A COMPARISON OF MELT RATE METHODS USED IN FORECASTING RUNOFF FROM AN ALBERTA MOUNTAIN ENVIRONMENT

by

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INTRODUCTION

Many different methods to predict snowmelt have been developed and tested over the years. These methods vary considerably in both complexity and capability. There is no universal method of forecasting snowmelt which can be applied with certainty to any one particular watershed. Depending on basin characteristics, what works well on one watershed, might perform terribly on another.

Three methods used to determine melt rate have gained prominence: the physically based energy budget approach, the simple degree day method, and the use of snow pillows.

Melt rate estimates are an important input reqiurement for simulating snowmelt runoff using the SSARR model. This paper addresses the question of which melt rate method provides the best input to the SSARR model for a small forested watershed in southwestern Alberta.

STUDY AREA AND INSTRUMENTATION

Marmot Creek experimental basin is located in the Rocky Mountains 80 km west of Calgary, Alberta. Marmot Creek covers an area of 9.4 km^2 and ranges in elevation from 1590 to 2800 metres. The average slope of the basin is 39% and it has an easterly aspect.

Average annual precipitation varies from 65 cm at the 1600 meter contour to about 100 cm at the 2400 meter contour, and 70 to 75 percent of the annual precipitation falls as snow (Storr, 1967). Peak stream discharge on Marmot Creek normally occurs in late May or June as a result of snowmelt. The maximum instantaneous discharge on record is 2.39 m^3/s on June 6, 1971. The tree line at Marmot Creek varies between 2135 to 2285 metres. Forest covered approximately 65% of the basin area (before 8% of one sub-basin was logged). Approximately 14% of the basin at higher elevations is bare rock and talus slopes.

The data network utilized in the study consists of two meteorological stations, five snow pillows, one radiation sensor and the gauging station on the main stem of Marmot Creek. The components of the network, their location and elevations are illustrated in Figures 1 and 2.

MELT RATE APPROACHES

1) Energy Budget Approach

A daily melt rate was calculated by dividing daily melt as determined by the U.S. Army Corps Engineers (1956) equation for partly forested areas by the number of positive degree days on the same day. Temperatures used in the computation of melt and melt rate were those of the middle elevation of the basin. Calculated in a manner

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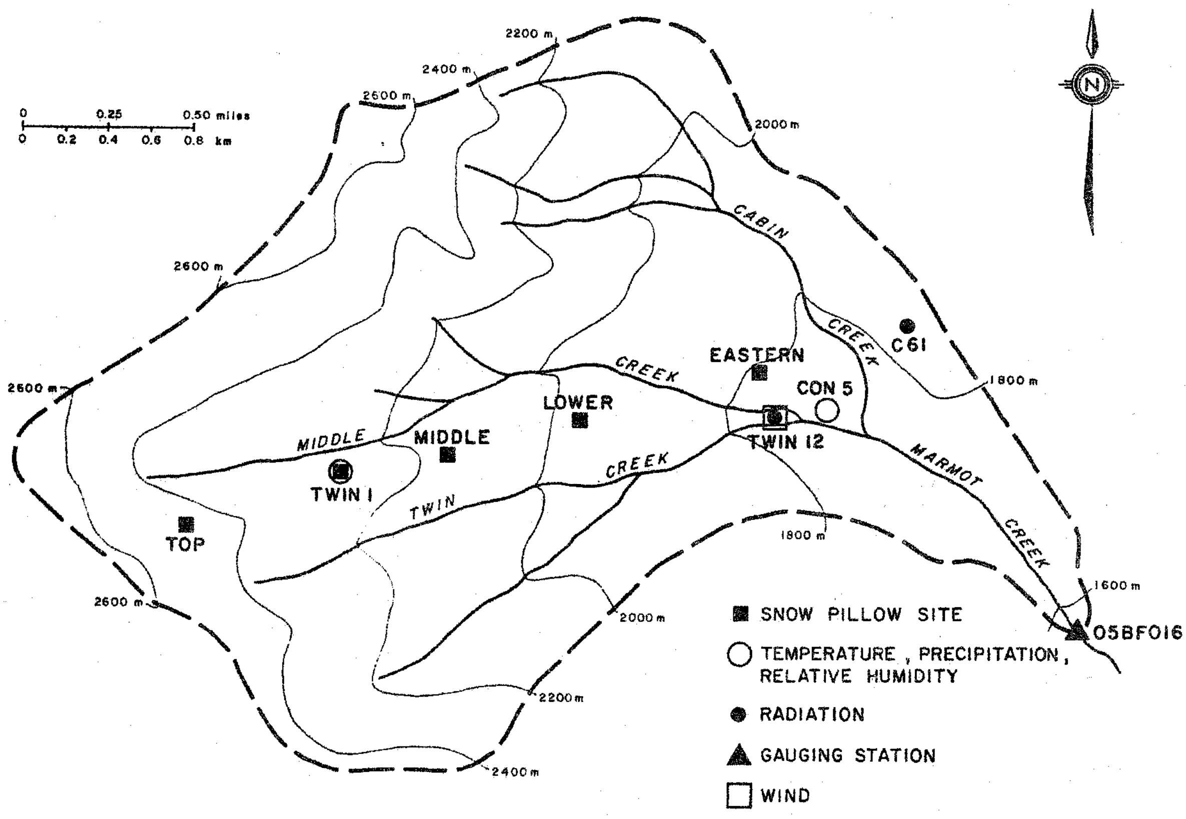

