STOCHASTIC TRANSFER FUNCTION TIME-SERIES MODELING OF SUSPENDED SEDIMENT CONCENTRATIONS AS A FUNCTION OF STREAMFLOW

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BY

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INTRODUCTION

The sediment rating curve approach has been widely used for the past forty years to predict changes in sediment concentrations within stream reaches. USDA(1977) summarizes the many research efforts involved. One of the more recent applications is contained in USDA (1980). Flaxman (1975) and Rosgen (1975) have developed the technique to predict variations in sediment production caused by changes in streamflow regime following silvicultural treatment.

Typically, suspended sediment is measured concurrently with stream discharge for a wide range of flows during a water year. A simple linear regression model is then fitted to the log-transformed data. If significant differences in sediment concentration result from the rising versus falling limbs of the hydrograph, two separate regression models are fitted to account for this so-called "hysteresis effect". In practice, it is often difficult, if not impossible, to cleanly separate rising and falling limbs, particularly when regime is snow-melt dominated and there are a number of melt cycles. There is a temptation to be quite selective in the choice of data sets, missing out those observations that fall on secondary rising or falling segments. Such a procedure is often rationalized on the basis that the data should be randomized to eliminate serial correlations. Even is this is done, however, there are often other problems such as early depletion of in-channel sediment sources caused by an initial minor hydrograph peak, a forerunner of the main peak. In snow-melt dominated systems, it is easy to see that, as spring runoff proceeds, sediment concentrations, even for high flows, gradually diminish.

Examination of streamflow and suspended sediment time-series frequently reveals that they are stochastic in nature, each observation appearing to depend to a certain extent on the previous ones. Time-series models would therefore more accurately describe the data.

Two watersheds in the West Kootenays near Creston, B.C. were selected to illustrate this alternative approach to the prediction of suspended sediment concentrations as a function of streamflow. The Water Survey of Canada, Inland Waters Directorate of Environment Canada publishes annual records of streamflow and suspended sediment on selected streams and river systems throughout Canada. Use of their data on Arrow creek near Erickson (Station No. 08NH084 Lat. 49 09 32 N, Long. 116 27 04 W) and Duck creek near Wynndel (Station No. 08NH016 Lat. 49 12 10 N, Long. 116 31 56 W) in this paper is gratefully acknowledged. These two watersheds have been extensively studied by the British Columbia Ministries of Forests and Environment in the recent past and are important sources of domestic, irrigation and commercial brewing water supplies. They are situated adjacent to each other, with generally southerly aspects rising from approximately 750 m to 2150 m elevation. Drainage areas above the monitoring stations are 78.7 km^2 for Arrow and 57.0 km^2 for Duck creek. They generally receive more than 60% of their precipitation in the form of snow from late November to early April. Streamflow rises during April and early May, peaking on average in late May followed by a gradual recession during the summer and fall months. The sediment sampling methodology is described in detail in Water Quality Branch (1983).

DISCUSSION OF DATA

Daily suspended sediment concentration peaked at 59 mg/l on May 18, 1982 for Arrow creek and 63 mg/l on May 25, 1982 for Duck creek during the first of two major melt cycles that occurred in that year. Peak values of 92 mg/l on June 14, 1982 for

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Arrow and 61 mg/l on June 15, 1982 for Duck were measured during the second major melt cycle. Twelve days of data values on both sides of the first cycle peaks were abstracted from the records, for the purpose of these analyses. Each time-series therefore consists of twenty-five sequential daily values, a rather typical length of time for the transport of the major component of annual available sediment in these source-limiting creeks, during spring snowmelt. Longer series could have been chosen but the shorter series serve to illustrate more simply the differences between the following models.

