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# INTRODUCTION

In his critique of methods for measuring liquid water in snow Colbeck (1978) resurrected a paper by Bader (1948) on alternatives to calorimetry. One of Bader's suggestions which we call the "dilution method" is especially attractive because of its speed and simplicity: an aqueous stock solution (at  $0^{\circ}$ C) containing a small (<1%) concentration of dissolved impurity is mixed thoroughly into a wet snow sample taking every possible precaution to maintain the mixture at  $0^{\circ}$ C. The mixture is decanted from the sample; the liquid water originally in the sample is computed from the impurity concentration ratio (mixture to stock). Bader proposed using a  $\sim 1/2\%$  maltose stock solution and measuring the concentration ratio with a polarimeter. He did not have a chance for experiments, but believed the dilution method should be highly accurate.

## DILUTION METHOD

Assume a 0°C stock solution of mass S and impurity concentration  $C_s$  is mixed thoroughly into a wet snow sample.  $C_s$  is small enough that the melting point depression of the mixture is small, but large enough to be well above the impurity concentration  $C_w$  of the liquid water in the sample. If the decanted mixture has an impurity concentration  $C_m$ , then the liquid W that was in the sample is computed from

$$(S + W)C_m = SC_S + WC_W$$
 (1)

which can be rearranged as

$$W = S \left( 1 - \frac{C_m}{C_s} \right) / \left( \frac{C_m}{C_s} - \frac{C_w}{C_s} \right)$$
 (2)

Note that  $C_s$ ,  $C_m$ , and  $C_w$  need not be determined since only ratios are required; this greatly simplifies the analytical chemistry. An accurate determination of  $C_m/C_s$  is crucial. The ratio  $C_w/C_s$  can be set very small (<<1.0) by selecting a suitable impurity.

## STOCK-MASS RATIO

If we divide both sides of (2) by the sample mass M, we obtain

$$\frac{W}{M} = \frac{S}{M} \left( 1 - \frac{C_m}{C_s} \right) / \left( \frac{C_m}{C_s} - \frac{C_w}{C_s} \right)$$
 (3)

where W/M is the liquid water per unit mass, usually expressed as a percentage 100%W/M, and typically in the range 0% to 30%. Note from (3) that increasing the ratio of stock solution to mass (S/M) in a measurement of fixed W/M drives  $C_m/C_s$  closer to unity, and hence increases the instrumentation accuracy needed to find W/M to its level of accuracy. Also the greater S/M, the greater the probability of phase change error during mixing since it is difficult to maintain the stock solution at precisely 0  $^{\circ}$ C. Thus, it seems advantageous to keep S/M as small as possible. However, the larger S/M, the greater the probability that the stock solution will mix homogeneously with the sample liquid water, and if S/M is too small it is difficult to mix and decant.

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The optimum range of S/M probably depends on snow structure -- high surface area structures such as newly fallen snow probably require relatively high S/M. Using coarse grained snow, we found it difficult to mix and decant with S/M < 1.0, and obtained best results with 1.0 < S/M < 2.0.

If S/M = 2.0 then  $C_m/C_s$  must be measured to accuracy about  $\pm 0.01$  if W/M is to be accurate to about  $\pm 0.02$  (i.e. 2g of liquid water in 100g of sample mass). If S/M = 1.0, a measurement of  $C_m/C_s$  to about  $\pm 0.01$  gives W/M to about  $\pm 0.01$ .

## INSTRUMENTATION POSSIBILITIES

 $C_m/C_s$  can measured using polarimetry on optically active solutions, as suggested by Bader; using spectrophotometry or fluorometry on dyes, as suggested in a recent paper by Davis and Dozier (in preparation); using differential refractometry; using interferometry; etc. With expensive and specialized instrumentation in a careful laboratory environment,  $C_m/C_s$  can be measured to accuracy  $\pm 0.001$  or better. However, the handling problem of mixing at precisely 0°C, and the problem of incomplete mixing limit the overall accuracy (to perhaps  $\pm 0.01$  or  $\pm 0.02$ ), irrespective of instrument sophistication.

