APPLICATION OF A SNOWMELT MODEL TO TWO DRAINAGE BASINS IN COLORADO

E. Bruce Jones,[1/] Bernard A. Shafer,[2/]
Albert Rango,[3/] David M. Frick[1/]

INTRODUCTION

Hydrologists, engineers and water managers charged with the responsibility for making forecasts of snowmelt runoff from mountainous watersheds on an operational basis have long sought improvements to traditional statistical models. Of special significance to snow hydrologists in recent years has been the success of investigations affirming the ability to detect and map snow-covered area on mountainous watersheds from spaceborne platforms (Rango and Peterson, 1979).

An Applications Systems Verification and Transfer (ASVT) project addressing the operational applications of satellite snow cover observations sponsored by the National Aeronautics and Space Administration (NASA) demonstrated that snow covered areas could not only be mapped on watersheds as small as 100 mi^2, but that the information was of immense value in forecasting snowmelt runoff (Shafer and Leaf, 1979). Various techniques were explored in the technology transfer project to define the potential of using snow-covered area as a useful parameter in snowmelt forecasting schemes. Both probabilistic and deterministic approaches incorporating Landsat derived snow cover met with success and showed a reduction in forecast error. In addition, Martinec and Rango (1979) documented results applying a relatively simple deterministic model for making snowmelt runoff simulations using satellite derived snow cover as a critical input variable.

Two watersheds in Southwest Colorado were chosen as test watersheds where the model's performance would be evaluated. The watersheds were South Fork Rio Grande and Conejos River. These watersheds were chosen because of the availability of a high quality data base which included snow cover maps, snow course, streamflow, temperature and precipitation information compiled during prior ASVT studies. The evaluation of the Martinec-Rango model is the focus of this paper.

MODEL DESCRIPTION

No model is "perfect" because the knowledge of the system is "imperfect." Since this is recognized, models are often simplified by using integrating factors which combine many variables, known and perhaps unknown, into a single variable. The Martinec model is of this category in that it uses the percentage of areal snow cover as a main variable. This rationale has been pointed out by Leaf (1967). The model is further simplified in that

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[1/] Resource Consultants, Inc., Fort Collins, CO.

[2/] USDA, Soil Conservation Service, Denver, CO.

[3/] NASA, Goddard Space Flight Center, Greenbelt, MD.

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it uses the concept of degree-days for the energy input to the model. These parameters are adapted to a specific watershed by dividing the area into elevation zones and obtaining the percentage of snow cover as a function of time over each zone. Further, watershed adaptation is obtained by using runoff coefficients and degree-day factors. Finally, the runoff is matched to the stream network by using historical recession coefficients.

The model utilized for this study was based upon previous work by Martinec and Rango (1979). Several modifications were made in order to generalize the model and refine its capabilities. Three changes were made from the version developed by Martinec and Rango. These were (1) modification of the precipitation runoff algorithm to allow runoff to be calculated for nonsnow-covered areas only; (2) modification of the input data required to allow input in degree-days and precipitation per zone providing flexibility for development of these data based on the location of climatological stations in the vicinity of the watershed; (3) incorporation of a lag factor into the daily generated runoff to account for watershed travel time of computed runoff.

The Basic Equation for the Martinec model as applied to the South Fork and Conejos watersheds is given below:

$$Q_{n+1} = c_n \Big[[a_{An}(T_A)\, S_{An} + P_{An}]\, A_A \cdot \frac{10^{-2}}{86400} + [a_{Bn}(T_B)\, S_{Bn} + P_{Bn}]\, A_B \cdot \frac{10^{-2}}{86400} + [a_{Cn}(T_C)\, S_{Cn} + P_{Cn}]\, A_C \cdot \frac{10^{-2}}{86400} \Big] (1 - k_{n+1}) + Q_n k_{n+1} \qquad (1)$$

where

Q	is the average daily discharge $[m^3 s^{-1}]$
c_n	is the runoff coefficient
a_n	is the degree-day factor $[in. \ °C^{-1} \cdot d^{-1}]$
T	is the calculated number of degree days in a zone
S_n	is the snow coverage (100% = 1.0)
P_n	is the precipitation contributing to runoff [cm]
A	is the area $[m^2]$
k_n	is the recession coefficient
n	is an index referring to the sequence of days
A,B,C	as subscripts refer to the three elevation zones
$\frac{10^{-2}}{86400}$	converts $cm \cdot m^2$ per day to $m^3 s^{-1}$

The output generated by the model in its current version consists of a printout of all input data, climatological data and watershed data along with a printout of the computed runoff which is generated in each zone on a daily basis. The actual streamflows and computed streamflows are printed on a daily basis for the 6-month period. An output file is generated for input to a plotter.

