# HYDROGRAPH RECESSION LIMBS--A CASE OF HYDROLOGIC DEJA VU

by

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## INTRODUCTION

Snowmelt-dominated hydrologic systems, such as the upper Columbia River Basin, have fairly consistent hydrograph characteristics from year to year. Peak flows in the upper Columbia Basin normally occur between mid-May and mid-June. Because summer and early fall are usually dry, flows recede at a uniform and constant rate.

The recession limb of streamflow hydrographs in such systems during periods of low precipitation resemble a decay curve. Recession flows are often represented by the following function (Nutbrown and Downing, 1976):

$$Q = Q_0 e^{-kt} \tag{1}$$

where Q is the flow at time, t, $Q_0$ is the flow at t=0, and k is the recession constant.

The recession constant is a function of watershed characteristics and climate and therefore varies from basin to basin and from year to year. In the absence of precipitation, flows in a specific watershed recede at a near constant rate, with the recession characteristics depending on the amount of snowmelt volume available during peak runoff (high snow years vs. low snow years).

If one analyses the crest and recession-limb segments of snowmelt-dominated watersheds in the upper Columbia Basin for a long period of record (50 or more years), it becomes obvious that some years are very similar to one another in terms of both shape and magnitude of the crest and recession-limb segments. In fact, I have discovered several pairs of near-identical hydrograph twins for many stations. Assume that one randomly chooses a snowmelt hydrograph from a substantial period of record (at least 40 years). There is a high probability that at least one other hydrograph in the record will very closely resemble the first. This sense of hydrologic "deja vu" is an interesting phenomenon and may provide a useful water-supply forecasting tool.

The challenge then is to find a method of searching through the streamflow record for the best "model-year" hydrograph.

## THE INDEX OF HYDROGRAPH SIMILITUDE

I developed an analysis procedure to find, in the period of record, a snowmelt hydrograph that most closely resembles the snowmelt hydrograph for a target year. The following objective function was used to find the best-fit model year:

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$$\iota_s = \min \frac{\sum_{i=1}^{N} (Q_i^{target} - Q_i^{model})}{\sum_{i=1}^{N} Q_i^{target}} \qquad (2)$$

where $\iota_s$ is termed the "index of hydrograph similitude," $Q_i^{target}$ is the monthly flow for the target year, $Q_i^{model}$ is the monthly flow for the model year, i is the month, and N the total number of months of the hydrograph segment. Nearly identical hydrographs would have a $\iota_s$ approaching zero.

The computer program calculates $\iota_s$ by searching the entire period of record for the best-fit model year. The choice of the objective function is critical. Since the goodness-of-fit depends on minimizing $\iota_s$ in Equation 2, the model year selected will appear very similar in pattern to the target year. The objective function represented by Equation 2 is biased towards a similitude in high flows. According to Fleming (1975) an objective function which utilizes logarithmic deviations will decrease the bias towards high flows. The following function may be used to reduce such bias:

$$\iota_s (\log) = \min \frac{\sum_{i=1}^{N} (\log_e Q_i^{target} - \log_e Q_i^{model})}{\sum_{i=1}^{N} \log_e Q_i^{target}} \qquad (3)$$

I used the following three river basins to demonstrate the phenomenon of hydrograph similitude:

Columbia River at The Dalles, Oregon (237,000 $mi^2$)
South Fork of the Flathead River below Hungry Horse Reservoir, Montana (1660 $mi^2$)
Moyie River at Eastport, Idaho (570 $mi^2$)

The results are shown in Tables 1 through 3. A striking aspect of the data in all three tables is that in all cases the same model year was chosen regardless of the hydrograph period used.

Table 1. Comparison of hydrograph similitude indices for various hydrograph periods, Columbia River at The Dalles.

| | Apr-Jul | | Apr-Aug | | Apr-Sept | |
|---|---|---|---|---|---|---|
| Target Year | Model Yr | $\iota_s$ | Model Yr | $\iota_s$ | Model Yr | $\iota_s$ |
| 1976 | 1951 | .048 | 1951 | .082 | 1951 | .096 |
| 1977 | 1944 | .142 | 1944 | .129 | 1944 | .127 |

Table 2. Comparison of hydrograph similitude indices for various hydrograph periods, South Fork Flathead River at Hungry Horse Reservoir.

| | Apr-Jul | | Apr-Aug | | Apr-Sept | |
|---|---|---|---|---|---|---|
| Target Year | Model Yr | $\iota_s$ | Model Yr | $\iota_s$ | Model Yr | $\iota_s$ |
| 1976 | 1951 | .092 | 1951 | .092 | 1951 | .096 |
| 1977 | 1944 | .148 | 1944 | .148 | 1944 | .158 |

