PREDICTING DEPOSITION OF BLOWING SNOW IN TRENCHES FROM PARTICLE TRAJECTORIES

681-81

By

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INTRODUCTION

Moisture is a limiting factor for revegetation on a large part of the western U.S. where we will strip mine coal in the next few decades. Relocation of winter snowfalls by wind compounds our revegetation problems by scouring moisture from ridges and concentrating the precipitation in ravines, producing evaporation losses in the process. In such regions, there is potential for restoration using techniques which distribute drifting snow in patterns that provide sufficient moisture for plant establishment on some fraction of the disturbed area. This paper presents one design consideration pertinent to the use of contour trenches for such a purpose.

Many factors limit the use of contour trenches, including the type of material (soil, overburden, rock), the steepness of slope, and intensity of summer thunderstorms. In addition, if trenches are to accumulate moisture from wind-blown snow, then land slope must be in the same or opposite quadrants as the wind direction which prevails during drifting. Once the portion of a restoration area that is suitable for contour trenches has been delineated, the design problem is to specify size and spacing of trenches to maximize the area over which the required soil moisture for plant establishment is provided. Inputs are the amount of relocatable precipitation, the number of storms in which drifting takes place, and the efficiency with which blowing snow is captured by the trench. This last item, trapping efficiency, is the problem addressed here.

This paper extends a model of snow-particle trajectories, proposed by Berg (1977), and computes snow deposition behind ground surface geometry that approximates the two-dimensional cross-section of a contour trench. Only the mean wind velocity and particle fall velocities are used to compute particle trajectories. Although several assumptions limit its range of application, useful estimates of trapping efficiency result, and the approach seems to hold promise for other snow deposition problems.

MECHANICS OF SNOW RELOCATION

To estimate the fraction of the solid phase that drops from the flow into a region of reduced velocity in the wind shadow of an obstruction, the distribution of mass with height in the flow must be known. At any given height, the composition of this mass according to particle size is then required, along with estimates of particle settling speed as a function of size. A brief discussion of the blowing snow phenomenon is presented to justify the assumptions made in the calculations.

Most drifting snow particles are smaller and more rounded than the precipitation crystals from which they derive, and as a result, the assumption of spherical particles of ice is a reasonable approximation. Effective diameters of the actual particles are distributed with some tendency toward greater frequency of smaller sizes, so that log-normal or two-parameter gamma probability density functions provide suitable

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mathematical descriptions (Budd, 1966). Distribution of mass with height above a horizontal surface shows an almost exponential decrease. Particle motion near the surface is by saltation; a hopping, jumping mode in which particles strike the surface after flights of several tens of centimeters, during which they rise only a few centimeters above the surface (D. Kobayashi, 1972). This mechanism is of fundamental importance in generating snow transport because the impact of saltating particles is necessary to overcome cohesion between particles at the surface (S. Kobayashi, 1979; Schmidt, 1980). Distances of several hundred meters are usually required to generate flow that is saturated with respect to snow particles (Dyunin, 1967; Takeuchi, 1980). As the drifting layer grows, turbulence carries particles out of saltation into a "suspension" mode that extends to heights of tens of meters. The settling speed of the particles determines the height to which they rise when vertical velocities are directed upward. Businger (1965) argues that settling speeds in turbulent flow may be less than in still air. Budd (1966) assumed a linear relationship between size and fall velocity.

Mean streamwise flow velocities in snow drifting over a uniform surface are approximated by the logarithmic wind profile equation, except in the saltation layer where speeds tend to be higher than predicted by the equation that describes velocities above the 10 cm level (S. Kobayashi, 1979). The velocity field downstream from a break in a horizontal surface is more complicated. In the case of a backward-facing step, a region of reverse flow may develop next to the surface just downwind of the step. Above this, the mean horizontal velocity adjusts across a region that increases with distance downstream from the step (Figure 1). Although regions of increased turbulence intensity are generated downwind of the step, particles appear to follow the mean flow field in wind tunnel tests with models of such obstacles (Finney, 1934).

