# PROBABILISTIC PREDICTION OF SNOW LOADS ON SLOPED ROOFS

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## INTRODUCTION

In many areas of the United States structural design is governed by the roof snow load. Prescribing exact design roof snow loads is not a simple matter. Ground snow loads are affected by geographic location, elevation, large-scale meteorological factors, and micro-meteorology. Roof snow loads vary because of exposure of the roof to wind and sun, thermal losses from the building, roof slope, material, and shape, obstructions on and around the roof, and various regional factors. Snow loads on sloped roofs can be reduced when the snow slides from a sufficiently steep and slippery roof.

Much of the snow load research to date has dealt with predicting roof snow loads using ground snow load data and the various factors affecting the snow load distribution on the roof. Building codes have attempted to define some of these factors and quantify them in the form of coefficients applied to the ground snow load. ANSI A58.1-1982 uses coefficients for roof exposure, building thermal effects, various statistical mean recurrence intervals (i.e., importance factors), and roof slope to determine a design snow load. Isyumov and Davenport (1974) were the first to attempt to predict flat roof snow loads by simulation using meteorological variables.

## MECHANICS OF SNOW SLIDING

Frictional forces, adhesive forces, tensile forces at the ridge, and compressive forces at the eave stabilize a snowpack against sliding on a sloped roof. A component of the weight of the snow acts parallel to the roof opposing these resisting forces. Frictional forces on the roof are affected by temperature changes and snowpack characteristics; they are much less than the adhesive forces introduced when the snow is frozen to the roof material. Tensile forces are developed from snow frozen to the ridge and snow stabilized over the ridge (Taylor, 1983). Therefore, a gable roof can retain considerably more snow than a shed roof. Water from melting snow runs down the warm surface of heated roofs and comes in contact with the cold eave where it refreezes and forms an ice dam. Ice dams can contribute to snow retention on heated roofs; ice dams typically do not occur on cold roofs.

Snow sliding is caused by a reduction in resisting forces at either the roof-snow interface or between snow layers. The presence of a lubricating layer of water on the roof surface is the greatest cause of change in frictional and adhesive forces. The lubricating layer of water is deposited by snowmelt and/or rain. According to Linsley, et al (1982) the energy for snowmelt comes from solar radiation, conduction and convection of heat from the air, condensation of water vapor on the snow, and heat supplied by rain. If the shear strength of the snow is exceeded snow slides can occur. Changes in the cohesive properties of the snow will cause shear strength of the snow to vary. According to Schaerer (1981) the most common cause of shear failure in a snowpack is added weight from precipitation.

## PREDICTIVE MODEL OF SNOW SLIDING

Snow generally slides from roofs because either frictional forces at the roof-snow interface are significantly reduced or shear failure between the snow layers occurs (Taylor, 1985; Schaerer, 1981). This seems to be borne out from our experimental data for the winters of 1983-84 and 1984-85 (see following section), which indicates that one or both of these effects probably occurred during every slide.

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Assuming that slides are caused by these two effects, we can write the following probabilistic equation:

$$P(S,t)=P(S_r,t)+P(S_s,t)-P(S_r,t)P(S_s,t) \qquad (1)$$

where $P(S,t)$= probability of sliding on any given day, $P(S_r,t)$= probability of sliding on any given day due to reduction in resistance forces, and $P(S_s,t)$= probability of sliding on any given day due to shear failure. Independence between the two causes of sliding is assumed because they exist under different meteorological conditions.

Sliding can be caused by water on the roof; therefore, a measure of snowmelt would provide an indicator of when sliding would occur due to a reduction in the coefficient of friction and other resisting forces. Predicting snowmelt, and hence sliding, can be based on a simplified approach (Linsley, et al, 1982) using an index of snowmelt such as temperature. Applying the law of total probability to a continuous case gives

$$P(S_r,t)=\int_0^\infty P(S_r,t|T'=t')P(T'=t',t)dt' \qquad (2)$$

where $P(S_r,t|T'=t')$ = probability of sliding on any day due to a reduction in resisting forces given the high temperature equals t', and $P(T'=t',t)$=probability that the temperature on any day equals t'. Several temperature variables (e.g., degree-hours, average temperature, or maximum temperature) could be used in this equation. Maximum temperature was used because snow sliding events for the experimental data closely followed the daily high temperature regardless of the minimum temperature of the previous evening. As the temperature increases, the probability that sliding will occur, given by Eq. (2), also increases.