FIGURE 1. INSTRUMENTATION NETWORK AT MARMOT CREEK

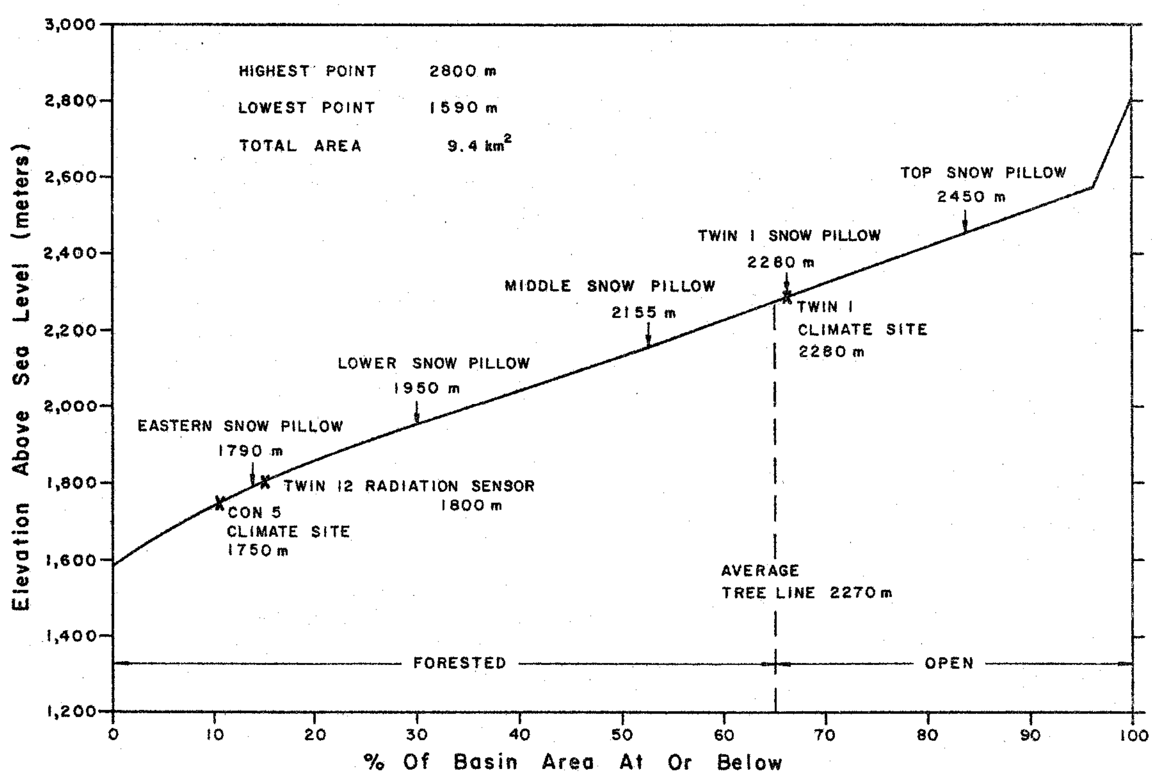

FIGURE 2. AREA - ELEVATION RELATIONSHIP, SNOW PILLOW AND CLIMATE SITE ELEVATIONS AT MARMOT CREEK

similar to the technique used by Storr (1977), the equation is applied as follows:

$$M = k' (1-F) (24.271 K) (1 - a) + F (1.32 T_a - 0) + k (.133 v) ((.396 T_a - 0) + (1.404 T_d - 0))$$

where:

M = daily melt in mm.

k' = shortwave radiation melt factor dependent on average slope and aspect of the basin in comparison with an unshielded horizontal surface (using data compiled by Ferguson et-al (1971), and techniques outline by Storr (1972), k' varied from 1.26 early in the melt season to 1.10 later in the melt season).

F = the average forest canopy cover effective in shading the site from solar radiation (values of .65 and .58 after logging assumed).

K = incoming shortwave radiation (Kjoules/cm^2 at Twin 12).

a = average snow surface albedo, U.S. Corps of Engineers (1956) curve for albedo reduction versus time applied.

T_a = air temperature (°C) (temperature values from the middle elevation of the basin used).

k = convection - condensation melt factor which depends on exposure to the wind (.5 assumed).

v = mean wind speed (measured at Twin 12 in km/hr and adjusted to a height of 15 m).

T_d = dew point temperature (°C) (determined from temperature and humidity measurements at Twin 12 which were applied to psychrometric tables).

2) Snow Pillows

Data from five snow pillows are utilized. As illustrated in Figure 2, the pillows are situated at 1790 m (Eastern pillow), 1950 m (Lower pillow), 2155 m (Middle pillow), 2280 m (Twin 1 pillow) and 2450 m (Top pillow).

Although having a similar aspect, each of the pillow sites is unique in terms of elevation, forest cover and snow accumulation. The Eastern pillow is in a partially open site. Both the Lower and Middle pillow sites are in a heavily treed area. The Twin 1 pillow is in a partially open site and the Top pillow is completely in the open, being situated well above the tree line.

The Eastern snow pillow is 3.0 metres in width and mounted flush with the ground. All the other pillows are 2.4 meters in width and placed on wooden platforms raised slightly above the ground.

A daily melt rate in mm/degree day C was calculated at each of the snow pillow sites. Daily melt was determined from decreases in water equivalence on the snow pillow charts (midnight readings). The degree day value for each day during the melt season was calculated using temperature data on a 6 hour interval for years with 6 hourly data, and on a daily basis for years with only maximum and minimum temperatures available.

An average melt rate from the snow covered area was also determined simply by averaging the melt rate of those pillows which still had snow on them. Knowledge of the areal extent of the snow cover is roughly provided by the five snow pillows located at various elevations.

3) <u>Accumulated Degree Day</u>

At each of the snow pillow sites on Marmot Creek, accumulated melt values at 20 mm increments were extracted along with the corresponding accumulated degree day values. These accumulated degree day values were then averaged and plotted against melt. The result, along with the exponential equation which best expresses the relationship is provided in Figure 3.

This one regression equation suggests that accumulated degree day values can be used to adequately estimate the amount of accumulated melt which takes place at any one of the snow pillow sites at Marmot Creek, regardless of the elevation or exposure of that site. For example, if 200 degree days have been accumulated at the Eastern pillow site (1790 m) one could expect roughly 460 mm of snowmelt to have accompanied this. Cooler temperatures at higher elevation may have provided an accumulation of only 100 degree days over the same period at the Twin 1 (2280 m) pillow site. With this one could expect roughly 170 mm of melt.

Daily melt is calculated as the difference in accumulated melt (as determined by the regression equation) from day to day. A daily melt rate is calculated by dividing the daily melt by the number of degree days on the same day. In the Marmot Creek study, temperatures used in the computation of melt and melt rate where those of the middle elevation of the basin. This was done since the temperature at the middle elevation is best representative of a "basin" temperature.