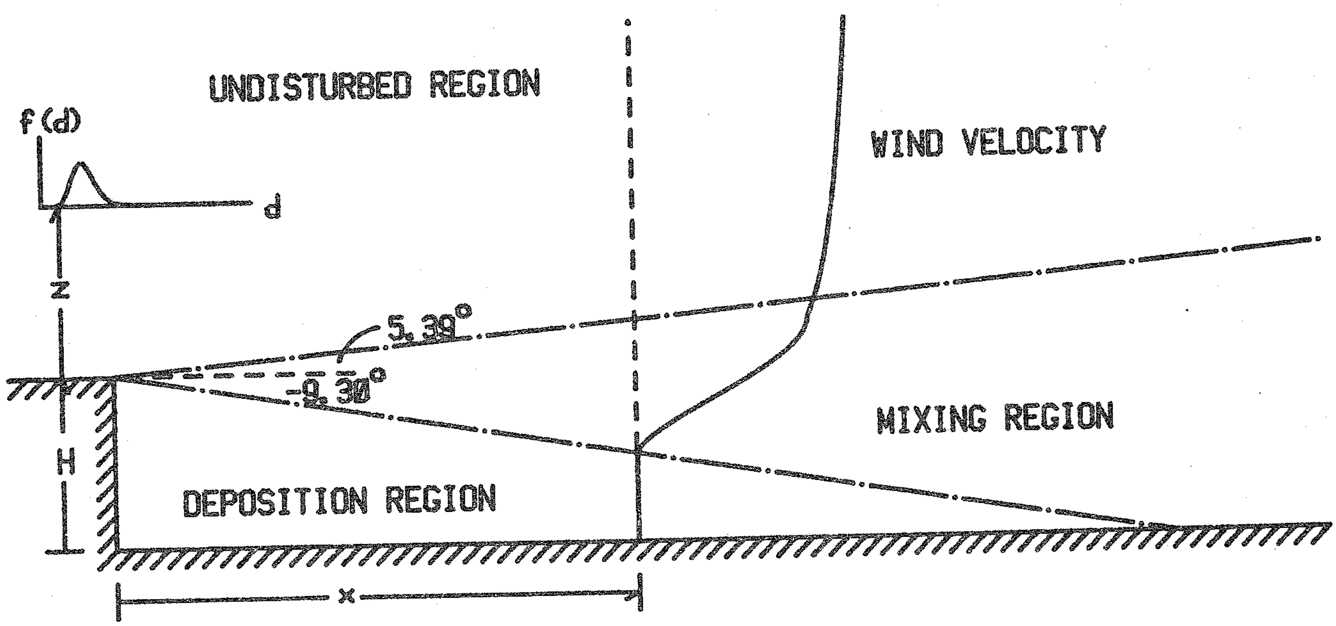

Figure 1.
Geometry and flow regions of the model. Example velocity profile has zero backflow in the deposition zone

Experiments that measured trapping of snow by trenches dug perpendicular to the wind provide a good picture of the deposition pattern to be expected under light and moderate drifting (Ōura et al., 1967). Figure 2 shows that during initial drifting (A), the largest mass is deposited close to the upwind wall, corresponding to trajectories of the great majority of particles, moving very near the surface. A secondary feature is the small cornice that formed near the center of the trench after the initial deposition period. Apparently as the trench filled, particles moved along the new surface by saltation or creep until they dropped off the new "step." Reverse flow may also play a part in the pattern of deposition.

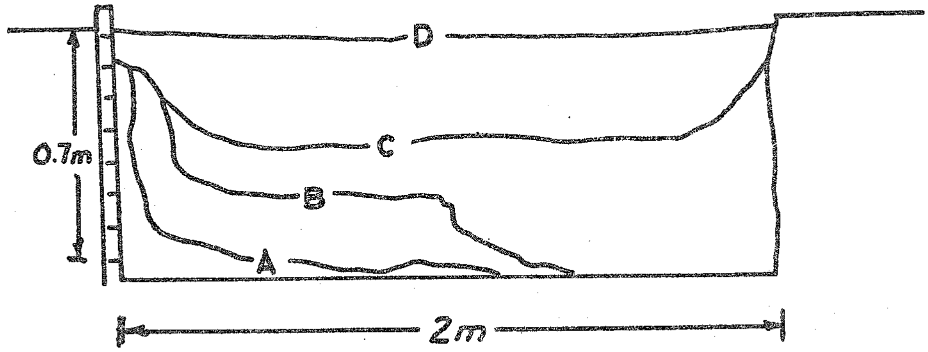

Figure 2.
Patterns of deposition in a trench under light to moderate drifting snow (Ôura et al., 1967). Deposition from A to B occurred in 2 hours.