A factor that affects the predicted snowmelt is the daily temperature variation caused by solar radiation and changing weather patterns. If temperature rises quickly from below freezing to a temperature t' above freezing and then drops rapidly, sliding may not occur. But if the temperature is maintained at t' for a long time, sliding could occur. If the temperature remains above freezing long enough, sliding will occur; thus, at any temperature above freezing there must be some probability of sliding.

Degree-hours appears to be a better indicator of sliding than temperature because time above freezing is included. The proportion of time that a given high temperature is maintained affects the probability of sliding at that temperature. We used temperature as the sliding indicator because meteorological records with daily temperature values are readily available, and standard records with calculated degree-hour values are nonexistent. Using temperature only should be re-examined if results are extrapolated to locations where daily temperature cycles vary markedly from those at our experimental site. For this preliminary model, temperature should yield acceptable trends.

Sliding due to shear failure is caused by a degradation of shear resistance (which is a function of temperature) within the snowpack or the introduction of large loads which overcome the existing shear resistance. Additional loading is introduced by precipitation, which is a good indicator of the likelihood of sliding due to shear failure. Using the effect of precipitation in the law of total probability yields

$$P(S_s,t)=\int_0^\infty P(S_s,t|R=r)P(R=r,t)dr \qquad (3)$$

where $P(S_s,t|R=r)$ = probability of sliding due to shear failure on any day that precipitation equals r, and $P(R=r,t)$ = probability that precipitation on any day equals r.

PROBABILITY OF SLIDING USING FIELD DATA

A study to obtain snow load data for cold, sloped roofs with slippery surfaces was begun in 1983-84 in McCall, Idaho. The structures monitored were six small unheated shed roofs with three slopes of 10°, 30°, and 45° and were covered with slippery metal roofing. Data were also collected for a group of unheated local buildings with either enamelled steel, galvanized steel, or aluminum roofing. These consisted of three shed and seven gable roofs with slopes ranging between 14° and 42°. Details of the experimental work are contained in Sack, et al (1985).

Field data from these experimental setups during the winters of 1983-84 and 1984-85 were used to develop the probabilities of sliding (for both degraded resisting forces at the roof surface and interlayer snow shear failure). Data were collected from the small test roofs and two independent relationships had to be developed for each roof slope. Slides were observed 19 times from the 10° roofs, 35 times from the 30° roofs and 36 times from the 45° roofs. Reasonable relationships were developed although only minimal field data existed.

A model of the form characterized by Eq. (1) was developed that takes both causes of sliding into account. The probability of sliding due to a reduction in resistance forces (the first term in Eq. (1)) was computed as follows. Each day that snow was observed on the roof the occurrence or non-occurrence of snow sliding was noted. If sliding occurred then the high temperature prior to sliding was noted. If sliding did not take place then the daily high temperature was noted. The temperature for each date from February 1984 through March 1984, and December 1984 through April 1985 were obtained, and the probability of sliding at each temperature was calculated.

The computed probabilities have a characteristic sigmoidal shape (S-shape) when plotted against daily high temperature. Probability analysis indicates that sigmoidal distributions may effectively have a normal distribution through the points (Finney, 1962). The probability of sliding was represented by fitting a normal curve to the points. An example for 45° roofs is shown in Fig. 1.