The model also computes a goodness of fit or efficiency parameter and prints it along with the total seasonal computed and observed volumes. The model efficiency parameter, R^2, proposed by Nash and Sutcliff (1970) is used to test how well the model simulates snowmelt runoff. R^2 is somewhat analogous to the coefficient of determination and is defined as follows:

$$R^2 = \frac{\frac{1}{n}\sum_{i=1}^{n}(q_i-\overline{q})^2 - \frac{1}{n}\sum_{i=1}^{n}(q_i-q_i')^2}{\frac{1}{n}\sum_{i=1}^{n}(q_i-\overline{q})^2} \qquad (2)$$

where

R^2 is a measure of model efficiency

q_i = observed discharge

q_i' = simulated discharge

$\overline{q}$ = mean of observed discharge

n = number of discharge values

WATERSHED DESCRIPTIONS

The South Fork Rio Grande and Conejos River are tributary watersheds of the Rio Grande Basin in the San Juan Mountains of Southwest Colorado (Figure 1). The South Fork Rio Grande was studied above the stream gaging station at South Fork, Colorado. Drainage area is 559 km^2 (216 mi^2). It has an elevation ranging from 2506 m (8,222 ft.) at the stream gaging station to 3914 m (12,841 ft.) at the highest point. The Conejos River above the stream gage near Mogote, Colorado, was the second basin chosen for study. It has a drainage area of 730 km^2 (282 mi^2). Its elevation ranges from 2521 m (8,272 ft.) at the stream gage to 4017 m (13,180 ft.).

Both watersheds are typical of the region where a permanent snowpack begins accumulating in late October and generally reaches a seasonal maximum near the first of April. During April, snowpack depletion is normally experienced at lower elevations (below 2896 m [9,500 ft.]) while accumulation may continue at middle and high elevations up to the first of May. Winter snowpack accumulation is a gradual process which results from many snowfall events as contrasted with areas where a few major storms contribute the bulk of the permanent snowpack. Snowpacks in both watersheds are sub-freezing throughout most of the winter. This heat deficit must be made up and the snowpack brought to isothermal conditions in the spring before appreciable snowmelt runoff can begin. Snowmelt runoff produces 75 to 85 percent of the average annual flow in both watersheds.

