

A METHOD OF MODELING THE FREQUENCY CHARACTERISTICS OF DAILY SNOW AMOUNT,

FOR STOCHASTIC SIMULATION OF RAIN-ON-SNOWMELT EVENTS

by
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INTRODUCTION: EXTREME EVENTS IN THE PACIFIC NORTHWEST

In much of the western Cordillera of North America, and particularly on the windward sides of the coastal mountain ranges between SE Alaska and central California, rain-on-snowmelt (R/SM) is a hydrometeorologic event of moderate frequency, yet capable of extreme effects. In this paper, I examine some of the characteristics of these events, and the questions that they provoke; describe the research strategy that I am following to address some of the questions, involving Monte Carlo simulation of R/SM events based on data from the Washington Cascades; and introduce a method of statistically analyzing and modeling snow data for use in stochastic simulation.

R/SM conditions occur when winter-season cyclonic storms riding in on warm air from the Pacific bring heavy rains, enhanced during orographic rise over the mountains. Rainfall, combined with melting of some of the snow that is likely to be present, produce water inputs to soil and streams that are typically greater than can be produced by winter storms without snowmelt, spring snowmelt without rain, or convective storms at any time. The occurrence and effects of R/SM events have been recorded at least since the 19th century (and probably have been recognized as long as humans have wintered in this region). Most of the Northwest's largest flood- and landslide-producing storms have been exacerbated by the combination of snowmelt with rain; four such storms crossed Oregon and Washington this past winter. Although precipitation is the main source of water input in most areas during these events, the contribution of snowmelt is significant in increasing their effects.

Western North America is particularly well situated to receive these storms. It straddles the winter storm track off the North Pacific, which typically hit the continent at 40-55° latitude; and it supports a high-pressure zone over California, which tends to divert storms to the north. Thus air flow is from the southwest, resulting in above-normal temperatures during storms. Tectonism has created multiple mountain ranges in the West, which force the incoming air to rise, cool, and dump its moisture. Furthermore, the mountains maintain widespread snowpacks in winter, especially at higher elevations and latitudes, so incoming warm winter storms are liable to find some snow to melt. The presence of mountainous topography makes landslides and rapid runoff likely consequences of these events. Thus, the circumstances suitable for heavy rain to combine with snowmelt occurs somewhere along the west coast almost every year, and typically several times per season. This combination of conditions, and the resultant hydrologic significance of R/SM events, is almost unique to this part of the planet. Most of the mountainous regions of the world experience their worst flood- and landslide-producing weather in convective storms or tropical cyclones during the warmer seasons, when snow is restricted to glacial environments. Only in western North America do the orientations of the climatic and meteorologic patterns, and of the cordilleran mountain ranges in their paths, conspire sufficiently to create an environment where R/SM events are so hydrologically and geomorphically significant.

These impressions of the importance of the R/SM phenomenon in the humid Pacific Northwest engender a number of implications and hypotheses. The amount of liquid water generated in an event is sensitive to the amount of snow already on the ground, the frequency, intensity, and duration (F-I-D) of precipitation, and the weather during the storm, all of which are controlled by elevation, latitude, time of year, etc. Within a limited geographic area, we can suppose that at low elevations, the amount of snowmelt contribution to the available water for runoff (AWR) will generally be small, so a F-I-D curve calculated for AWR will not be much different from that for precipitation (fig. 1). However, since snow can accumulate to sea level in the Northwest, there can be a significant contribution to the AWR hydrograph during individual R/SM events. At high elevations, storm precipitation falls as snow in

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most storms, and the water that is rarely generated is likely to be refrozen into a deep snowpack; thus, despite the fact that these areas can possess steep frequency curves for precipitation, the F-I-D curves for AWR might be lower, indicating that the ground there is actually receiving less water, for a given return period, than the standard curve indicates.