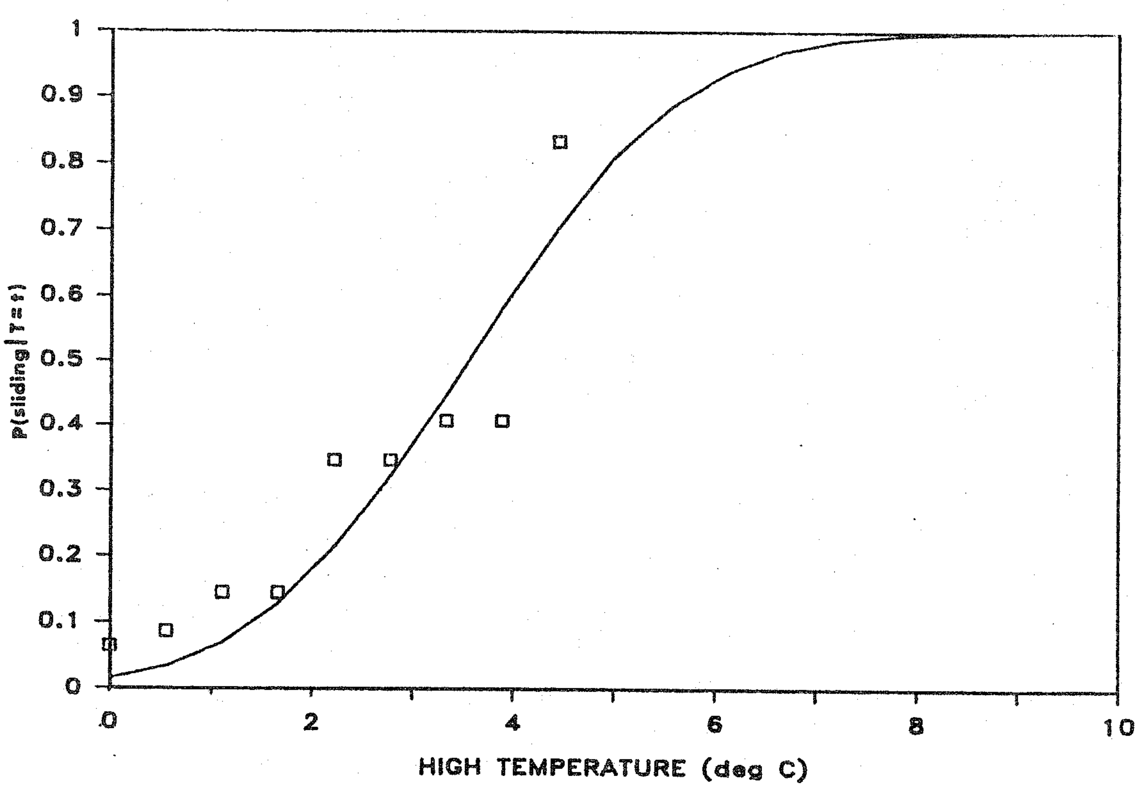


Fig. 1 Conditional probability of sliding for 45° roofs.

The calculations for sliding due to shear failure proceeded in a fashion identical to those for slides caused by a reduction in resistance forces except the precipitation for each day was used. A normal curve was fit to these points producing a plot similar to Fig. 1. The probabilities calculated for the reduction in resistance forces were corrected using two assumptions: (a) all slides occurring at or below freezing were caused by shear failure; and (b) all slides occurring when there was no precipitation were caused by a reduction in resistance forces. These assumptions are not entirely accurate because energy from convection, radiation, rainfall and condensation may cause snowmelt at temperatures below freezing. The data used in this study are sufficient to illustrate the approach, but accuracies would be increased by using a larger number of data points.

Fitting a normal distribution yielded the following equation for the probability of sliding on 45° roofs based on temperature.

$$P(S_r,t|T'=t') = \int_0^{t'} \frac{1}{(1.66)\sqrt{2\pi}} \exp\left[-\frac{(x-3.54)^2}{2(1.66)^2}\right] dx \qquad (4)$$

where x is expressed in °C. The conditional probability of sliding due to shear failure is

$$P(S_s,t|R=r) = \int_0^{r} \frac{1}{(0.60)\sqrt{2\pi}} \exp\left[-\frac{(y/10-1.28)^2}{2(0.60)^2}\right] dy \qquad (5)$$

where y is expressed in mm of water.

Relationships similar to those for the 45° roofs were also developed for both 10° and 30° roofs. Observations from the field data showed that no slides were caused by precipitation (i.e., no shear failures occurred) on the 10° roofs. This observation was further verified by the fact that avalanches, which are caused by shear failure, usually occur on slopes greater than 25° (Schaerer, 1981). For the 10° roofs Eq. (1) reduces to one term.

PROBABILITY DISTRIBUTIONS FOR METEOROLOGICAL DATA

Distributions for meteorological data were developed to complete the probability relationships. A large volume of meteorological data is generally available for such variables as temperature and precipitation. According to Isyumov and Davenport (1974) temperature:

a) varies seasonally about a long term average due to seasonally varying solar radiation;
b) varies daily about the seasonally varying mean (this variation is due to changes in solar radiation during the day, with the magnitude of variation about the mean changing throughout the year; the daily temperature is also affected by passing weather systems); and
c) has a short term variation due to air turbulence and mixing of air.