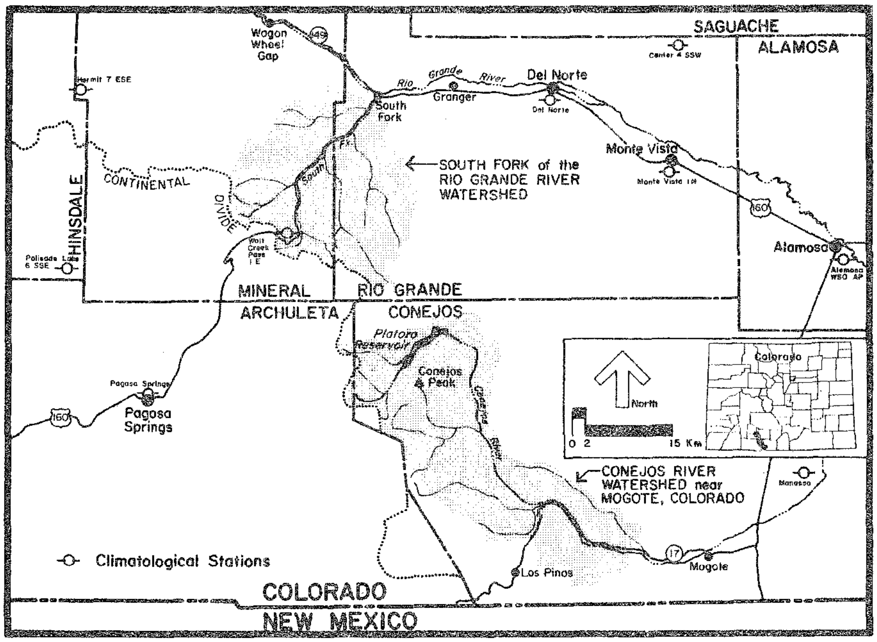

Figure 1. General Location Map for Study Watersheds

Streamflow records for each of the watersheds for the major runoff period April-September for 1973-79 show that during this 7-year period wide extremes in runoff were observed. A frequency analysis of streamflow reveals the drought conditions which prevailed during 1977 had a recurrence interval of nearly 100 years. On the other hand, 1979 was an extremely heavy snowpack year which produced runoff that would only be expected to occur once in every 20 years. Other years fell somewhere between these two extremes.

STREAMFLOW LAG CHARACTERISTICS

An analysis of the daily hydrographs for both watersheds was made in an effort to evaluate the effect of the shape factor and the time it took meltwater to reach the stream gage. For South Fork Rio Grande near the time of the seasonal peak in early June it was determined that approximately 70 percent of the flow of day n is a result of melt on the previous day with only about 30 percent coming from day n's generated melt. A similar analysis was performed for the Conejos River. In that watershed the proportions were found to be 83 percent and 17 percent, respectively.

SNOW COVER DEPLETION CURVES

Basin snow-covered area is a critical variable in the Martinec model. Daily values of the proportion of each zone covered by snow (S) are required to drive the model. To obtain this information Landsat imagery was analyzed by photointerpretive techniques, and a map was constructed showing the location of snow covered areas in each zone. These areas were then manually planimetered and a value was obtained for snow cover in each zone. All available imagery for each year was mapped and a series of snow-cover depletion curves were developed similar to those shown in Figure 2 for the South Fork Rio Grande for 1979. Daily values of zonal snow cover are extracted from the depletion curve and used as input to the model.

A comparison of the snow cover depletion curves for a given watershed over a period of years shows them to be basically similar in shape but shifted in time. This time shift is primarily a function of the volume of snow stored in the watershed. Figure 3 shows snow-cover depletion curves for South Fork Rio Grande for 1977. When one compares the curve of 1979 (Figure 2), which was a very big snow year, with those of 1977, which was a drought year, a displacement of nearly 2 months in Zones A and B meltout is observed and of about a month for Zone C. Snow-cover depletion curves for other years included in this study fall between these two extremes.

The slope of the snow-cover depletion curves is controlled by the climatological regime imposed on the basin during the meltout period. Abnormally cool periods tend to flatten the slope of the curves while abnormally warm temperatures act to steepen it.

TEMPERATURE LAPSE RATES

Climatological data for this study was obtained from published records for the Del Norte and Wolf Creek 1E stations. Since Del Norte at an elevation of 2403 m (7,884 ft.) is located near the lower end of the South Fork of the Rio Grande watershed and the Wolf Creek 1E station is located on Wolf Creek Pass in the South Fork watershed at an elevation of 3244 m (10,642 ft.), actual daily lapse rates calculated between these two stations were utilized to develop degree days for the hypsometric mean elevations of each zone. These same climatological stations were used to develop zone data for the Conejos watershed, with adjustment for the different hypsometric mean elevations of each zone.

PRECIPITATION

Precipitation input for the model was based on data from Wolf Creek 1E and Del Norte. Precipitation in each zone was determined by two different methods. To account for orographic type storms which occur in the winter and early spring, the precipitation data were calculated by the same elevation ratios used for determining lapse rates for temperatures in each zone. This method was utilized for precipitation occurring in April and May of each year. The second method used to develop precipitation for each zone accounted for convection type storms which occur during the late spring and summer. For this case the precipitation occurring at Del Norte was utilized directly for Zones A and B, and Wolf Creek 1E data were used for Zone C. This method was used for the months of June through September.