Table 3. Comparison of hydrograph similitude indices for various hydrograph periods, Moyie River, northern Idaho.

| | Apr-Jul | | Apr-Aug | | Apr-Sept | |
|---|---|---|---|---|---|---|
| Target Year | Model Yr | $\iota_s$ | Model Yr | $\iota_s$ | Model Yr | $\iota_s$ |
| 1976 | 1951 | .093 | 1951 | .105 | 1951 | .105 |
| 1983 | 1978 | .098 | 1978 | .113 | 1978 | .112 |

For comparison purposes, the target year and model year hydrographs are plotted in Figures 1 through 6. Water year 1976 was a wet year in the Columbia Basin, while water years 1977 and 1983 were considerably drier.

Applying the different objective functions occasionally resulted in different model years being chosen for some target years and hydrograph periods. This occurred for the April-July hydrograph period in 1977 at Hungry Horse. Since the criteria of goodness-of-fit are expressed in different units in Equations 2 and 3, the results of applying the two objective functions to the same data sets cannot be compared directly. Figure 7 demonstrates the goodness-of-fit for model year 1940. The use of $\iota_s$(log) results in a closer fit (better similitude) at the lower flows on the recession limb as compared to model year 1944 in Figure 4.

## APPLICATIONS

Is hydrograph similitude just an interesting phenomenon or does it have some practical value as well? The high degree of similitude between target and model years suggests that a predictive relationship could be developed from the two sets of data. For example, a regression model based on the April-July period could be used to forecast August streamflow for the target year. To demonstrate this predictive ability, I made forecasts of August, September, and October, 1977, streamflows for the Columbia River at The Dalles, Oregon. Forecasts were made using a linear regression model (Y = a + bX) developed from the monthly flows of the model year (1944) and the target year (1977). The forecast was simply the predicted value of Y computed from X, the observed flow of the same month in the model year. Forecast errors were determined using the following equation:

$$F_i = \frac{Q_i\text{(observed)} - Q_i\text{(forecast)}}{Q_i\text{(observed)}} \times 100 \qquad (4)$$

where $F_i$ is the forecast error in percent. The forecast results are shown in Table 4. Forecast errors vary with forecast month but are generally encouraging, especially when one compares the error squared against the variance of observed monthly flows for the period of record.

Table 4. Forecast errors for three low-flow months at The Dalles, 1977 water year.

| Month | Forecast Flow (cfs-days x $10^3$) | Observed Flow (cfs-days x $10^3$) | Forecast Error, % | Forecast Error Squared | Variance of Observed Flow |
|---|---|---|---|---|---|
| August | 3031.9 | 3278.3 | +7.5 | $6.1 \times 10^{10}$ | $1.0 \times 10^{12}$ |
| September | 2469.4 | 2362.3 | -4.5 | $1.1 \times 10^{10}$ | $4.0 \times 10^{11}$ |
| October | 2226.4 | 1969.7 | -13.0 | $6.6 \times 10^{10}$ | $3.8 \times 10^{11}$ |

In an operational forecast mode, the forecast value would be based on the lower limit of a confidence interval around the predicted value, rather than the predicted value itself. The forecast then would represent an assured streamflow at a selected level of probability.

As a test of the relative value of the two objective functions, $\iota_s$ and $\iota_s$(log), their ability to choose model years having good predictive characteristics was compared for the April-July hydrograph period, with 1977 as the target year and August as the forecast month. A linear regression model was used, although better correlations could have been obtained with non-linear models.

Table 5. Comparison of streamflow forecast errors for August, 1977 at Hungry Horse Reservoir. Linear regression models based on the April-July hydrograph period were computed separately for the model years 1944 and 1940, representing the $\iota_s$ and $\iota_s$(log) objective functions, respectively.

| $\iota_s$ | | | $\iota_s$(log) | | |
|---|---|---|---|---|---|
| Model Year | Forecast Month | Forecast Error, % | Model Year | Forecast Month | Forecast Error, % |
| 1944 | August | -82.4% | 1940 | August | -42.7% |

The use of $\iota_s$(log) as the objective function improved the forecast considerably.

## CONCLUSIONS

Recession limb flows in both dry and wet years could be forecasted with a high degree of accuracy, as suggested in the above results. Streamflow forecasting models currently in use do an acceptable job of predicting flows in "normal" years, but many models have difficulty with the recession limbs of hydrographs in either low or high snowmelt runoff years. The similitude method of predicting recession limb flows requires only historical streamflow data and is very simple to use.