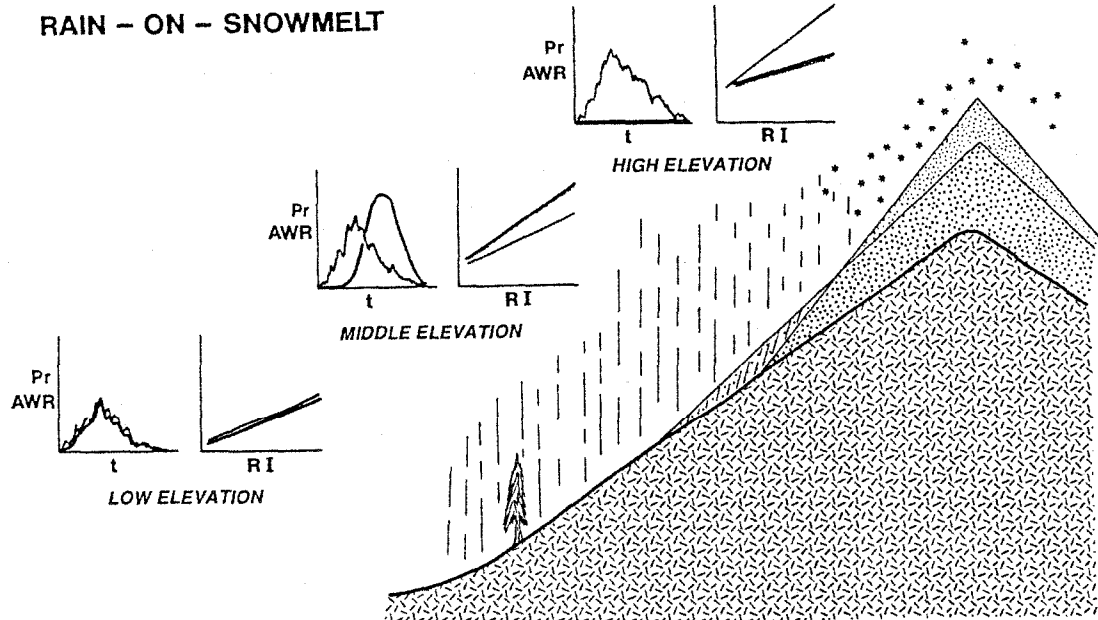


Figure 1. Conditions during a hypothetical R/SM event: hyeto-/hydrographs (left) and F-I-D curves for different elevations. Lighter lines indicate precipitation (Pr); heavier lines indicate water delivered to the soil (AWR). The storm causes melt of the preexisting snowpack (heavy stipple) at low levels, and new accumulation (light stipple) uphill.

Somewhere in between these extremes, there is an elevation zone in which above-normal storm temperatures, heavy rain, and the presence of a moderate amount of snow combine so that the water delivered to the soil is maximized. At the upper end of the frequency scale, a small augmentation in water input translates into a considerable increase in return period; so, over time (assuming climate remains fairly constant), there should be an area where these conditions occur most often, and where the F-I-D curve for AWR is a maximum amount (possibly above that for precipitation). Thus, the combination of these various probabilistic factors should create preferred elevation zones where the combination of effects, averaged over time, is most hydrologically significant; it is inferred that this mid-elevation zone could coincide with the elevations at which snow accumulates and melts several times in a winter, the so-called transient snow zone (TSZ). How can we determine the location of this zone in a region? How can we estimate the difference between precipitation and the AWR that might be expected at some return interval, particularly in the zone of maximum effect? Are the intensity and duration of AWR such that these zones should be more susceptible to storm-triggered mass movement? Will any such increased landsliding result in significant differences in landform, compared to areas lower and higher on the mountainsides? Will there be noticeable effects on runoff and channel morphology, at least in small basins of limited altitudinal range? If so, will these effects be detectable in the larger streams below?

While the hydrologic and geomorphic questions are interesting in themselves, the answers will also have important implications for the management of mountainous lands in susceptible areas, particularly for forest operations that affect hillslope hydrology and erosion processes. Timber harvesting and associated road construction can significantly alter the hydrologic and strength characteristics of the near-surface environment, resulting in locally increased subsurface flow of water during storms, higher peak flows in small streams, and mass erosion [Sidle and others, 1985]. As Harr [1981] suggested, if the creation of clearings in the forest allows more snow to accumulate and then melt during warm storms, as seems (on the basis of theoretical and observational evidence) to be the case, it could change the frequency, intensity, and/or duration at which water moves through soils and streams. If water inputs are sufficiently increased, channel and slope processes might also be altered.