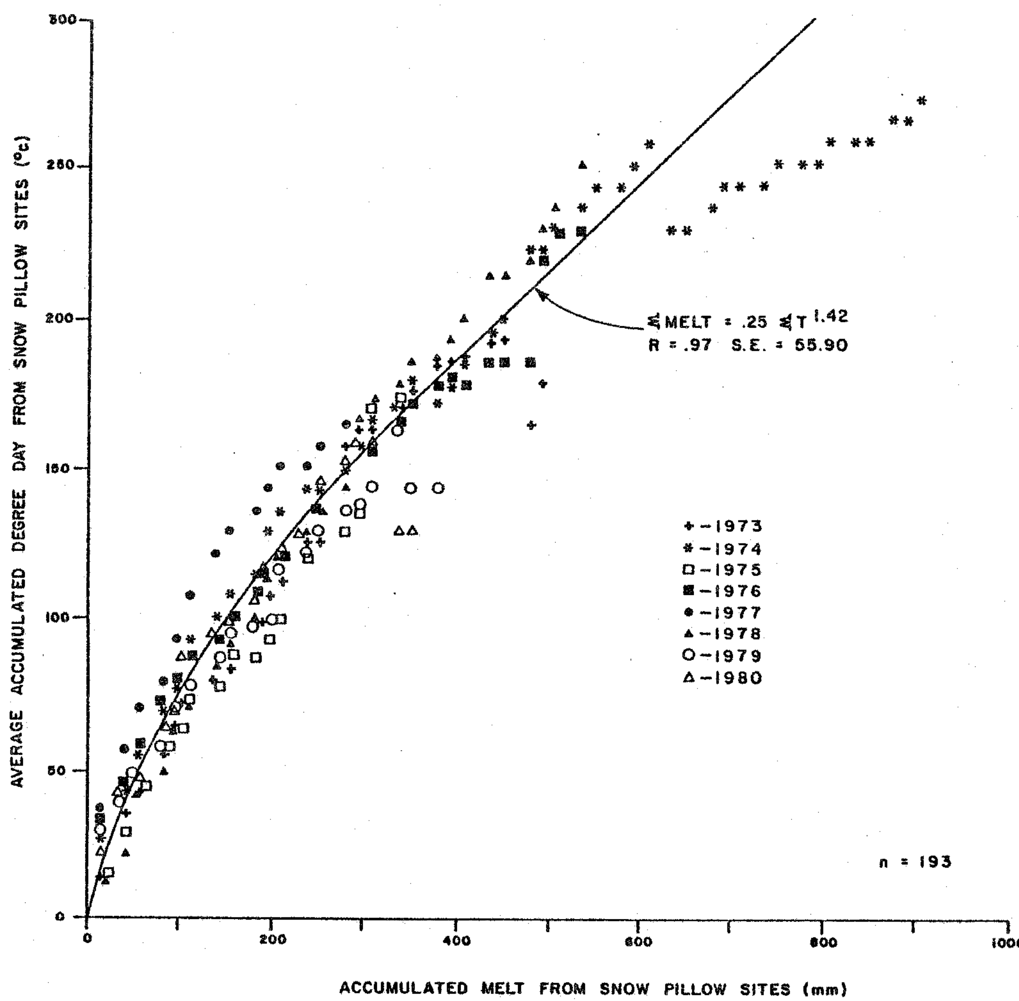

FIGURE 3. RELATIONSHIP BETWEEN ACCUMULATED MELT AND AVERAGE ACCUMULATED DEGREE DAY AT FIVE PILLOW SITES, MARMOT CREEK, 1973 - 1980.