# ELECTROLYTIC CONDUCTIVITY

We selected the method of electrolytic conductivity to measure  $C_m/C_s$  and  $C_w/C_s$ . Digital conductance meters are available for 1/10th the cost of the above mentioned optical instruments with comparative resolution and repeatability. (We obtained poor results in terms of resolution and repeatability using low cost spectrophotometers and fluorometers). Moreover, a conductance measurement is relatively fast since the operator simply dips a cell into the mixture, whereas in most optical measurements one must carefully transfer the mixture into small tubes or cuvettes, and sacrifice the advantage of averaging over a large volume of the mixture in a single reading.

Our stock solution was 0.01 N HCl ,which has a specific conductance of ~4.1 m  $\Omega^{-1}$ /cm at 25 °C, and a meltpoint depression of < 0.05 °C. The snowmelt of our samples had specific conductance < 25  $\mu$   $\Omega^{-1}$ /cm, hence  $C_{\mathbf{w}}/C_{\mathbf{s}}$  was less than 0.01.

We corrected for the intrinsic nonlinear relationship between concentration and conductance using

$$\left(C_{m}/C_{s}\right) = \left(G_{m}/G_{s}\right)^{q} \tag{4}$$

where specific conductance of mixture and stock solutions are respectively  $G_m$  and  $G_s$ , and q is an empirical constant. For HCl in the concentration range  $0.005\,N$  to  $0.01\,N$ , interpolation from tables of equivalent conductance at  $T=25\,^{\circ}\text{C}$  (the "compensation temperature" of our meter) gave q=1.0133.

In preliminary dilution tests using water and stock solution (no snow), we found that  $C_m/C_s$  could be measured with a resolution of  $\pm 0.01$  maximum error band.

# SAMPLE PREPARATION

Dry snow samples were collected from 3 - 4 month old, coarse grained layers which were located 10 - 50 cm above soil in a 150 cm deep mountain snowpack in the Canadian Rockies. The snow was compacted into cylinders to densities in the range  $400 - 500 \, \text{kg/m}^3$ . At time of collection, snow temperatures were in the range  $-2\,^{\circ}\text{C}$  to  $-5\,^{\circ}\text{C}$ . The cylinders were transported in insulated boxes (filled with precooled snow) to a refrigerated laboratory, and then stored for several hours at  $0\,^{\circ}\text{C} \pm 1\,^{\circ}\text{C}$  prior to testing. The mass of the "dry" snow sample  $M_d$  was determined to  $\pm 0.1 \, \text{g}$ . This measure was used to select the amount ( $W_a$ ) of distilled water to be added to the sample, and the amount (S) of  $0.01 \, \text{N}$  HCl stock solution.  $W_a$  and S were weighed to  $\pm 0.1 \, \text{g}$  in polyethylene bottles, which were then stored in  $0\,^{\circ}\text{C}$  ice baths in the refrigerated laboratory.

A total of 172 samples were prepared in 17 runs performed from 28 January to 19 March 84. The left-hand columns of Table 1 list the number of samples per run N, the average dry snow mass  $\overline{M}_d$  in the run, the average stock solution-dry snow mass ratio  $\overline{S/M}_d$ , and the range of added water  $W_a$ .

Table 1. Sample data and errors.