SIMPLE LINEAR REGRESSION MODELS

Simple linear regressions were first fitted to the unlogged data. The results were

$$\hat{S}_t = -3.19 + 2.50\, Q_t, \qquad R^2_{1,23} = 0.428 \tag{1}$$

and

$$\hat{S}_t = -39.9 + 20.7\, Q_t, \qquad R^2_{1,23} = 0.653 \tag{2}$$

for Arrow and Duck creeks respectively, where $\hat{S}_t$ is predicted suspended sediment (mg/l), Q_t is discharge (m^3/s) and $R^2_{1,23}$ is the coefficient of determination with 1 and 23 degrees of freedom. Using $\log_{10}$-values, the results were

$$\hat{S}_t = 0.401\, Q_t^{1.68} \qquad R^2_{1,23} = 0.669 \tag{3}$$

and

$$\hat{S}_t = 1.05\, Q_t^{2.65} \qquad R^2_{1,23} = 0.758 \tag{4}$$

for Arrow and Duck creeks respectively. These increased R^2-values clearly demonstrate the widely recognized strong exponential relationship between S_t and Q_t. The data can be separated into a rising stage, i.e. the first 13 pairs of values in each case, and a falling stage, i.e. the last 12 pairs of values. The log-value regressions obtained were

$$\hat{S}_t = 0.055\, Q_t^{3.12} \qquad R^2_{1,11} = 0.841 \tag{5}$$

and

$$\hat{S}_t = 0.123\, Q_t^{2.09} \qquad R^2_{1,10} = 0.558 \tag{6}$$

for Arrow creek rising and falling stages, and

$$\hat{S}_t = 1.16\, Q_t^{2.61} \qquad R^2_{1,11} = 0.735 \tag{7}$$

and

$$\hat{S}_t = 1.03\, Q_t^{2.63} \qquad R^2_{1,10} = 0.810 \tag{8}$$

for Duck creek rising and falling stages. Equations (5) and (8) are improvements over equations (3) and (4) but only at the expense of equations (6) and (7).

TIME-SERIES MODELS

Box and Jenkins (1970) time-series analysis was applied to the two data sets for Arrow and Duck creeks to demonstrate an alternative approach. This procedure was used by Salway (1979) to predict snow avalanche activity from meteorological data for the Rogers Pass area of British Columbia. The steps are described in some detail in Salway (1979). A brief description only will be given here.

1) Sample autocorrelation functions for the logged input series Q_t and the logged output series S_t are computed, together with the cross-correlation function and examined for "stationarity". If the correlation functions decay to insignificance at or before the third lag, the series are regarded as stationary and differencing is unnecessary. This was the case with the Q_t and S_t series used in these examples. Deviations from the series means are used throughout the model development procedure for computational convenience.

2) A first order autoregressive (Markov) model was identified and least-squares estimates of the parameters obtained for each of the Q_t series. The results were

$$\text{Log } Q_t = 0.901 \text{ Log } Q_{t-1} + \alpha_t \tag{9}$$

and

$$\text{Log } Q_t = 0.777 \text{ Log } Q_{t-1} + \alpha_t \tag{10}$$

for Arrow and Duck creeks respectively. Logarithms are to the base 10 and α_t is the "white noise" series (random and uncorrelated).

3) The input and output series are then "pre-whitened" using the operators (1 - 0.901 B) for Arrow creek and (1 - 0.777 B) for Duck creek. B is the "backward-shift" operator. Prewhitening of the input series produces more efficient estimates of the transfer function (Box and Jenkins, 1970, p.379).

4) The transfer function is then identified and least-squares estimates of the parameters obtained. The results were

$$\beta_t = 1.21\ \alpha_t \tag{11}$$

and

$$\beta_t = 2.79\ \alpha_t \tag{12}$$

for Arrow and Duck creeks respectively. β_t is the transformed output series.

5) The transfer function is then subtracted from the output series to obtain the stochastic noise component

$$N_t = \text{Log } S_t - 1.21 \text{ Log } Q_t \tag{13}$$

and

$$N_t = \text{Log } S_t - 2.79 \text{ Log } Q_t \tag{14}$$

for Arrow and Duck creeks respectively. N_t is the stochastic noise series.

6) A first order autoregressive (Markov) model was identified and least-squares estimates of the parameters obtained for each of the N_t series. The results were

$$N_t = 0.767\ N_{t-1} + a_t \tag{15}$$

and

$$N_t = 0.598\ N_{t-1} + a_t \tag{16}$$

for Arrow and Duck creeks respectively. a_t is the residual error.