4) Antecedent Temperature Index

The last melt rate technique examined in this study is the antecedent temperature index (ATI). This technique attempts to account for the effects of recent weather, as reflected by mean daily temperature, on the current rate of melt (i.e., yesterday's value effects today's value). With this approach a daily melt rate is calculated by:

$$M_N = k'(T_{AN} - T_B + k(ATI_{N-1}))\ (SAF_M)$$

where:

M_N = melt rate today (mm/degree day C)

T_{AN} = mean daily air temperature today (°C)

T_B = base temperature (0°C)

k = a constant (.5 was used)

ATI_{N-1} = ATI value for yesterday

$(T_{AN} - T_B + k(ATI_{N-1}))$ = ATI value for today

(SAF_M) = monthly seasonal adjustment factor to account for seasonal changes in the angle of incidence of solar radiation (April = .71, May = .92, June = 1.00)

k' = constant to adjust ATI values to a melt rate in mm/degree day C for input into SSARR (0.1 was used)

The ATI procedure has been used by the River Forecast Centre of Alberta Environment in recent years to provide an index of daily melt for areas without snow pillows. ATI melt rates using Con 5 (1750 m) mean daily temperatures were used in this study for the calibration of SSARR.

PROCEDURE

The parameters of the SSARR model were calibrated with ATI (Con 5) melt rates. The initial snow volume input to SSARR each year was a basin average determined from a network of 12 snow courses. Soil moisture index values were proportionally set to the streamflow discharge at the start of the simulation period using the relationship developed by Storr (1974). The melt rate determined by the various techniques was the only input parameter changed in the model which was run for the April to June period, 1973 - 1980.

The extent to which a computer model can best replicate snowmelt runoff by the alteration of only one input parameter is evaluated. Each of the melt rate techniques outlined above were applied to a split watershed version of the SSARR model ((U.S. Army Corps of Engineers (1972), modified by Kuhnke & Nguyen (1977)) in order to determine which provides the best forecasting result.

RESULTS AND DISCUSSION

The best forecasting result is determined by comparisons of the standard error of the computed hydrograph to the recorded hydrograph, direct comparisons of the computed snow volume to that measured at snow courses, and direct comparisons of the computed flood peak to the timing and magnitude of the recorded annual flood peak. These are used as measures to evaluate the optimum melt rate approach used in SSARR.

The standard error of the computed hydrograph is determined by subtracting the computed flow from the observed flow. This difference is then squared, summed, and divided by the total number of observations (Chu and Bowers, 1978). It is calculated on a six hourly basis, the computation interval used in SSARR.

The computation of the standard error begins when the observed flow on Marmot Creek begins to show on appreciable rise. Although the timing is variable from year to year, this is normally in the month of May. To make results from the index methods directly comparable to those of snow pillow data, the standard error calculation ends when the last pillow on the basin is out of snow.

A comparison of the standard error of the computed hydrograph for each of the melt rate techniques is provided in Table 1.

The average melt rate from all 5 snow pillows provides the lowest average standard error, and using this as the criteria provides the best result. This is followed in order of decreasing accuracy by the use of data from the Middle snow pillow, the accumulated degree day regression equation, and the ATI (Con 5) approach. The energy budget approach does not provide as good a result, but provides a better result than snow pillows sited at a lower elevation on the basin.

The basin estimate of snow volume from snow course measurements provides the standard to which the snow volume registered in SSARR is compared. Even for a basin only 9 km^2 this exercise is rather tenuous, due to the limited accuracy of basin estimates of snow volume. In any event, the comparison is made to illustrate how well SSARR responds to depletions or additions of snow volume over time and how it compares to a basin estimate of snow volume.

The average snow volume for the basin, the date of the snow course measurements, and the snow volume registered in SSARR on same date is plotted in Figure 4. It is evident that the melt rate techniques show different snow volume depletion trends. The ATI (Con 5) approach tends to register a higher snow volume in SSARR than any other melt rate approach examined, and in all years, it registers a snow volume greater than that reported from the snow course measurements.

Two properties of the recorded annual flood peak are examined and compared to the SSARR computed hydrograph. In Table 2, the difference in the timing of the recorded maximum instantaneous annual flood peak to the computed annual flood peak as determined by SSARR is provided.

Based on the number of cases in which the computed peak is within plus or minus one day of the recorded peak, 4 of the melt rate approaches provide a comparable result. The use of an average melt rate from 5 snow pillows, the middle pillow, the top pillow or the ATI (Con 5) approach provide a computed flood peak within plus or minus one day of the recorded peak in 5 of the 8 years modelled. Using the average number of days difference from the recorded annual flood peak as an additional criterion of melt rate performance, the use of an average melt rate as determined from 5 snow pillows provides the best result. Over the 8 year period examined, this technique has the lowest average deviation in the timing of peak runoff.