TRAJECTORY CALCULATIONS

Berg (1977) discusses very completely the concepts used in these calculations. His predictions, assuming snow particles of uniform size, agreed well enough with observed deposition downwind of terrain barriers on an alpine ridge, to encourage the senior author to pursue this approach. Accounting for the distribution of particle sizes with height was undertaken by the junior author as a Master of Science degree problem with the direction of Dr. Wilson Brumley, Associate Professor of Mathematics, Colorado State University (Hagberg, 1979). Later she expanded this work to include the variable approach slope and trapping efficiency calculations which are reported in this paper.

Assuming the particles have streamwise velocity equal to the flow velocity, but vertical velocity equal to settling speeds in still air, trajectories were calculated by a Runge-Kutta method. The following assumptions were required:

(1) At any height z, upwind of the barrier, distributions of particle diameter d are given by the two-parameter gamma probability density function

$$f(d) = \frac{d^{\alpha-1} e^{-d/\beta}}{\beta^{\alpha}\,\Gamma(\alpha)} \qquad \alpha>0 \quad \beta>0 \qquad (1)$$

Shape of the distribution is measured by α, and β is the scale parameter. The mean diameter is $\bar{d} = \alpha\beta$, and $\Gamma(\alpha)$ denotes the complete gamma function at α. Distributions measured in saltation are presented by Schmidt (1981).

(2) Shape parameter α increases with height according to the relationship

$$\alpha = a \ln z + b \qquad (2)$$

where a and b are empirical constants, taken as a = 1.5 and b = 17 (Schmidt, in preparation). These values are determined from an array of photoelectric particle counters (Schmidt, 1977) exposed to moderate blowing snow on a nearly horizontal short-grass plain.

(3) Data from the same experiment shows that the decrease in mean diameter with height can be approximated by a power law

$$\bar{d} = cz^{f} \qquad (3)$$

where c and f are again empirically determined. For moderate drift conditions, representative values were $\bar{d}$ = 200 μm at z = 1 cm, $\bar{d}$ = 80 μm at z = 1 m.

(4) The number of particles F passing through a unit area normal to the direction of flow, per unit time, was assumed to vary inversely with height. This gives

$$F = F_1/z \qquad (4)$$

were F_1 is the number flux at z = 1 cm.

With these assumptions, the mass of particles at each height could be computed by size classes.

(5) Particle fall velocities were computed for the mid-point of each size class by means of Carrier's (1953) relation between drag coefficient C_D and particle Reynold's number, R_e (Lee, 1975)

$$C_D = (24/R_e)(1 + 0.0806\ R_e) \qquad (5)$$

where R_e = dV/n, settling velocity being denoted by V, kinematic viscosity by n.

An iterative solution determined C_D from V by the definition $C_D \equiv D/(\rho V^2 A/2)$,

where D is the force due to gravity, A is the projected area of the particle and ρ is air density.

(6) To specify streamwise velocities, the flow field was divided into (a) the undisturbed stream, (b) a mixing region, and (c) the eddy zone or deposition region (Figure 1). The logarithmic wind profile was assumed to apply across the entire transport layer upwind, and in the undisturbed region downwind of the step.

$$U = (u_*/k)\ \ln(z/z_o) \qquad (6)$$

where streamwise velocity U is proportional to friction velocity u_* and z_o is an integration constant, termed roughness parameter.

(7) The mixing zone is defined on the upper boundary by z = x tan 5.39° where x is distance measured from the step. The lower boundary of the mixing region is taken as z = -x tan 9.3° following Berg (1977). Streamwise velocities in the region are computed from a formula suggested by Naib (1966) from his results in a water flume. The location of flow reattachment at the surface, 6 to 7 step heights distance downwind of the step, is considered the limit of the deposition zone. This point is computed from the intersection of the lower boundary of the mixing layer with the surface.

$$U_{z,x} = U_{m,x} - (1-N^{1.5})^2\ (U_{m,x} - U_{b,x}) \qquad (7)$$

where

$N = (z-z_{b,x})/(z_{m,x} - z_{b,x})$

and $U_{m,x}$ = velocity at the upper boundary of the mixing zone

$z_{m,x}$ = height above the step at which $U_{m,x}$ occurs,

$U_{b,x}$ = velocity at the lower boundary of the mixing zone

$z_{b,x}$ = height at which $U_{b,x}$ occurs,

$U_{z,x}$ = velocity at present height,

z = height.

