

ERROR ASSESSMENT OF AIRBORNE  
SNOW WATER EQUIVALENT MEASUREMENTS

by

Thomas R. Carroll\* and Warren K. Jones\*

INTRODUCTION

The National Weather Service has conducted research since 1969 to develop a technique using natural terrestrial gamma radiation attenuation to measure snow water equivalent and soil moisture from a low-flying aircraft. As a result of early research and development, the National Weather Service has recently implemented and currently maintains an operational Airborne Gamma Radiation Snow Survey Program in North Dakota, South Dakota, western Minnesota, and a portion of Saskatchewan (Peck, et al., 1980). The snow survey program is intended primarily to provide real-time snow water equivalent data to the National Weather Service Forecast Offices in the region and River Forecast Centers in Kansas City and Minneapolis for use in the spring flood outlooks and flood forecasts made for the upper Midwest (Carroll and Larson, 1981). Additionally, it is possible to use the resources of the Airborne Snow Survey Program to calculate reliable, real-time, mean, areal soil moisture values for large areas of the upper Midwest at critical times during the hydrologic and agricultural cycles.

A network of 300 flight lines, each 15 to 20 km long, has been established in the region. A twin-engine Aero Commander flies at an altitude of 150 m and measures natural terrestrial gamma radiation over a path 300 m wide. Consequently, radiation data collected over each flight line represent mean areal measurements over approximately 4.5 to 6 km<sup>2</sup>. The ability to measure mean areal snow water equivalent or soil moisture is attractive in light of the extensive ground sampling procedure required to estimate mean areal values with reasonable accuracy.

AIRBORNE SNOW WATER EQUIVALENT AND SOIL MOISTURE MEASUREMENT TECHNIQUE

The technique used to measure natural terrestrial gamma radiation attenuation and subsequently infer snow water equivalent and soil moisture values has been described recently by Carroll and Vadnais (1980) and Carroll (1981); consequently, only a brief overview will be given here.

---

\*Airborne Gamma Radiation Snow Survey Program, National Weather Service-NOAA, 6301 - 34th Avenue South, Minneapolis, Minnesota 55450

Presented at the 50th Annual Western Snow Conference, Reno, Nevada, 1982 April 20-23.

The gamma radiation flux near the ground originates primarily from the natural  $^{40}\text{K}$ ,  $^{238}\text{U}$ , and  $^{232}\text{Th}$  radioisotopes in the soil. In a typical soil, 96 percent of the gamma radiation is emitted from the top 20 cm. After a measure of the background (no snow cover) radiation and soil moisture is made over a specific flight line, the attenuation of the radiation signal due to the snow pack overburden is used to calculate the amount of water in the snow cover over approximately 6 km<sup>2</sup>. Three snow water equivalent values are calculated by measuring the attenuation of the gamma radiation flux using data from the K window (1.36-1.56 MeV), the Tl window (2.41-2.81 MeV), and the gross count (GC) energy spectrum (0.41-3.0 MeV). The potassium photopeak is consistently the strongest in the energy spectrum and has been used successfully to measure snow water equivalent in Canada and in the U.S. The gross count window accumulates an order of magnitude more counts than the K and Tl photopeak windows. Consequently, gross counts are useful when measuring the variability of snow cover along a flight line or a snow cover with 15 to 25 cm of snow water equivalent.

The gamma radiation attenuation technique can also be used to measure soil moisture in the upper 20 cm. The gamma radiation emitted from the radioisotopes in the soil is attenuated by soil moisture near the surface. Consequently, airborne radiation data collected over bare ground reflect both the radioisotope and soil moisture concentration near the surface. Recent tests show a high correlation between airborne and ground soil moisture measurements (Jones and Carroll, 1982).

The principal sources of error in calculating snow water equivalent or soil moisture values using any of the three windows are incorporated in: (1) the measurement of mean areal soil moisture for a flight line, (2) the measurement of air mass (i.e., temperature, pressure, and radar altitude), and (3) radiation counting statistics. Nonetheless, the airborne technique is capable of measuring snow water equivalent with an error of less than 1 cm (Carroll and Vadnais, 1980) and soil moisture with an error of approximately 3.9 percent soil moisture (Jones and Carroll, 1982).

#### ERROR ANALYSIS

Airborne snow water equivalent measurements are made using the following relationship:

$$\text{SWE} = \frac{1}{\alpha} \left[ \ln \frac{C}{C_0} - \ln \left( \frac{100 + 1.11 M}{100 + 1.11 M_0} \right) \right] \text{g cm}^{-2}$$

where:

$C$  and  $C_0$  = Uncollided terrestrial gamma count rates over snow and bare ground,

$M$  and  $M_0$  = Percent soil moisture over snow and bare ground,

$\alpha$  = Radiation attenuation coefficient in water,  $\text{g cm}^{-2}$ .