MELT RATES

Daily snowmelt depths are calculated by the model using the degree-day method. Although several approaches in applying this method have been advanced by various authors including Gartska (1958), Martinec (1960) and Linsley and Franzini (1979) they all relate the amount of snowmelt in a basin to a degree-day index factor. The degree-day factor for snowmelt computations is normally the positive departure of the mean daily

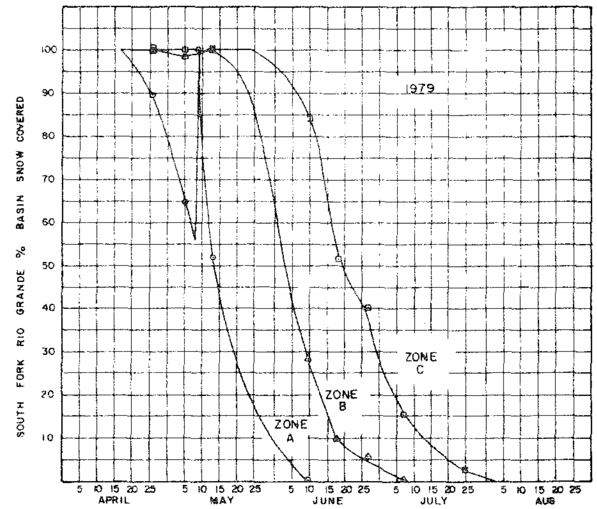

Figure 2. Snow-Cover Depletion Curves for South Fork Rio Grande for 1979. Note minor effect of May 8 storm on snowpack of Zone A.

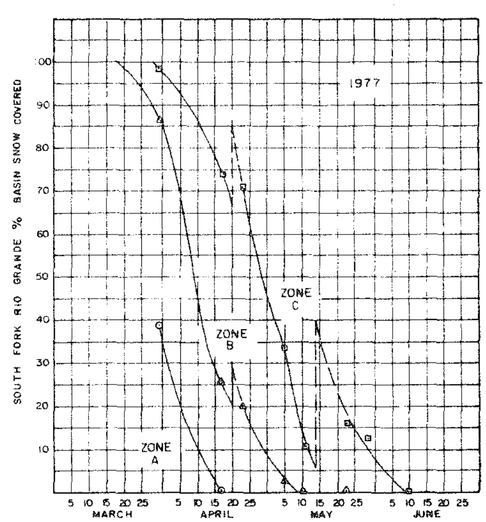

Figure 3. Snow-Cover Depletion Curves for South Fork Rio Grande for 1977. Compare the temporal displacement of this set of curves with those for 1979 (Fig. 2).

temperature above a base temperature of 0^{o} C. The factor is thus an empirical measure of the amount of energy available to melt snow based on air temperature alone. However, other variables including solar radiation, wind and humidity also influence melt rates. For this reason a wide variability in melt rates based on degree-days can be expected. Melt rates ranging from .23 cm $^{o}C^{-1}d^{-1}$ to .91 cm $^{o}C^{-1}d^{-1}$ (0.05 to 0.20 in. $^{o}F^{-1}d^{-1}$) have been observed at the Upper San Juan SNOTEL site.

It is not possible to completely eliminate problems of attempting to approximate mean daily temperatures from available climatological records. Even with its obvious shortcomings, the degree-day method does provide a useful tool for modeling. Since it is only an index and not an exact relationship, it is not critical in the model. It is important, however, that whatever degree-day method is once chosen, it be consistently applied thereafter.

For the purposes of the current investigation a degree-day was calculated on the following basis:

$$T_d = \frac{T_{max} + T_{min}}{2} - 0°C \qquad (3)$$

where:

T_d = degree day
T_{max} = maximum daily temperature (°C)
T_{min} = minimum daily temperature (°C)

Melt rates for Zone A in both watersheds generally ranged between .05 inches/degree-day in April to 0.32 cm $^{o}C^{-1}d^{-1}$ (.07 in. $^{o}F^{-1}d^{-1}$) for the May-September period. Zone B melt rates generally ranged from 0.14 cm $^{o}C^{-1}d^{-1}$ (.03 in. $^{o}F^{-1}d^{-1}$) in April to a maximum of 0.55 cm $^{o}C^{-1}d^{-1}$ (0.12 in. $^{o}F^{-1}d^{-1}$ in late May and early June. Zone C melt rates ranged from 0.09 cm $^{o}C^{-1}d^{-1}$ (.02 in. $^{o}F^{-1}d^{-1}$) in early April to a maximum of 0.73 cm $^{o}C^{-1}d^{-1}$ (0.16 in. $^{o}F^{-1}d^{-1}$) in early June.