Another trend is evident from Table 2. With an overestimation of the melt rate early in the season, the computed annual flood peak will occur much earlier than the recorded annual flood peak. Melt rate approaches which tend to overestimate melt early in the season are characterized by a large negative difference (an early flood peak). The eastern pillow, lower pillow, and in certain years the energy budget approach provide a good example of this occurrence.

In contrast, the ATI approach is characterized by a consistent positive difference (a delayed peak). The ATI melt rate approach has a tendency to generate a late computed flood peak.

In Table 3, the percentage error of the magnitude of the recorded maximum instantaneous annual flood peak to the computed discharge as determined by SSARR at approximately the same time is provided. The accumulated degree day approach provides the best result. As shown in Table 3, four of the eight years modelled using this approach have a computed discharge within 20% accuracy of the recorded annual flood peak. In addition, the average error in the magnitude of the computed peak over the eight year period is the lowest of any method at 20%.

TABLE 1. A COMPARISON OF THE STANDARD ERROR (m^3/s) BETWEEN THE USE OF SNOW PILLOW MELT RATES AND MELT RATES AS DETERMINED BY INDEX METHODS FOR APPLICATION TO SSARR. COMPARISONS ARE BASED ON TIME PERIOD FOR WHICH SNOW WAS STILL ONTHE SNOW PILLOWS AND A MELT RATE COULD BE CALCULATED. THE STANDARD ERROR IS DETERMINED BY SUBTRACTING COMPUTED FLOW FROM OBSERVED FLOW. THIS DIFFERENCE IS THEN SQUARED, SUMMED, AND DIVIDED BY THE TOTAL NUMBER OF OBSERVATIONS. IT IS CALCULATED ON A 6 HOURLY BASIS, THE COMPUTATION INTERVAL USED IN SSARR.

		STANDARD ERROR OF THE COMPUTED HYDROGRAPH (m^3/s)								
YEAR	DATE	EAST PILLOW	LOWER PILLOW	MIDDLE PILLOW	TWIN 1 PILLOW	TOP PILLOW	MEAN OF 5	ATI CON 5	ENERGY BUDGET	ACCUM DEGREE DAY
1973	May 11-June 21	6.14	3.99	3.28	3.72	N/A	2.88	3.61	1.79	1.72
1974	May 22-June 24	21.91	14.85	1.77	4.67	N/A	1.32	3.04	5.68	3.58
1975	May 9-June 20	2.59	1.56	.56	2.15	1.41	0.89	0.46	1.84	0.67
1976	May 1-June 17	3.89	2.04	1.10	1.93	2.35	1.11	2.18	1.41	1.33
1977	May 1-June 5	0.78	0.85	.53	0.58	.66	0.28	0.42	0.49	0.53
1978	May 8-June 28	5.22	N/A	1.91	2.07	2.03	1.86	1.63	3.77	2.93
1979	May 11-June 15	2.89	1.94	0.80	2.30	2.26	0.84	1.00	1.81	1.18
1980	May 1-May 18	1.05	1.15	0.21	0.65	1.29	0.40	0.22	0.27	0.36
MEAN		5.56	N/A	1.27	2.26	N/A	1.20	1.57	2.13	1.54