Evidence is beginning to accumulate that such is indeed the case. Several studies suggest that timber harvest is affecting peak flows in streams of a range of sizes [Christner and

Harr, 1982; Lyons and Beschta, 1983; Harr, 1986]. Observations regarding any similar zonation of landsliding in the transient snow zone is still mostly anecdotal, due largely to the vast range of controls on individual landslides, so this question is still open. Nevertheless, the possibility of great damage and injury that can result from even a single R/SM-induced debris torrent makes the possibility worth more than academic interest. As a consequence of these management considerations, the behavior of rain-on-snowmelt events, and the ways that forestry and engineering should be conducted in susceptible areas, are receiving much attention by forest hydrologists and geomorphologists in the Northwest.

Problems and Approach

Despite the moderate frequency of R/SM events, observational studies of their behavior has been limited by their lack of consistency and predictability. Because the magnitude of an event depends on many meteorologic and hydrologic conditions obtaining before and during the event, an appreciation of an its effects, and by extension the effects of such events over many years, depends on the measurement or estimation of the frequencies of occurrence of the critical controlling factors. Field studies intending to measure such variables have been undertaken at various times and scales, beginning with the Corps of Engineers' snow hydrology studies [U.S.A.C.E., 1956]. More recently, measurements during R/SM have been made in humid, forested areas of SW British Columbia [Beaudry and Golding], western Oregon [Berris and Harr, 1987], and NW Washington [Harr and Coffin, in progress]. Such field studies, combined with theoretical considerations, have resulted in a number of insights regarding R/SM. The validity and general usefulness of the quasi-empirical snowmelt equations of the Corps of Engineers have been generally confirmed by Beaudry and Golding [1982], Kattelman [1985], and Berris and Harr [1987], particularly for situations in which data are limited.

But rain-on-snowmelt events are basically stochastic phenomena, the results of the interaction of a large number of contributory processes, most of which are at least partly probabilistic. Precise description of an event depends on knowledge (by measurement or estimation) of the magnitude of the contributing elements; and definition of F-I-D characteristics depends on definition of the frequency properties of the critical factors involved. This information is generally lacking, especially for mountainous terrain: the dearth of meteorological instrumentation, and the wide dispersion and short history of the stations that exist, make it difficult to generalize the frequency patterns of precipitation, wind, temperature, snowpack, etc. over broad regions of complex topography and vegetation. The empirical relationships among hydrometeorological factors and such simple geographic features as elevation, aspect, and canopy density are ill-defined at best; the problems of estimation of the more complicated ones, such as air flow through irregular mountains, are daunting. Empiricism can be augmented by physical models, but these tend to be more data-intensive and thus site-specific in their applications, and poorly suited for application in regions where measurements have not been made. Therefore, the evaluation of the F-I-D characteristics of precipitation from observations in mountainous terrain is hard enough; that of estimation of a more complex yet less frequent phenomenon such as rain-on-snowmelt is even more difficult. Prediction of the likely magnitudes of R/SM inputs remains a challenging problem.