(8) Two separate cases were considered for flow in the eddy zone: (a) the horizontal velocity is taken as zero, (b) a constant reverse flow was implemented (from the work of Naib). In case (b), the maximum backflow was taken to be 0.25 of the maximum forward flow in the mixing region (Berg, 1977). The horizontal velocity in the eddy zone was then found by the formula:

$$U = -.25\ (u_*/k)\ \ln(z_{m,x}/z_o) \qquad (8)$$

(9) An additional assumption extended the model to a greater range of step geometries. The mean flow field was assumed to apply when the approach to the step was varied through a range of ± 10°, by rotation of the coordinate system. Height z was taken normal to the approach surface. If this approach angle is ϕ, then the length of the deposition zone, measured from the step is $L = -H/\tan(\phi - 9.3°)$, where H is the step height (Figure 3).

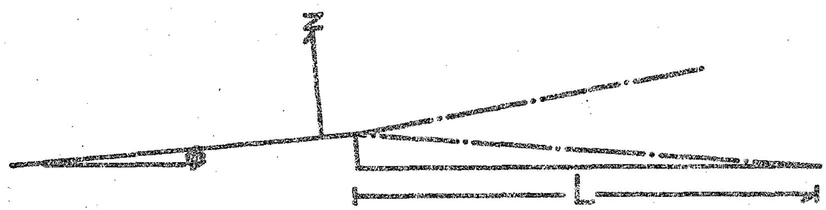

Figure 3.
Definition diagram of approach angle ϕ.

Calculations begin with specification of friction velocity and roughness parameter for the velocity profile. The distribution of particles with height is computed and each size class, at each height increment, is routed into the deposition zone by solving the trajectory equations for increments of time since the particles passed the step. As deposition occurs, the wind profile changes, and new catchment geometry must be computed (Berg, 1977). Because most deposition occurs near the step, a slightly lower backward-facing step with new approach slope approximates the new geometry. However, the trapping efficiency calculations presented are for deposition of the mass transported during the first one-second interval of flow over the empty step. The time required for the particles to complete their trajectories into the deposition zone is longer than one second, in most cases.

Size class widths were 10 μm, height increments were 1 cm, and time increments for trajectory iterations were adjusted to give location to the nearest 1 cm in the deposition zone. The deposition zone was divided into catchment "bins," to accumulate the computed mass. These increments were excessively small for the first runs with fixed wind speed, giving an irregular deposition plot (Figure 4). The smoothed features are the desired result, but the small integration intervals were retained in anticipation of further developments in the procedure.

RESULTS AND DISCUSSION

Hagberg (1979) tested the effect of changing particle size parameters α, β, and $\bar{d}$, on the disposition of mass in the accumulation region. In the range $3 \leq \alpha \leq 7$ and $200 \leq \bar{d} \leq 300$, for size distributions in saltation, only small changes in the predicted deposition pattern occurred. The effect of reversed flow in the deposition region, on the other hand, had a larger effect in accumulating particles closer to the step.