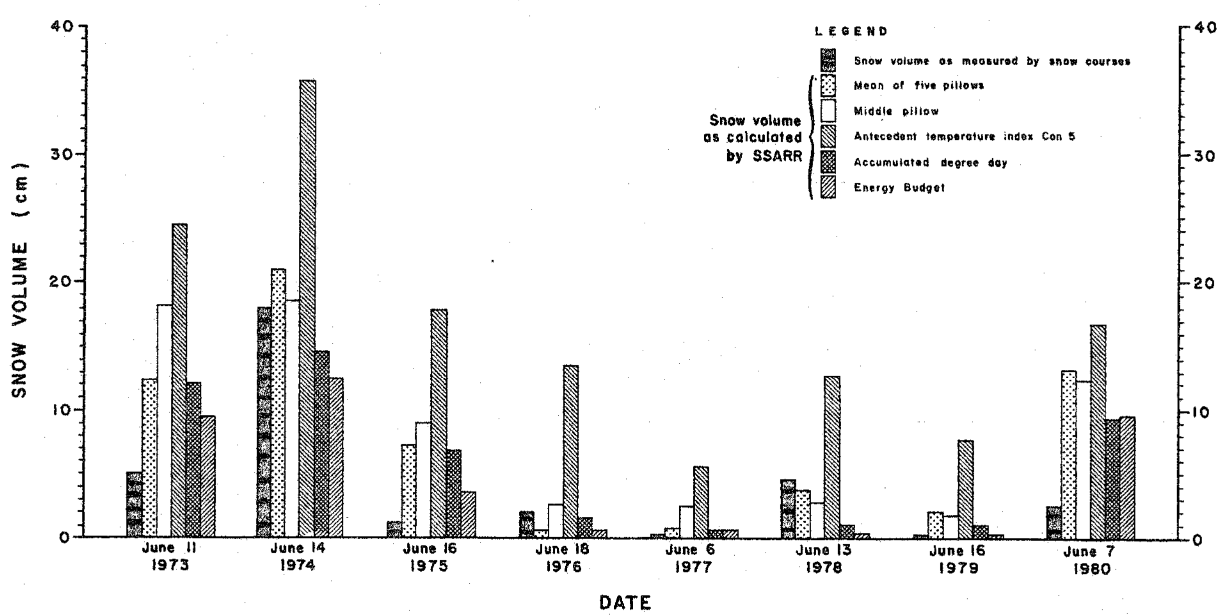

Figure 4. Comparison of snow volume as measured by snow courses and snow volume as calculated by SSARR for Marmot Creek in June 1973 - 1980.

TABLE 2. DIFFERENCE IN THE TIMING OF THE ANNUAL RECORDED MAXIMUM INSTANTANEOUS FLOOD PEAK (DAYS) AND THE TIMING OF THE COMPUTED ANNUAL FLOOD PEAK AS DETERMINED BY SSARR FOR MARMOT CREEK, 1973 - 1980. THE TIMING OF THE COMPUTED PEAK DISCHARGE WAS DETERMINED BY THE USE OF SNOW PILLOW MELT RATES AND MELT RATES AS DETERMINED BY INDEX METHODS FOR APPLICATION TO THE SSARR MODEL.

			DIFFERENCE (DAYS)								
YEAR	TIME/DATE	RECORDED ANNUAL FLOOD PEAK (m^3/s)	EAST	LOWER	MIDDLE	TWIN 1	TOP	MEAN OF 5	ATI CON 5	ENERGY BUDGET	ACCUM DEGREE DAY
1973	1910 June 23	1.05	- 38	- 23	- 1	- 17	N/A	- 1	+ 1	- 5	- 5
1974	1710 June 17	2.14	- 20	- 3	- 1	- 4	N/A	- 2	0	- 1	- 2
1975	1800 June 24	0.74	- 40	- 22	- 8	- 22	- 20	- 21	0	- 40	- 21
1976	1600 May 27	0.66	- 19	- 13	- 13	- 16	0	- 0	+ 23	- 16	- 10
1977	2200 June 1	0.54	- 17	- 17	0	20	0	0	+ 7	- 25	- 23
1978	1800 June 5	1.17	- 19	N/A	+ 1	+ 1	+ 1	+ 1	+ 1	- 1	- 1
1979	1700 May 26	0.77	- 11	0	+ 18	- 2	0	+ 9	+ 18	- 2	+ 7
1980	1035 June 11	0.92	- 9	- 9	- 1	- 1	- 1	- 1	0	- 1	- 1
AVERAGE			21.6	N/A	5.4	10.4	N/A	4.4	6.2	11.4	8.7
NUMBER OF CASES ± 1 day			0	1	5	2	5	5	5	3	2

TABLE 3. % ERROR OF THE MAGNITUDE OF THE ANNUAL RECORDED MAXIMUM INSTANTANEOUS FLOOD PEAK (m^3/s) TO THE MAGNITUDE OF COMPUTED DISCHARGE (m^3/s) AT APPROXIMATELY THE SAME TIME AS DETERMINED BY SSARR FOR MARMOT CREEK, 1973 - 1980. COMPUTED DISCHARGE WAS DETERMINED BY THE USE OF SNOW PILLOW MELT RATES AND MELT RATES AS DETERMINED BY INDEX METHODS FOR APPLICATION TO THE SSARR MODEL.