7) The transfer function and noise models are then combined. The result for Arrow creek was

$$\text{Log } S_t = 1.21 \text{ Log } Q_t + \frac{a_t}{(1 - 0.767\ B)} \tag{17}$$

Multiplying throughout by (1 - 0.767 B) gives

$$\text{Log } S_t = 1.21 \text{ Log } Q_t - 0.928 \text{ Log } Q_{t-1} + 0.767 \text{ Log } S_{t-1} + a_t \tag{18}$$

for deviations from the mean. Using the raw data, the final model reduces to

$$\hat{S}_t = 0.993\ Q_t^{1.21}\ Q_{t-1}^{-0.928}\ S_{t-1}^{0.767} \qquad R^2 = 0.838 \tag{19}$$

The result for Duck creek was

$$\text{Log } S_t = 2.79 \text{ Log } Q_t + \frac{a_t}{(1 - 0.598\ B)} \tag{20}$$

Multiplying throughout by (1 - 0.598 B) gives

$$\text{Log } S_t = 2.79 \text{ Log } Q_t - 1.67 \text{ Log } Q_{t-1} + 0.598 \text{ Log } S_{t-1} + a_t \tag{21}$$

Using the raw data, the final model reduces to

$$\hat{S}_t = 0.939\ Q_t^{2.79}\ Q_{t-1}^{-1.67}\ S_{t-1}^{0.598} \qquad R^2 = 0.859 \tag{22}$$

8) Tests of model adequacy are performed, as described by Box and Jenkins (1970, pp. 392 - 5). Among other things, insignificant autocorrelation in the residuals is confirmed.

R^2-values have been quoted for the time-series models without the associated degrees of freedom, since they will be slightly less than 3 and 21, due to the auto- and cross correlation properties of the series. Throughout the model development, partial

F-values were used to determine whether extra terms should be included.

CONCLUSIONS

The final time series models, equations (19) and (22) represent significant improvements over the simple regression models (3) and (4). If predicted values are compared with actual values for the two data sets, it can readily be seen that inclusion of a few extra time series terms has resulted in better predictions. These two data sets varied relatively uniformly with only minor secondary peaks, suggesting that a more typical set should be tried. Daily suspended sediment concentration peaked at 43 mg/l on May 26, 1981 for Arrow creek. Twelve days of data values on both sides of this peak were abstracted from the records. The streamflow hydrograph peaked twice during this interval and displayed some minor fluctuations. Suspended sediment concentrations exhibited lag-effects, short-term time-decay effects during relatively constant flows and longer-term depletion effects. A simple linear regression model was fitted to the logged data. The result was

$$\hat{S}_t = 0.574\, Q_t^{1.54} \qquad R^2_{1,23} = 0.418 \qquad (23)$$

The time-series approach was applied to obtain

$$\hat{S}_t = 1.27\, Q_t^{0.974}\, Q_{t-1}^{-0.685}\, S_{t-1}^{0.703} \qquad R^2 = 0.722 \qquad (24)$$

Predicted values using equation (24) are clearly better than those generated by equation (23).

REFERENCES

Box, E.P., and Jenkins, G.M., 1970: Time series analysis, forecasting and control. Wiley.

Flaxman, E.M., 1975: The use of suspended sediment load measurements and equations for evaluation of sediment yield in the west. Proc. Sediment-Yield Workshop, Oxford, Miss. U.S. Dept. Agric. Agric. Res. Serv. Rep. ARS-S-40. 285 p.

Rosgen, D.L., 1975: Preliminary report--procedures for quantifying sediment production. U.S. Dept. Agric. For. Serv., Sandpoint, Idaho.

Salway, A.A., 1979: Time-series modelling of avalanche activity from meteorological data. Journal of Glaciology, Vol. 22, No. 88, p. 513-528.

U.S.Department of Agriculture, Forest Service, 1977: Non-point water quality modeling wildlife management. A state-of-the-art assessment. EPA-IAG-D5-0660. Washington, D.C.

U.S. Department of Agriculture, Forest Service, 1980: An approach to water resources evaluation of non-point silvicultural sources. EPA-600/8-80-012, Athens, Georgia.

Water Quality Branch, 1983: Sampling for Water Quality. Environment Canada, Ottawa.