The latter effect is generally negligible in an analysis of this type, because temperatures must be maintained for a considerable period before sliding occurs.

The seasonally varying daily high temperature about the long term average and the standard deviation about the average daily high temperature can be accurately fit using a two-term Fourier series. Fitting the data for McCall, Idaho gives the following:

$$\bar{T} = 12.1256 - 12.882\cos\frac{2\pi t}{365} - 0.435\cos\frac{4\pi t}{365} - 4.899\sin\frac{2\pi t}{365} + 1.735\sin\frac{4\pi t}{365} \qquad (6)$$

$$\sigma = 7.635 - 4.5089 \cos \frac{2\pi t}{365} + 1.136 \cos \frac{4\pi t}{365} - 1.874 \sin \frac{2\pi t}{365} + 0.772 \sin \frac{4\pi t}{365} \quad (7)$$

where $\overline{T}$ and $\sigma$ are expressed in °C and t is time in days.

Since air temperature varies about the seasonally varying mean, we assume that the daily variation in temperature is normally distributed about the average high temperature (Isyumov and Davenport, 1974).

The seasonally varying likelihood of having precipitation on any day can be calculated from long term precipitation records. The probability of receiving an amount of precipitation, r, on any day is

$$P(R=r,t)=P(R=r,t|R\neq 0)P(R\neq 0,t) \quad (8)$$

where $P(R=r,t|R\neq 0)$ = probability that precipitation on any day will equal r given that precipitation will occur, and $P(R\neq 0,t)$ = probability that there will be precipitation on any given day. The probability that precipitation will occur on any given day has been computed from long-term records (Gifford, et al, 1967, and Heermann, et al, 1971) for a number of locations in Idaho. Weekly probabilities of receiving precipitation were obtained for McCall, Idaho and were also fit using a Fourier series as follows:

$$\overline{p} = 4.3118 - 1.360 \cos \frac{2\pi t}{365} + 0.7316 \cos \frac{4\pi t}{365} - 0.8050 \sin \frac{2\pi t}{365} + 0.7893 \sin \frac{4\pi t}{365} \quad (9)$$

where $\overline{p}$ is the probability that precipitation occurs on a given day t.

The model contains the assumption that the probability of receiving precipitation on one day is independent of the occurrence or non-occurrence of precipitation on the previous day. This assumption greatly simplifies calculations but should not drastically affect the results. Cumulative distributions of monthly precipitation are typically skewed, and Gamma distributions are generally accepted as appropriate for fitting the data. The two parameters, $\alpha$ and $\beta$ of the Gamma distribution (that vary seasonally) were fit with Fourier series equations (see Pinkard, 1985).

Substituting the appropriate relationships into Eqs. (2) and (3) yields probability equations for the 45° roof with a form similar to that of Eq.(1), i.e.,

$$P(S_r,t)= \int_0^\infty \int_0^{x_2} \left[ \left[ \frac{1}{(1.66)\sqrt{2\pi}} \exp\left[ - \frac{(x_1 - 3.54)^2}{2\,(1.66)^2} \right] \right] * \left[ \frac{1}{(\sigma)\sqrt{2\pi}} \exp\left[ - \frac{(x_2 - \overline{T})^2}{2\,(\sigma)^2} \right] \right] \right] dx_1 dx_2 \quad (10)$$

$$P(S_s,t)= \int_0^\infty \int_0^{p_2} \left[ \left[ \frac{1}{(0.60)\sqrt{2\pi}} \exp\left[ - \frac{(p_1/10 - 1.28)^2}{2\,(0.60)^2} \right] \right] * \left[ \frac{\overline{p}}{\beta^\alpha\, \Gamma(\alpha)} p_2^{(\alpha-1)} \exp\left[ - \frac{p_2}{\beta} \right] \right] \right] dp_1 dp_2 \quad (11)$$

where $x_1$ and $x_2$ are temperature variables and $p_1$ and $p_2$ are precipitation variables. Similar equations were developed for the 10° and 30° roofs.

Using relationships such as these, the probability of snow sliding can be predicted for any day. The probabilities of sliding for each day during the winter months in McCall, Idaho were computed and are plotted in Fig. 2. Different meteorological distributions must be developed if the probabilities of sliding for other geographical locations are desired. Extrapolation to locations with climates vastly different from McCall may not be valid because of the assumptions associated with using the temperature index instead of the more precise temperature-time index. Fig. 2 implicitly includes the effects of roof slope, temperature, and precipitation on the likelihood of sliding.