			% ERROR								
YEAR	TIME/DATE	RECORDED ANNUAL FLOOD PEAK (m^3/s)	EAST	LOWER	MIDDLE	TWIN 1	TOP	MEAN OF 5	ATI CON 5	ENERGY BUDGET	ACCUM DEGREE DAY
1973	1910 June 23	1.05	86%	86%	59%	72%	N/A	68%	14%	23%	47%
1974	1710 June 17	2.14	91%	85%	31%	2%	N/A	14%	31%	47%	27%
1975	1800 June 24	0.74	85%	85%	76%	78%	78%	76%	9%	36%	1%
1976	1600 May 27	0.66	79%	75%	26%	27%	161%	73%	57%	29%	21%
1977	2200 June 1	0.54	79%	79%	79%	74%	89%	39%	38%	43%	46%
1978	1800 June 5	1.17	82%	N/A	9%	17%	7%	6%	8%	41%	6%
1979	1700 May 26	0.77	83%	23%	12%	41%	69%	0%	38%	18%	4%
1980	1035 June 11	0.92	87%	87%	37%	15%	8%	3%	28%	20%	12%
AVERAGE			84%	N/A	41%	41%	N/A	35%	28%	32%	20%
NUMBER OF CASES ≤ 20%			0	N/A	2	3	N/A	4	3	2	4

SUMMARY OF RESULTS

The four techniques which were used as a measure of forecasting accuracy include:

i) Standard error of the computed hydrograph
ii) Snow volume depletion
iii) Timing of the annual flood peak
iv) Magnitude of the annual flood peak

Each of the melt rate approaches have been ranked on the basis of forecasting accuracy as determined by these four techniques. The results are summarized in Table 4 below.

TABLE 4. A RANKING OF THE PERFORMANCE OF EACH OF THE MELT RATE APPROACHES BASED ON CRITERIA OUTLINED IN THIS STUDY. 1 = BEST, 10 = WORST.

MELT RATE APPROACH	RANK			
	STANDARD ERROR OF THE COMPUTED HYDROGRAPH	ACCURACY OF THE SNOW VOLUME DEPLETION IN JUNE	TIMING OF ANNUAL FLOOD PEAK	MAGNITUDE OF ANNUAL FLOOD PEAK
EASTERN PILLOW	7th	-	7th	7th
LOWER PILLOW	-	-	-	-
MIDDLE PILLOW	2nd	4th	2nd	6th
TWIN 1 PILLOW	6th	-	6th	5th
TOP PILLOW	-	-	-	-
MEAN OF 5 PILLOWS	1st	3rd	1st	2nd
ATI (CON 5)	4th	5th	3rd	3rd
ENERGY BUDGET	5th	1st	5th	4th
ACCUM DEGREE DAY	3rd	2nd	4th	1st

CONCLUSIONS

1) An average melt rate from the snow covered area of the basin, as determined from five snow pillows at sites of different elevation and exposure, provides the best forecasting result.

2) If only one snow pillow is installed to provide melt rate information for application to SSARR, it is best to locate that snow pillow at the middle elevation of the basin in a heavily forested area.

3) If no snow pillows are available, or the snow pillows used to calculate a melt rate are not optimally positioned on the basin, index techniques can be used to provide a comparable forecasting result.

4) The best index technique applied to SSARR is the exponential regression equation relating accumulated melt to the average accumulated degree day value at the five snow pillow sites.

5) The ATI approach and the energy budget approach provide a better forecasting result than a poorly sited pillow, but not as good as the three techniques listed above.

6) The use of melt rates as measured from snow pillows, or a temperature index approach developed from snow pillow melt provide a better forecasting result than the energy budget approach.

7) Calibrating the SSARR model with ATI melt rates appears to have no limiting effect on results, since improvements in the forecasting result were made by the use of methods other than the ATI approach.

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