Because soil moisture varies over the length of a flight line and only a limited number of samples can be taken,  $M_0$  and  $M$  are subject to statistical errors that can affect the computation of snow water equivalent. Approximately 25 ground samples are typically taken over the length of a 20 km flight line; the average coefficient of variation is 26 percent. The standard error of the mean varies from survey to survey and averages 1.46 percent soil moisture. The influence of this error on the final measurement of snow water equivalent can be estimated by:

$$\text{var}(\text{SWE}) = \left( \frac{\partial \text{SWE}}{\partial M_0} \right)^2 \text{var}(M_0) + \left( \frac{\partial \text{SWE}}{\partial M} \right)^2 \text{var}(M)$$

where:

$$\frac{\partial \text{SWE}}{\partial M_0} = \frac{1}{\alpha(M_0 + 90)} \quad \text{and} \quad \frac{\partial \text{SWE}}{\partial M} = \frac{-1}{\alpha(M + 90)}$$

Consequently, the variance of the snow water equivalent measurement due to errors in the soil moisture measurements can be expressed as:

$$\text{var}(\text{SWE}) = \frac{1}{\alpha^2} \left( \frac{\text{var}(M_0)}{(M_0 + 90)^2} + \frac{\text{var}(M)}{(M + 90)^2} \right)$$

It is clear that the values of  $M_0$  and  $M$  will not have a major effect on the error estimate. Choosing a median value of  $M_0 = M = 25$  percent soil moisture and letting  $\text{var}(M_0) = \text{var}(M) = 2.13$  (based on the average standard error of the mean) gives  $\text{var}(\text{SWE}) = .093$  cm due to errors in the ground soil moisture calibration.

Additional sources of error in the snow water equivalent calculation include those due to counting statistics and analog measurements of air pressure, air temperature and radar altitude. Errors in counting statistics reflect the limited time with which to collect radiation data over a specific flight line. The analog data are used to calculate the air mass between the aircraft and ground which also attenuates the radiation signal. These airborne errors are difficult to partition and have been estimated numerically for airborne soil moisture measurements using a larger sample than is currently available for snow measurements (Jones and Carroll, 1982). Airborne errors contribute substantially more than ground soil moisture errors in airborne soil moisture measurements. It is possible to use the airborne to ground error relationship to estimate the airborne error which contributes to the snow water equivalent measurement. Table 1 gives a summary of the major theoretical RMS errors associated with the airborne snow water equivalent measurements.

Table 1

THEORETICAL AIRBORNE SNOW WATER EQUIVALENT  
MEASUREMENT ERRORS

Total theoretical RMS error	0.71 cm
RMS error contributed by ground sampling	0.30 cm
RMS error contributed by airborne estimates	0.64 cm

During the winter of 1982, we had an opportunity to check nine airborne snow water equivalent measurements against two sets of intensive ground snow cover data collected along the length of two calibration flight lines. The results are given in Table 2.

Table 2  
OBSERVED SNOW WATER EQUIVALENT  
MEASUREMENT ERRORS

Flight Line	Ground SWE	Airborne SWE
ND130C	8.4 + 1.5 cm N̄ = 29	8.0 + 0.3 cm N̄ = 5
ND117C	7.4 + 1.8 cm N̄ = 27	8.3 + 0.3 cm N̄ = 4
* * * * *		
RMS error	0.74 cm	Percent bias 2.85%
Average absolute error	0.61 cm	Cases 9
Average bias	0.23 cm	

#### CONCLUSION

Mean areal airborne snow water equivalent measurements can be made with an error of approximately 0.75 cm.

#### REFERENCES

- Carroll, T. R. (1981) Airborne soil moisture measurement using natural terrestrial gamma radiation. Soil Science 132(5):358-366.
- Carroll, T. R. and L. W. Larson (1981) Application of airborne gamma radiation snow survey measurements and snow cover modeling in river and flood forecasting. Proceedings of the Western Snow Conference, St. George, Utah, 14-16 April 1981.
- Carroll, T. R. and K. G. Vadnais (1980) Operational airborne measurement of snow water equivalent using natural terrestrial gamma radiation. Proceedings of the Western Snow Conference, Laramie, Wyoming, 15-17 April 1980.
- Jones, W. K. and T. R. Carroll (1982) Airborne soil moisture measurement using natural terrestrial gamma radiation - Part 2. Agricultural Meteorology (in press).
- Peck, E. L., T. R. Carroll, and S. C. Van Demark (1980) Operational aerial snow surveying in the United States. Hydrological Sciences Bulletin 25(1):51-62.