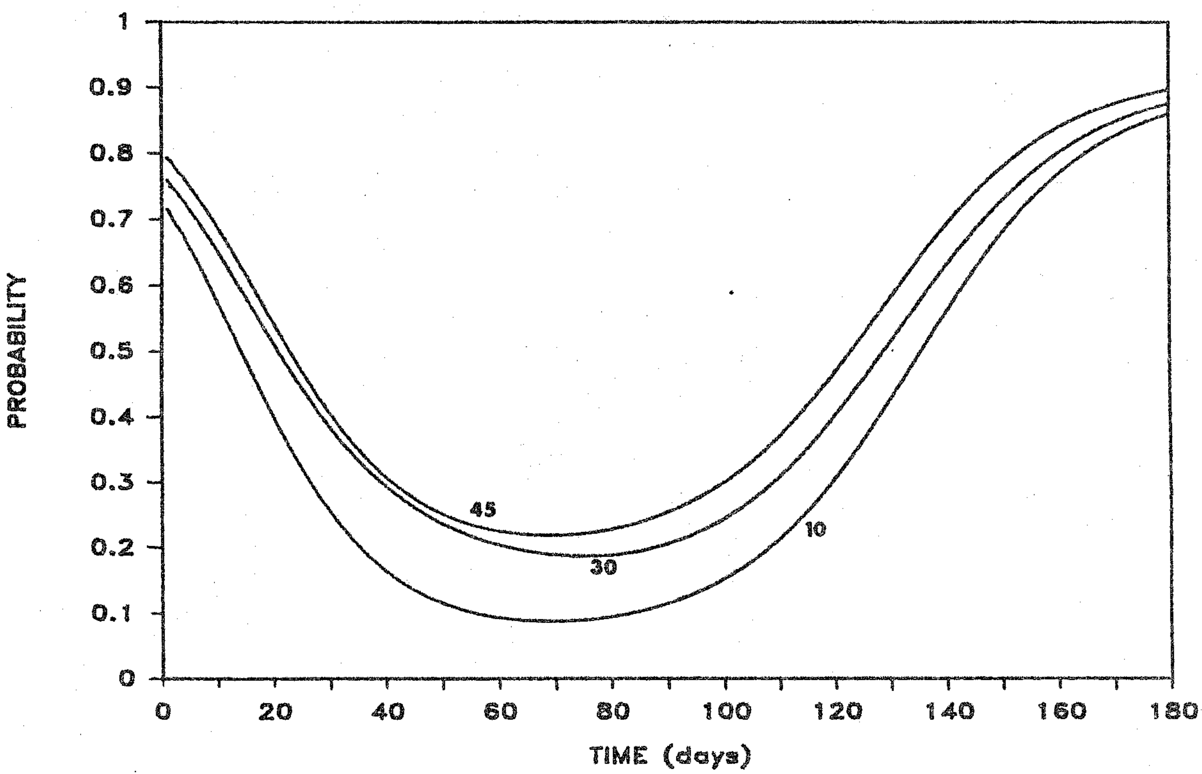


Fig. 2 Probability of sliding for 10°, 30°, and 45° roofs in McCall, ID.

SIMULATION OF ROOF SNOW LOADS

Although it is valuable to know the seasonally varying probability of sliding, the roof snow load is the critical design parameter for structural engineers. Using the derived probabilities of sliding, roof snow loads can be predicted using simulation techniques. A stratagem that follows logic devised by Isyumov and Davenport (1974) and uses simulated meteorological data has the advantage that a large number of winters can be simulated to determine design roof loads. An alternative approach that uses actual meteorological data collected in previous years appears to provide a better representation of actual roof loads. The meteorological distributions were developed assuming there is no correlation between: (a) temperature on successive days; (b) precipitation on successive days; and (c) temperature and precipitation. Furthermore, we recognize that temperatures and precipitation can vary widely from winter to winter. Trying to explain all the variations and correlations simply to use the theoretical meteorological distributions in a simulation seems unwarranted when years of actual data are available. Moreover, extreme value statistics should yield an accurate design roof load based on predicted roof loads from a small number of winters if the correct distribution is used.