RUNOFF COEFFICIENT

A proper evaluation of the zonal runoff coefficient, c, is necessary to achieve a reasonable degree of success with the model. An explicit evaluation of c is not easily performed. Rather, an estimate based on prior experience in other watersheds and consistent with hydrologic conditons prevailing in the study watershed is normally made. For both the South Fork Rio Grande and Conejos, estimates of c were adjusted between zones and seasonably varied for each year to achieve as close a fit as possible between the observed and simulated hydrographs for each year.

In general, the runoff coefficient was progressively varied from about 0.75 in early April to 0.20 in late June or July and remained constant thereafter. These values are similar to those estimated by Rango and Martinec (1979) for two watersheds in Wyoming.

The runoff coefficient is a very sensitive parameter in the model and can easily cause major problems in simulation attempts if it is not correctly evaluated. During trial simulations it is relatively simple to vary the runoff coefficients for each zone on a systematic basis consistent with hydrologic principles to arrive at a reasonable fit.

HYDROGRAPH RECESSION ANALYSIS

In order to correctly apportion the observed daily streamflow discharge between the current day's snowmelt runoff contribution and the effect of previous day's melt requires an evaluation of the recession coefficient "k." Martinec and Rango (1979) have shown that this coefficient is related to discharge by the following exponential function:

$$k = x \cdot Q^{-y} \qquad (4)$$

where: k = recession coefficient
x and y are parameters unique to a given basin
Q = current discharge in m^3/sec.

A direct evaluation of this relationship is possible for each watershed. To obtain the values of x and y, values of Q_n were plotted against Q_{n+1} for snowmelt recession periods over a number of years and a lower envelope line and an average line drawn through the points.

RESULTS OF MODEL RUNS

The model was initially run on each watershed for the 1973-79 period and melt-rate factors and runoff coefficients optimized for each year. For these runs, the recession coefficient was calculated using the lower envelope recession. Results from these runs were encouraging and demonstrated the model's capability to provide reasonable simulations of the snowmelt runoff hydrograph on watersheds of this size. Table 1 shows the simulation results for the years 1977, 1978, and 1979 which correspond to a drought year, near average year, and a maximum of record year, respectively.

To test the model's sensitivity to estimates of melt rate factors and runoff coefficients, the model was re-run for the 1973-79 period with average parameter values. The average values were for 15-day periods calculated from the initial optimized runs. This analysis was performed to predict what results might be anticipated if the model was run by a person who had little skill in the selection of reasonable parameter values. Table 1 presents the results of this analysis for the same 3-year period as the initial optimized runs. As might be expected, the results revealed a marked degradation in performance when compared with optimized runs for all years with the exception of Conejos in 1979, where an improvement was observed in the R^2 value. This analysis implies that a certain degree of hydrologic knowledge about a watershed's mean areal water equivalent, residual soil moisture storage, and normal runoff patterns should improve the model's ability to simulate the snowmelt hydrograph. This knowledge can be acquired by running the model and optimizing parameter values for a wide range of hydrologic conditions.

During the initial simulation runs and those with average parameter values, a consistent over-prediction in the recession portion of the hydrograph was observed. An adjustment in the recession coefficient, k, was indicated by these observations. To accomplish this adjustment, the mean recession line relationship for the South Fork Rio Grande and Conejos, respectively, were substituted for the lower envelope recession line employed in the initial optimization runs and runs with average parameter values.

When the model was run with the revised recession relationship and optimized parameter values, a substantial improvement in all simulations for both watersheds was observed. Table 1 gives results of these runs for the 1977-79 seasons. Table 2 shows results of runs with optimized parameter values for 1973-79.

Figures 4 and 5 show plots of model runs for South Fork Rio Grande for 1977 and 1979, respectively. Figures 5 and 6 show the same plots for Conejos.