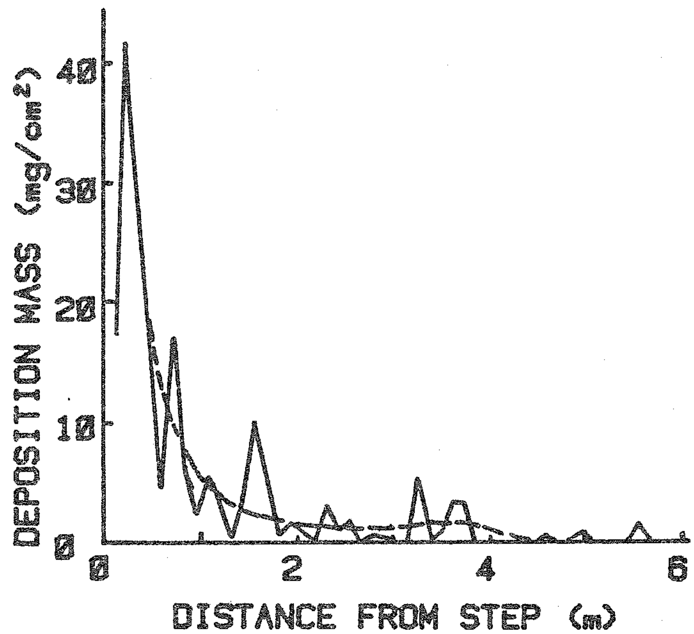

Figure 4.
Example of deposition mass for $u_* = 50$ cm/s, $z_o = 0.01$ cm, from a one-second interval in moderate drift conditions as assumed in the text.

The computed trapping efficiency E = 100 G/Q is the percentage of transport rate Q that accumulates as drift deposition. Blowing snow transport rate Q is the total mass of snow, per unit time, blown through an area of unit width across the flow, extending perpendicular to the approach surface (e.g., kg/s m). Deposition rate G is the sum of all snow mass accumulated downwind of the step, per unit time, per unit width. Trapping efficiency was plotted as a function of approach angle, for step heights of 1 and 2 m (Figure 5).

The results certainly did not support our preconceived notions of the efficiency of such barriers in accumulating blowing snow. First, doubling step height made almost no difference in computed trapping efficiency, while our intuition predicted a significant increase with increasing height. Second, an efficiency less than 50% for a backward facing step with horizontal approach was much smaller than expected. Third, although the decreasing efficiency with increasing approach angle was expected, the increase again, above $\phi = 5°$ was a surprise. Part of our problem here is the difficulty in separating expectations of deposition efficiency from observations of equilibrium drift capacity. A larger step, for example, definitely traps a larger volume of snow, but perhaps with no greater efficiency than one having similar geometry, but smaller height.

Before we accept these results, the shortcomings of the model should be reviewed. Assuming a constant wind velocity is unrealistic when trajectories are being computed over intervals of several seconds. However, computations that summed the deposition, as friction velocity varied by ± 10%, only smoothed the deposition curves and did not change trapping efficiency substantially. The assumption that only mean streamwise and settling velocities determine the location of deposition ignores any effect of turbulent fluctuations. Measurements in turbulent flow of water (Naib, 1966; Raithby et al., 1978) and in air (Plate and Lin, 1965) show a region of high turbulence that extends downstream from the crest of the step or model hill, as well as non-zero mean vertical velocities. The effect of ignoring these facts is not clear, even in a qualitative way, at this point. Vertical velocities are generally toward the surface in the lower part of the mixing region, which should improve trapping efficiency, but high turbulence would extend trajectories streamwise, thus reducing efficiency.