	Samples	Mean dry	Mean stock to	Water added	Water measured	Error per 100 g of dry sample mass 2/		
Run	in run N	$\frac{\text{snow mass}}{M_{d}} \frac{1}{4}$	$\frac{\text{mass ratio}}{\text{S/M}_{d}}$	to samples $\frac{W}{a} \frac{1}{a}$	in samples W <sub>m</sub> <u>1</u> /	Mean of absolute 3/	Mean of algebraic 4/	Worst case 5/
		(g)	(g/g)	(g)	(g)	(g/100 g)	(g/100 g)	(g/100 g)
1	15	88	2,28	0 to 45	1 to 43	2.65	+2.65	+8.74
2 .	15	105	0.95	0 to 45	1 to 35	3.89	-3.08	-12.76
3	15	117	1,51	0 to 47	4 to 36	3.95	+0.16	-9.71
4	15	108	1.75	0 to 47	2 to 44	2.03	-0.88	-5.37
5	15	112	2.01	0 to 46	4 to 47	2.21	+0.83	-4,22
6	15	112	1.74	0 to 43	-1 to 43	1.67	-1.41	-5.23
7	15	111	1.76	0 to 42	-1 to 39	1.25	-0.86	-3.12
8	15	109	1.74	0 to 43	1 to 43	0.65	-0.03	-2.29
8	7	474	1.75	0 to 152	4 to 165	2.53	+2.53	+3.51
10	7	451	1.49	0 to 149	5 to 158	1.16	+1.16	+1.98
11	7	440	1.26	0 to 149	1 to 155	0.88	+0.88	+1.53
12	7	445	1.26	0 to 150	1 to 158	0.81	+0.81	+1.69
13	6	887	1.25	0 to 256	8 to 273	1.65	+1.65	+1.85
14	6	863	1.26	0 to 252	3 to 214	2.18	-1.60	-4.54
15	4	1469	1.25	272 to 310	296 to 318	1.13	+0.89	+1.77
16	4	1920	1.24	369 to 415	360 to 412	0.59	-0.02	-1.06
17	4	1824	1.26	350 to 371	388 to 404	1.93	+1.93	+2.53

 $<sup>\</sup>underline{1}/$  Measurements were to nearest 0.1 g

$$\underline{2}$$
/ Error = E<sub>i</sub> = 100  $\left[ W_{m}^{(i)} - W_{a}^{(i)} \right] / M_{d}^{(i)}$ 

$$3$$
/ Mean of absolute errors =  $\left[\sum_{i=1}^{N} |E_i|\right]/N$ 

$$\underline{4}$$
/ Mean of algebraic errors =  $\left[\sum_{i=1}^{N} E_{i}\right]/N$ 

5/ Worst of N sample errors

#### TEST PROCEDURES

The procedure for runs 1 - 8, which were restricted to relatively small  $M_d(\sim 100~g)$  was to place each dry snow sample in a stainless steel beaker surrounded by a 0 °C ice bath in a Dewar flask. The snow was disaggregated and stirred within the beaker to raise the snow temperature to 0 °C from the initial storage temperature < 0 °C. The distilled  $W_a$  was poured onto the snow, and as quickly as possible (to minimize phase change) the stock solution S was added to the wet snow  $(M_d + W_a)$  in the beaker. The mixture was stirred vigorously for at least one minute, then decanted into polyethylene bottles through coarse filter paper which withheld ice particles that floated in the mixture.

 $\rm M_d$  was increased in runs 9 - 17 where mixing and sometimes vigorous shaking was performed in a variety of containers at the ambient laboratory temperature (0 °C  $\pm$  1 °C) without taking precaution to use an ice bath around the container.

In preliminary tests, we found accuracy improved if all the available mixture was decanted. The decanted mixture was stored for several hours until reaching room temperature, close to the compensation temperature (25 °C) of our conductance meter. In order to further minimize the conductance variation with temperature,  $G_{\rm s}$  and  $G_{\rm m}$  were measured in rapid sequence:

 $G_s^{(1)}$ ,  $G_m^{(1)}$ ,  $G_s^{(2)}$ ,  $G_m^{(2)}$ , ...etc.

The ratio  $G_{\mathbf{w}}/G_{\mathbf{s}}$  was about 1/400, but could be a more significant factor depending on the ionic impurities in the snowmelt.