OPERATIONAL APPLICATION

The model has been proven to be reliable in explaining most of the variability in daily discharge from snowmelt runoff during trial simulations previously discussed. To be really functional for water-supply forecasting, however, the model must be adapted for use in an operational predictive mode.

SUMMARY

The model has been successfully applied to both the Conejos and South Fork Rio Grande watersheds in a simulation mode for a 7-year period. Several minor modifications were made to the original model version used by Martinec and Rango (1979) to better approximate the hydrologic conditions of the two watersheds examined. Overall, the model performed remarkably well during all of the simulations with the exception of 1977 which was the minimum year of record in terms of both snowpack and streamflow. Additional improvement in model performance would be expected if a denser network of climatological stations were available to better approximate the precipitation regime.

For the 7-year period analyzed, the model accounted for an average of 89 percent of the observed variation in streamflow on the South Fork Rio Grande and 87 percent on the Conejos. Individual yearly simulations accounted for streamflow variations of from 69 to 97 percent on the South Fork and 60 to 95 percent on the Conejos. Seasonal streamflow volume simulations had an average error of 1.8 percent on the South Fork and 1.1 percent on the Conejos for the combined analysis period.

ACKNOWLEDGMENT

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Table 1. Summary of Model Trial Runs Using Various Coefficients

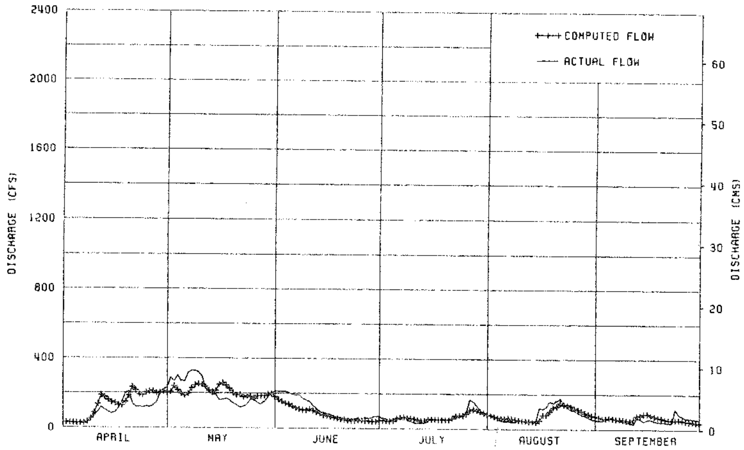
	South Fork of Rio Grande			Conejos River		
	1977	1978	1979	1977	1978	1979
Initial optimized melt and runoff coefficients						
Nash-Sutcliffe R^2	0.5668	0.8952	0.9556	0.5111	0.9168	0.9122
Seasonal volume difference in percent	0.71	-0.28	0.97	3.17	-0.67	0.21
Average melt and runoff coefficients, 1973-1979						
Nash-Sutcliffe R^2	-1.4214[1]	0.7670	0.9299	-0.0331[1]	0.8641	0.9384
Seasonal volume difference in percent	38.40	1.92	4.76	28.47	-8.81	0.66
Optimized melt and runoff coefficients with revised recession coefficients						
Nash-Sutcliffe R^2	0.6910	0.9168	0.9747	0.5950	0.9405	0.9534
Seasonal volume difference in percent	-0.06	0.21	0.04	3.23	0.04	0.35

[1]Computed R^2 less than zero indicate that the variation from mean discharge is less than the variation between observed and computed flows and is therefore meaningless (see equation 2).