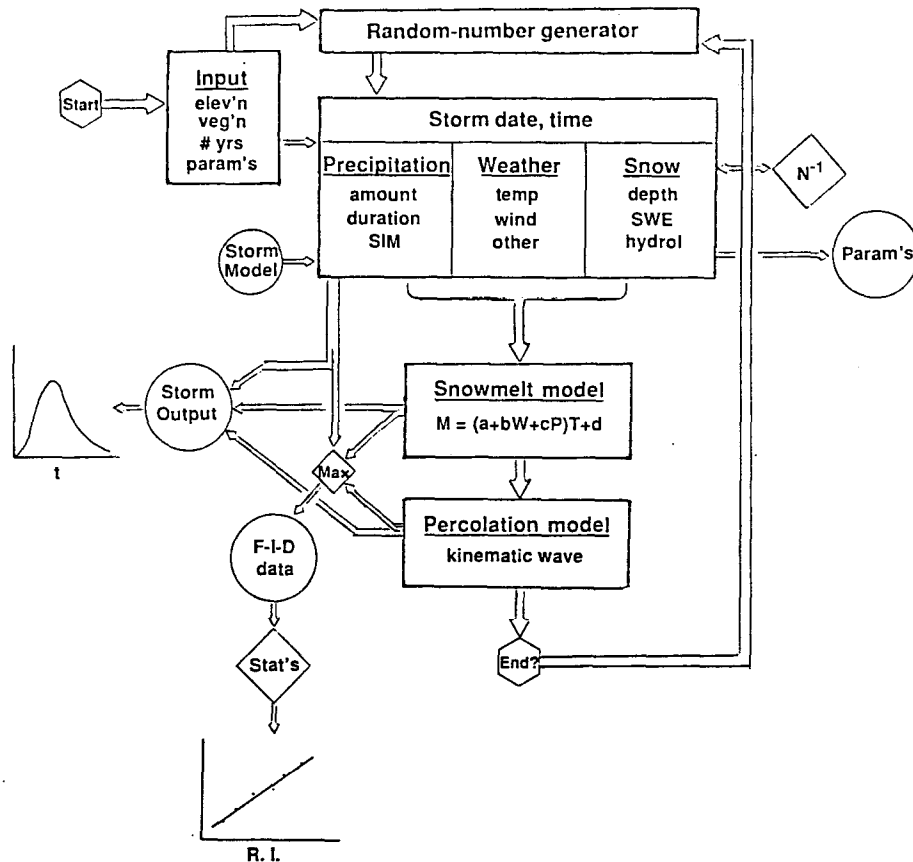
An alternate approach is to appreciate the stochastic nature of R/SM events, and design a probabilistic means of examining them. In such a scheme, the records of the individual contributing factors are used to estimate their statistical properties, which become sources of input for multiple executions on a computer-based Monte Carlo experiment. Random numbers become probabilities, and values of the physical factors are calculated from the cumulative distribution functions (cdf) for those factors. For each "model storm", the computer creates a set of factor realizations, and these are combined in a physical-deterministic model (a set of equations rendered into computerese) that simulates the processes of interest. The results of hundreds or thousands of runs of the computer model, each using randomly-generated factor values, can be analyzed statistically to indicate the nature and long-term consequences of the stochastic processes. In other words, a Monte Carlo experiment simulates the way two or more physical properties interact in nature, by allowing their probability distributions to interact within a computer model; the results of the experiment are the statistical properties of the realizations.

The rain-on-snowmelt phenomenon seems an appropriate candidate for stochastic simulation. I have been attempting to build a computer model of R/SM events, and collating the observations and probability distributions necessary for its operation. Such a model could be useful for estimation of water reaching the soil, during individual events (for input into subsurface-flow, slope-stability, or runoff models), or as the result of many events over a long time period (Monte Carlo mode). Some of the questions I would like to address include:

1. Does the stochastic method work, i.e. does it adequately reflect reality ?
2. How do the F-I-D characteristics of AWR compare with the precipitation F-I-D curves for weather stations ?
3. Is there an elevation zone in which R/S/M is particularly significant in terms of water input to soil ? How can the TSZ best be defined and determined ?
4. How does vegetation, particularly the contrast between coniferous forest and clearcuts, affect the magnitude and intensity of water delivery during R/S/M events ? Is any difference due more to changes in accumulation or melt rates ?

Figure 2 shows the structure of the Monte Carlo version of the model. The program is designed to run on personal computers, so efficiency of operation is a priority. Each run represents an event, and conditions between storms are not determined. Since R/S/M events are being modeled, some simplification is allowed, because certain weather conditions (e.g. bright sunshine) need not be regarded. It is assumed that the distributions are independent, which is not strictly the case, but they are based on data from a limited range of storm types, so most can safely be modeled as independent.

A. Model Architecture



B. Stochastic Value Calculation

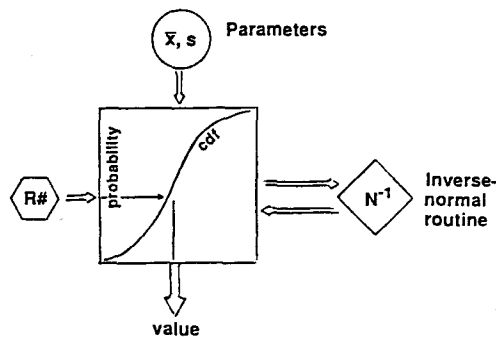


Figure 2. Structure of computer program for a stochastic model of R/S/M. A. Flow chart: major computational elements represented by rectangles, subroutines by diamonds, I/O files by circles. B. Detail: calculation of values from random probability, to cdf (defined by mean and standard deviation, from input), using inversion subroutine.