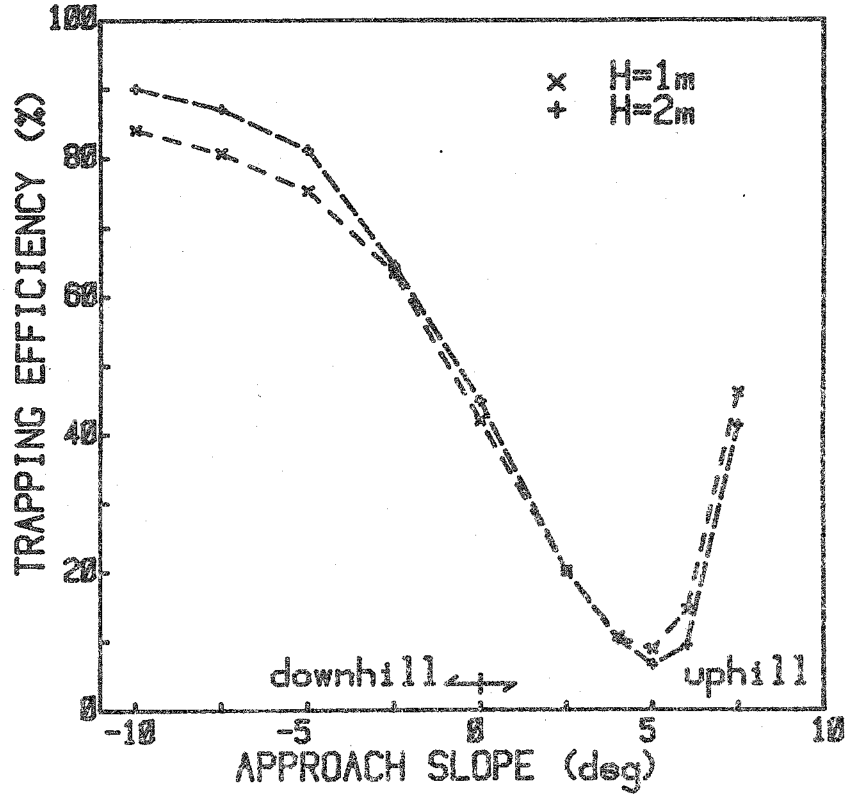

Figure 5.
Initial trapping efficiency of backward-facing steps is greater for approaches sloping downward toward the step. H = step height. Values of other parameters were $u_* = 50$ cm/s, $z_o = 0.01$ cm, $\alpha = 3$, $\bar{d}_r = 200$ µm and $F_1 = 1000$/s. Computations are for flux during a one-second interval.

The gamma distribution theoretically extends to infinity along the size axis, and when the mass distribution is computed by the cube of diameter, large particles in the tail of the distribution, though few in number, present a large mass, relative to the remainder of the distribution. Some thought was given to the physical reality of particles larger than 600 µm moving in saltation, at the moderate wind speeds used in these calculations. Measured distributions (Schmidt, 1981) do not show the frequency of large particles predicted by equation (1). Theoretical calculations of aerodynamic forces on surface particles (Schmidt, 1980) also suggest that there is a cut-off to the size of particle moving in saltation. The question only applies to the lowest few centimeters of the transport layer. If particles in the range 600-1000 µm are excluded from these calculations, the small particles that do not enter the deposition zone from these lowest few layers represent an important amount of the total transport.

Because the estimated trapping efficiency is so closely related to the length L, assumption (9), that L/H is independent of step height, must be examined. Tabler (1975) predicts that the ratio of drift length to height, for an equilibrium drift behind a step, increases rapidly with decreasing step height, for H < 5m. In the present calculations, L denotes the length of the flow separation bubble, or the location of flow reattachment, taken equal the limit of accumulation during initial snow deposition. One may reason that initial length of the deposition zone could be less than the final drift length, given drift development by successive steps. However, the present procedure would predict a shortening of the deposition region as the drift develops a negative approach slope. Outdoor modeling experiments (Tabler and Jairell, 1981) will test these trapping efficiency calculations and help determine the relationship between initial deposition zone and equilibrium drift length.

If we agree to skepticism born of several oversimplifications, then the calculated results may be explained as follows:

(1) There is little change of trapping efficiency with step height because the major portion of the drift is near the surface, deposits near the step face, and this is almost independent of height.

(2) The low trapping efficiency predicted at zero approach angle depends on the lack of particles larger than 600 μm diameter moving in saltation, for the friction velocity of 50 cm/s used in the calculations. Based on the available measured sizes, this result seems justified.

(3) The increase in trapping efficiency with increasing approach angle, above $\phi = 5°$ is the result of extension of the deposition zone that begins to outweigh the initial vertical angle of particles passing the step.

These tentative conclusions can be suggested for trench design based on the calculations. First, specification of berm height can be based primarily on snow storage capacity required, without compromising trapping efficiency, within the usual berm height range of 1 to 2 m. Second, trapping efficiency of trenches on lee slopes is greater than trenches on windward slopes. A general observation made in developing this computational model is that short distances between rows of barriers minimize the growth of the drifting layer, and this in itself tends to increase efficiency by keeping drift close to the surface.

CONCLUSIONS

A computational procedure that accounts for the distribution of drift density with height and particle size, estimates snow deposition behind a backward-facing step in general agreement with field observations of the location of deposition in trenches. The results suggest that snow trapping efficiency of contour trenches is independent of berm heights in the range 1-2 m.

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