A computer program was written in Fortran to simulate roof snow loads. Meteorological data were obtained from the Hydrologic Information Storage and Retrieval System (HISARS) available from the University of Idaho computer system. For a detailed discussion of the analysis see Pinkard (1985). This simulation process was used for the entire snow season. In McCall the snow season extends from November 1 through April 30. The simulation was run six times for each winter's data to check for accuracy and observe differences in loads due to the occurrence or non-occurrence of slides at critical times. The simulation was run for 25 winters and the resulting maximum roof snow load for each winter was noted. Extreme value analysis was carried out using the maximum predicted loads from each winter to predict a design roof snow load. The simulation was carried out for four Idaho locations: McCall (for which the model was developed), Fairfield, Sandpoint, and Island Park. Each site has snowfall with different characteristics from those in McCall. The accuracy of the probabilistic model was verified by comparing simulated loads with actual measured loads on the test structures and the ten case-study roofs for the winter of 1984-85.

The results are displayed in Fig. 3. Note that there is excellent agreement between the observed and predicted loads on the 45° roofs and between the observed and predicted loads on one 30° roof. Observed and predicted loads vary significantly for all 10° roofs and one 30° roof. Since the model was developed using data from the 1984-85 winter there should be close agreement between the observed loads and the predicted loads. The partial correlation indicates that the simulation process may be a useful method of predicting roof snow loads.

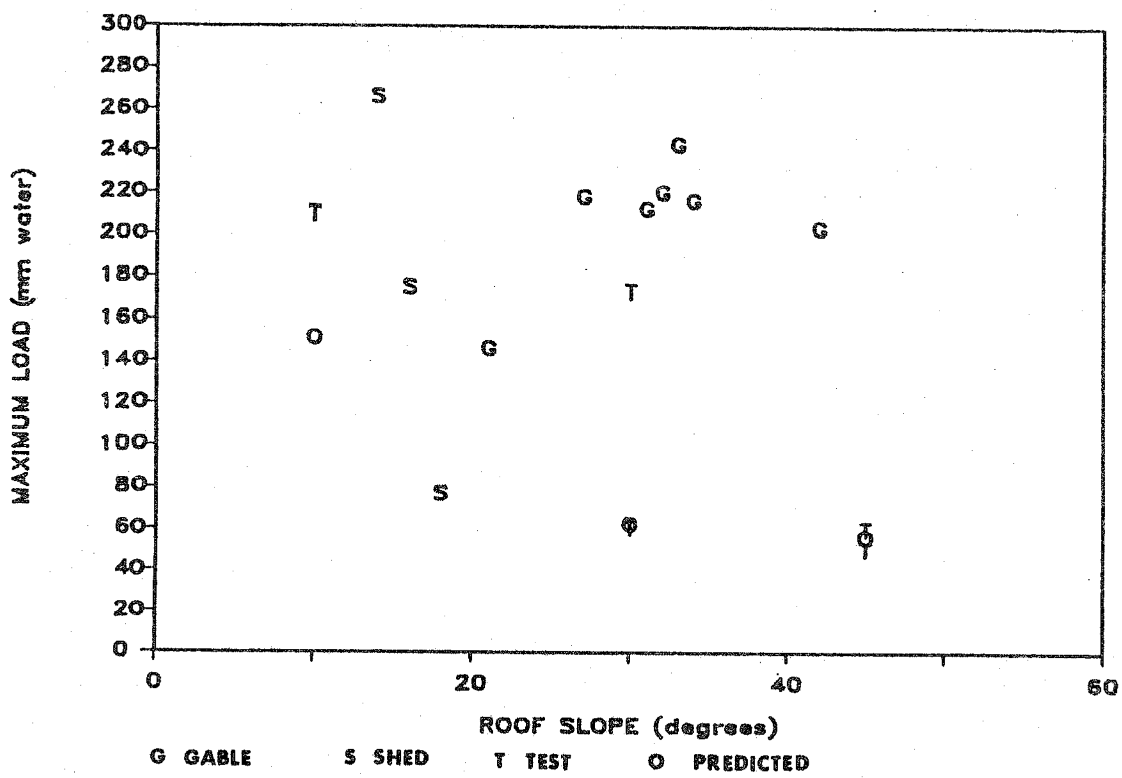


Fig. 3 Simulated and measured maximum roof snow loads in McCall, ID - 1984-85.