At the beginning of execution, a set of parameters is furnished. Then, for each simulated event, the computer uses a set of randomly-generated probabilities and an inversion routine to produce the values of the hydrometeorological "conditions" for that event. Simplified models are used to generate hourly values of precipitation and temperature; these are supplied to a snowmelt model adapted from the Corps of Engineers' equations, modified for hourly calculation. Percolation of water through the snowpack is modeled using the kinematic wave approximation, as developed by Colbeck [1972; Colbeck and Davidson, 1973] for vertical flow through snow and ice; despite the simplifying assumptions employed in this model, it has produced good agreement with field measurements [e.g. Dunne and others, 1976]. Input parameters and output realizations, especially of the precipitation and AWR hydrographs and the maximum amounts of each for various durations, can be stored on disk files.

The data utilized to generate probability distributions were gleaned from the records of weather stations and snow courses in the central-western Cascades of Washington (fig. 3). The area contains rugged terrain at elevations of 300-1500 m, originally covered by coniferous forests. The quality of measurements and records is variable; an attempt has been made to fill gaps in some series, but most observations have been accepted at face value. In order to answer the questions listed above, I have tried to determine the relation between the descriptive parameters and such local conditions as elevation and vegetation type for some of the storm characteristics, particularly those (precipitation, snow, temperature) that are strongly controlled by elevation. Limitations of data sources require inference and simplification for most of them, and consideration of other properties has been impossible.

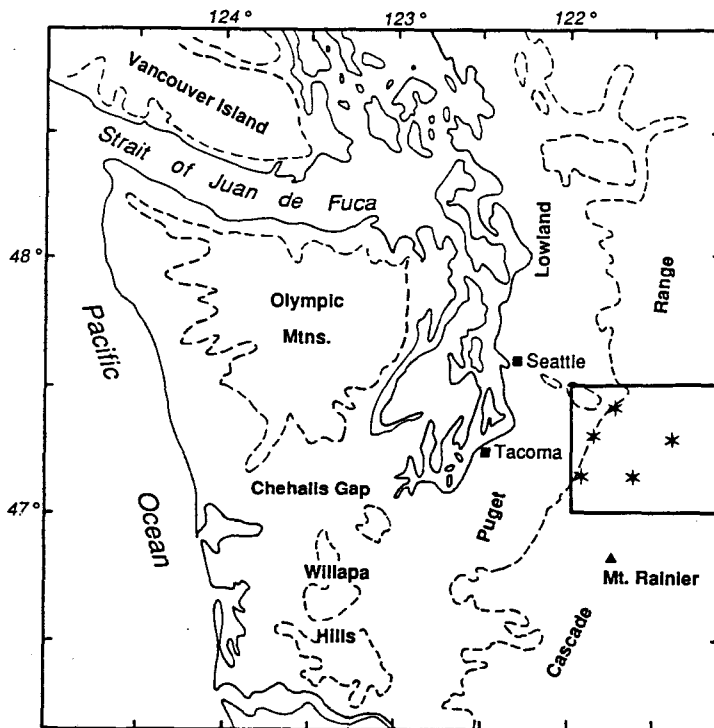


Figure 3. Map of western Washington, showing the study area; * indicate weather stations.

SNOW MODEL

The Mixed Probability Distribution

One goal of this study was to design a mathematical model that can be used for stochastic prediction of snow quantities, given some exceedance probability. I needed a means of generating realistic stochastic snow volumes for any day in the seasons when winter storms can occur (1 Sept to 31 May), so the probabilities must be sensitive to the date of a model event. Both the depth and water equivalent (SWE) of snow on the ground at the beginning of an event are required: melt is calculated in terms of SWE, but the rate of percolation depends on the snow depth. And because snow quantities are so strongly controlled by it, the parameters must also be functions of elevation. These criteria, specific to an event-based model, are different from those that might be required for other uses, such as estimation of maximum snow loads or continuous runoff.

