the snowpack runoff-melt equation.