Table 2. Summary of Final Model Runs

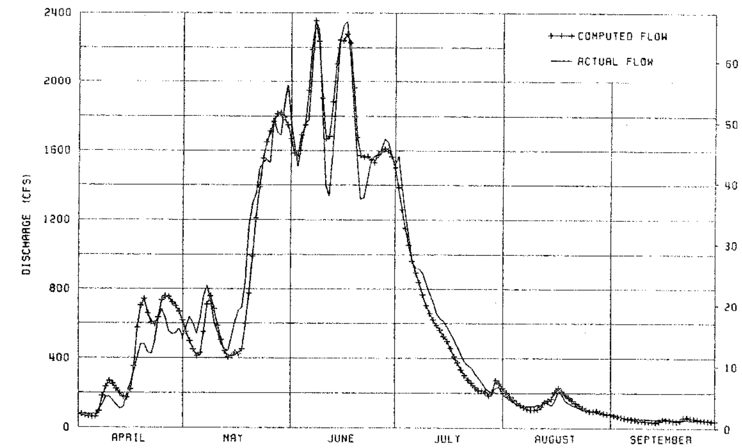
	1973	1974	1975	1976	1977	1978	1979
South Fork of the Rio Grande at South Fork							
Nash-Sutcliffe R^2	0.9371	0.8694	0.8999	0.9440	0.6910	0.9168	0.9747
Seasonal volume difference in percent	-0.77	-0.80	-8.91	-1.51	-0.06	0.21	0.04
Conejos River near Mogote							
Nash-Sutcliffe R^2	0.9485	0.8608	0.8941	0.8979	0.5950	0.9405	0.9534
Seasonal volume difference in percent	1.67	-0.53	0.22	-1.58	3.23	0.04	0.35

Figure 4



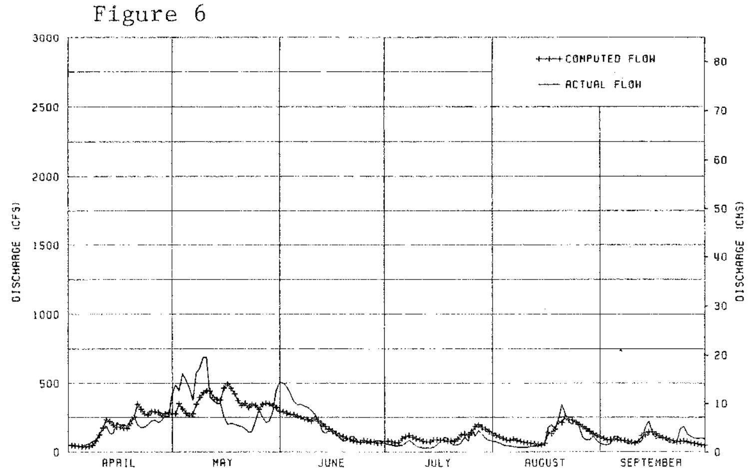
SOUTH FORK OF THE RIO GRANDE RIVER AT SOUTH FORK
MODEL RUN FOR 1977 RUN MADE OCTOBER 24, 1980

Figure 5



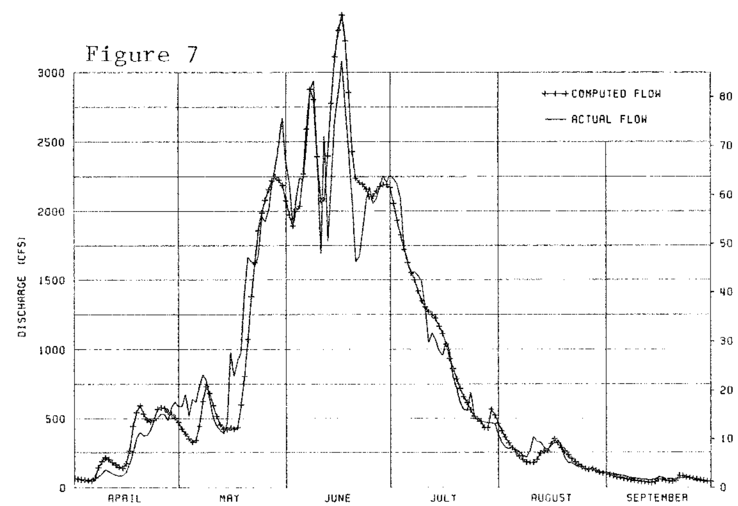
SOUTH FORK OF THE RIO GRANDE RIVER AT SOUTH FORK
MODEL RUN FOR 1979 RUN MADE OCTOBER 24, 1980

Figure 6



CONEJOS RIVER NEAR MOGOTE
MODEL RUN FOR 1977 RUN MADE OCTOBER 23, 1980

Figure 7



CONEJOS RIVER NEAR MOGOTE
MODEL RUN FOR 1979 RUN MADE OCTOBER 23, 1980

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