The biggest error appears to occur on the 30° roofs; this is probably the case because the two 30° roofs behaved very differently during the 1984-85 winter. Most slides occurred during the late winter when dense, wet snow fell on unloaded roofs; the effect of a temperature rise is much more pronounced when there is a small layer of snow than when there is a deep layer. It appears as though the depth and density of the snowpack play important roles in the sliding phenomenon; since neither factor was incorporated into the model the role of the temperature and precipitation may be exaggerated. The simulation process generally predicted snow slides during midwinter earlier than actual slides occurred.

Comparison between the loads on the test structures and on the full-scale roofs indicates the importance of non-frictional forces on stabilizing the snowpack. Note from Fig. 3 that the gable roofs in general had much greater loads than shed roofs. This is probably due to the large tension force in the snow at the ridge and obstructions on the roof such as gutters, vents, and chimneys. It would be unwise to use this simulation model for gable roofs since it was developed from shed roof data.

## PREDICTION OF DESIGN ROOF SNOW LOADS

The ANSI A58.1-1982 (1982) relates roof snow loads to ground snow loads based on a 50-year mean recurrence interval (i.e., the roof load has a probability of 0.02 of being exceeded during any given year). For Idaho, ground loads with a 50-year mean recurrence interval were obtained by Sack and Sheikh-Taheri (1986) using a log-Pearson type III extreme value distribution. It is instructive to compare the simulated roof loads to calculated design loads. ANSI A58.1-1982 (1982) utilizes a dimensionless coefficient ($C_s$) which is the ratio of sloped roof to flat roof snow loads. The log-Pearson type III distribution was applied to the simulated annual maximum roof loads to determine predicted design loads.

Values of $C_s$ were computed using both the simulated design roof snow load and the calculated design roof snow load. The values of $C_s$ are shown in Fig. 4. Because meteorological conditions vary from site to site, the calculated values of $C_s$ differ between locations. It is interesting to note that there is considerable difference in $C_s$ for 10° roofs at the four simulation locations, but there is a very small difference in $C_s$ for 30° and 45° roofs at the four locations.