# RESULTS

Errors for each run are shown in the right-hand columns of Table 1. The worst results occurred in the first three runs, and are probably due to inexperience with the method, -- the selections of S/M were not optimum, the mixing was probably too timid and incomplete, and full volume of the mixture was not decanted. The method seems to require a vigorous mixing and/or shaking for over a minute, although we cannot say when mixing is too prolonged in connection with phase change errors. All 172 sample points are plotted in Figure 1.

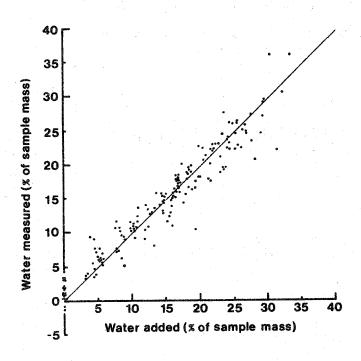


Figure 1. Dilution method results, 172 samples.

The ordinate represents the "measured water per unit mass", computed as

$$Y_i = 100\% w_m^{(i)} / (M_d^{(i)} + w_a^{(i)})$$
 (5)

and the abcissa represents the "added water per unit mass", computed as

$$X_i = 100\% W_a^{(i)} / (M_d^{(i)} + W_a^{(i)})$$
 (6)

If we define the "algebraic error" as  $Y_i$  -  $X_i$  and the absolute error as  $|Y_i - X_i|$ , then for all 172 points:

•	Mean algebraic error	+0.1%
•	Mean absolute error	1.7%
•	Standard deviation of error	2.3%
•	Worst positive error	+6.1%
0	Worst negative error	-8.8%

These errors are somewhat less than errors shown in Table 1 because "per unit mass" is the total wet sample mass  $(M_d + W_a)$ , whereas in Table 1, errors are computed per 100 g of dry sample mass.

# COMPARISON WITH CALORIMETRY

Calorimetry may provide more accurate measurements, at least at small sample size and in a laboratory setting. Akitaya (1978) has achieved an average absolute error of about 1% using hot water calorimetry on 20 g to 50 g samples (Figure 2). According to Jones et al (1983), the error band of freezing calorimetry on 200 g samples is within  $\pm 2\%$ .

On the other hand, calorimetry error may increase with sample size, whereas dilution method errors do not appear to increase with increasing sample mass in the range  $100\,\mathrm{g}$  to  $2000\,\mathrm{g}$ . Moreover, the dilution method is much quicker and simple to use in the field.

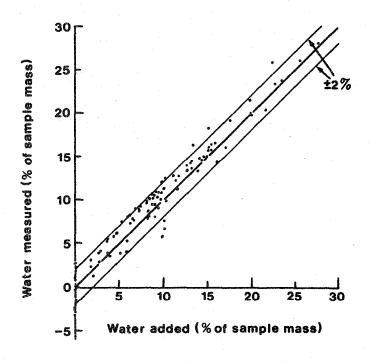


Figure 2. Akitaya's (1978) results using hot water calorimetry.

#### COMMENT

We have experimented with the dilution method, and although we cannot support Bader's claim for high accuracy, we found the method to be quick, simple, and applicable to a wide range of sample sizes. It should be given serious consideration as an alternative to freezing calorimetry for calibrating large scale dielectric measurements.

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## REFERENCES

- Akitaya, E., 1978: Measurements of free water content of wet snow by calorimetric method, Low Temperature Science, Ser. A, 36, pp. 103-111.
- Bader, H., 1948: Theory of non-calorimetric methods for the determination of the liquid water content of wet snow, Schweizerische Mineralogische und Petrographische Mitteilungen, Bd. 28, Ht. 2, pp. 355-361.
- Colbeck, S.C., (1978): The difficulties of measuring the water saturation and porosity of snow, <u>Journal of Glaciology</u>, Vol. 20, No. 82, pp. 189-201.
- Davis, R. E. and J. Dozier, (in preparation): Snow wetness measurement by fluorescent dye dilution. J. Glaciology.
- Jones, E.B., A. Rango, and S.M. Howell, 1983: Snowpack liquid water determinations using freezing calorimetry, Nordic Hydrology, pp. 113-126.