## LITERATURE CITED

Fleming, G. 1975. Computer simulation techniques in hydrology. American Elsevier Publishing Company, Inc. New York. 333 pp.

Nutbrown, D. A. and R. A. Browning. 1976. Normal-mode analysis of the structure of baseflow-recession curves. Journal of Hydrology 30:327-340.

Figure 1
Dalles Recession Flows

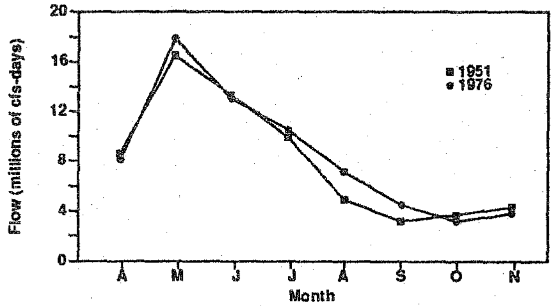



Figure 2
Dalles Recession Flows

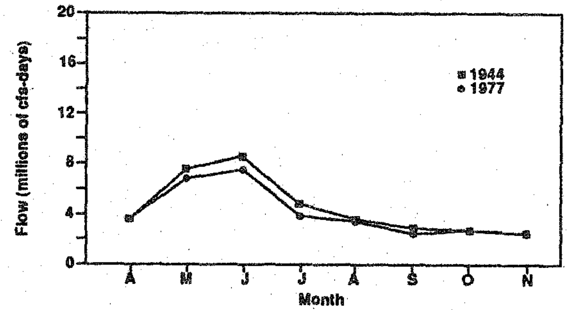



Figure 3
Hungry Horse Recession Flows

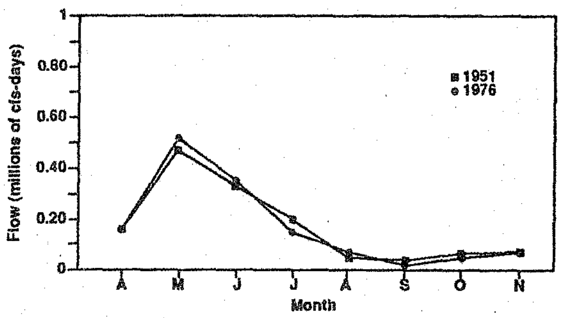



Figure 4
Hungry Horse Recession Flows

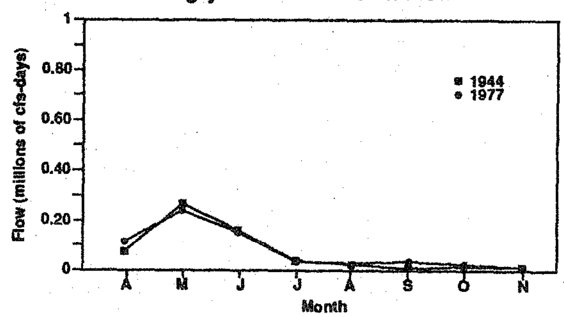



Figure 5
Moyie Recession Flows

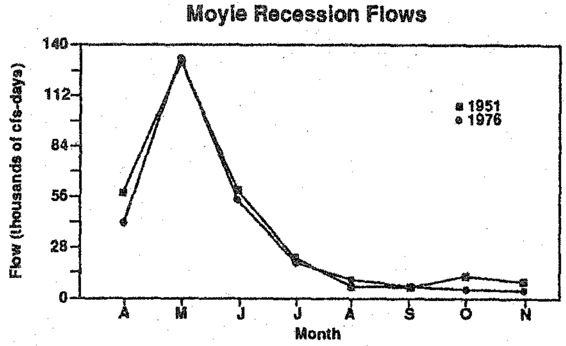
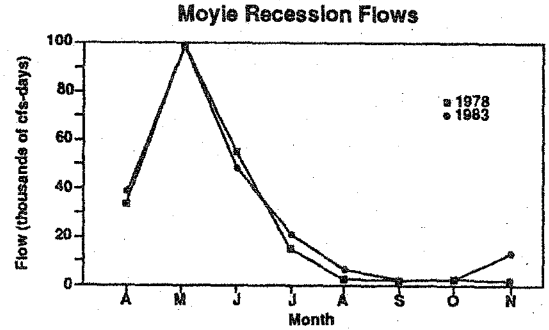



Figure 6
Moyie Recession Flows



## Figure 7
## Hungry Horse Recession Flows