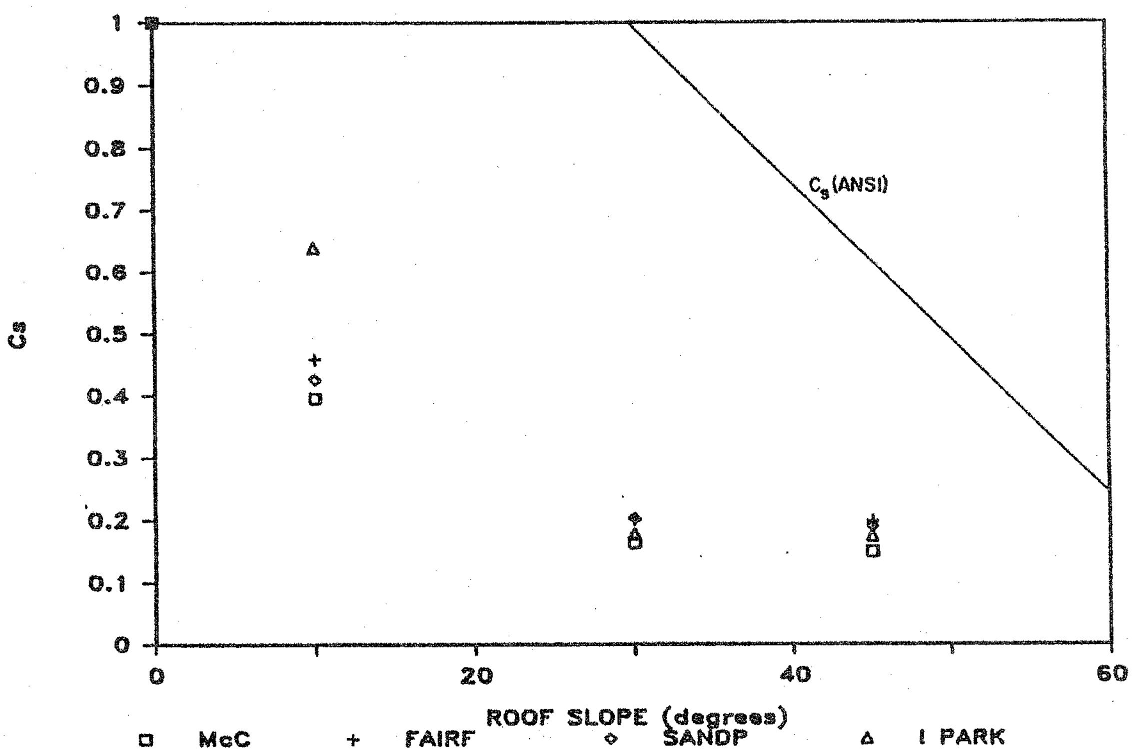


Fig. 4 Calculated ANSI roof slope coefficient ($C_s$) from simulated snow loads.

Since the simulated snow loads on the 45° roofs were very accurate the design equations seem to overpredict the actual snow load (at least in McCall). We observed from the McCall data that the simulated loads for the 10° and 30° roofs were less than

than actual values; therefore, the apparent overprediction by the ANSI equation in Fig. 3 is exaggerated.

## DISCUSSION

We found that the model for predicting snow load on sloped roofs gives values lower than those that actually occur especially on the 10° and 30° roofs. The major reason for the underprediction seems to be that no variables of snowpack characteristics, such as density or water content, were included in the model. The effect of a temperature rise or precipitation is more pronounced when there is a wet, dense snowpack than when there is a deep layer of dry, cold snow on the roof. The model does not account for the variability in snow conditions and could be improved by incorporating the effect of the snowpack characteristics.

Martinec (1976) stated that snowmelt, which influences the reduction of resistance forces, can be related to a temperature index by a factor a. Moreover, the factor a can be related to density which is a satisfactory index of the thermal properties of snow. That paper also related degree-days to melt by the equation:

$$M = aT_d \tag{13}$$

where M = melt (mm), $T_d$ = degree-days (°C day), and a = 1.1p (mm/°C) where p = density (in percent). It should be possible to develop similar relationships for other temperature variables. Therefore, multiplying the temperature index by density might improve the model substantially by explicitly relating the melt processes to the thermal quality of the snow.

The amount of liquid water held by a snowpack is influenced by the water content (Raudviki, 1979). A deep snowpack will hold more liquid water than a shallow snowpack; therefore, adding a specific amount of liquid water will not affect the deep snowpack as much as the shallow snowpack. When the temperature index is divided by the water content it is a measure of the energy input for each unit weight of snow.

The modifications suggested should improve the model. Increasing the data base will improve the existing model, but including snowpack properties appears necessary. The probability of sliding due to a reduction in resistance forces may be more accurate using the equation

$$P(S_r t) = \int_0^B P(\text{sliding}|B=b)P(B=b)db \tag{14}$$

where B is a variable that is a function of temperature, snow density and water content.

An alternative approach for improving the predictive capability involves developing the model on a month to month basis. This would enhance the model because the snowpack properties tend to be similar during specific times of the year; its applicability to other locales would be in question.

## SUMMARY AND CONCLUSIONS

A probabilistic model to predict the conditions for sliding of roof snow was developed using data collected for small roofs with slopes of 10°, 30°, and 45°. Daily high temperature and precipitation were the independent variables. The probabilistic model was incorporated into a simulation program for predicting roof snow loads on 10°, 30°, and 45° roofs. Actual meteorological data collected at four sites in Idaho (McCall, Fairfield, Sandpoint, and Island Park) were used. Roof loads were simulated at each location for each winter day for a period of 25 years. Actual data from the 1984-85 winter in McCall, Idaho were input to the simulation program, and the results were compared with observed snow loads during that winter.

A log-Pearson type III extreme value distribution was fit to the maximum yearly roof loads. For each location and each roof slope a design load with a 50-year mean recurrence interval was calculated. Comparisons were made between the predicted design loads and the design loads computed using the ANSI (1982) equation. Values of the ground-to-roof-conversion factor, $C_s$, were also computed and compared with the values of $C_s$ given by ANSI. From this study we can conclude that:

a) the simulation procedure is a useful method for predicting snow loads on sloped roofs;
b) the model in its present form underpredicts actual snow loads;
c) the ANSI equations overpredict roof snow loads on shed roofs with slopes between 10° and 45°;
d) slippery gable roofs tend to hold more snow than shed roofs;
e) climatological differences cause values of $C_s$ to vary with geographical locations.

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