Monte Carlo simulation requires that cumulative distribution functions, described by equations and associated simple statistical parameters, must be found or estimated for all of the stochastic factors. Therefore, an appropriate distribution function had to be chosen

for snow. Any physical quantity that is the outcome of a large number of causative factors or events, each of which is independent and has a relatively small effect on the outcome, ought to follow a normal distribution [Yevjevich, 1972]. Snow accumulation seems to fit these criteria: depth on a given day is the result of the snowstorms and melt events that have occurred up to that time. However, the distributions of the samples of snow depth for a given date vary tremendously, and some do not seem to be Gaussian at all. Although the mid-season data at high elevations are quite symmetrical, those for early and late in the season, and at low-elevation sites for any date, have pronounced skew. In addition, the range of the data is great, especially for snow depth: accumulations on the higher mountains can exceed 5 m, while depths less than a few centimeters are more common on the lower sites.

Thom [1966] dealt with similar problems in the distribution of annual maximum snow accumulation by dividing his data into zero- and non-zero samples, then applying a log-normal distribution to the latter. This mixed-distribution method reduces the skew of the sample data and the range of numbers that must be digested by the model, advantages that make it useful in stochastic simulations. Figure 4 shows the effect of this transformation on a hypothetical data set. The true distribution, with many null values and few measurements at the high end, is highly skewed and not well modeled by the normal curve suggested by the sample. Consideration of just the non-zero values, and transformation of the non-zero data into logarithms preserves the mean of the parent distribution, and reduces the skew coefficient almost to zero; the transformed histogram is reasonably well modeled by the log-normal curve.

The mathematics of the method was outlined by Thom [1966]. $P[0]$ is the probability that there is no snow on the ground, so the probability of having snow is $P[>0] = 1 - P[0]$. It is assumed that the natural logarithm of snow depth or SWE is normally distributed: for $u = \ln d$, the normal distribution is described by $N[u; \mu(u), \sigma(u)]$, or simply $N[u]$. The mixed distribution function is then:

$$M[u] = P[0] + P[>0] * N[u]$$

Solving for $N[u]$ and transforming it into the unit normal distribution gives:

$$\frac{M[u] - P[0]}{P[>0]} = N \left[\frac{u - \mu(u)}{\sigma(u)} \right]$$

Inverting the normal and solving for u :

$$u[M] = s(u) * N^{-1} \left[\frac{M[u] - P[0]}{P[>0]} \right] + \bar{x}(u)$$

for $u[M]$ = the quantity corresponding to a certain mixed probability
 N^{-1} = the inverse normal distribution
 $\bar{x}(u)$, $s(u)$ = sample mean and standard deviation

The characteristics of the mixed distribution are demonstrated in figure 5, the cumulative distribution curve for the sample data of figure 4. The lower cdf corresponds to the log-normal distribution of figure 4C. But 13 of the 50 measurements are zero, so $P[0] = 0.26$; and the upper cdf represents the mixed distribution: for any probability greater than 26%, the corresponding snow depth is indicated by the abscissa value of the curve at the appropriate point; and for probabilities $\leq 26\%$, snow depth is zero. Note that the size of $P[0]$ has a great degree of control on the relation between probability and snow depth.

The applicability of this method to annual maximum snow accumulation was demonstrated by Thom [1966]; and Isyumov and Davenport [1974] utilized the probability distribution of daily snowfall to create a stochastic model of snowpack growth through the winter. However, this combination of methods have not (to my knowledge) ever been used to model the frequency distribution of daily amounts of snow on the ground. There is some empirical evidence that this method is appropriate. In most cases, the logarithms of depth and SWE are closer to being normally distributed than are the original data: separation of null values and transformation into logs almost always reduces the skew. But this model is not perfect: for sample distributions that are nearly symmetrical, transformation results in negative skew; and for samples with large coefficients of variation, the parameters indicated for the parent distribution can be unreasonably large. However, many of these problems are countered by the averaging involved in the parameter calculations (below). Also, a very important advantage for a Gaussian function is that the mean and standard deviation (plus $P[0]$) are the

only parameters required to describe the distribution, while most other distributions that can describe snow volume use these statistics to estimate other parameters, thus removing the math further from the data. Ultimately, though, the suitability of this method will be judged by its ability to simulate the frequency characteristics of the field measurements.

Data Collection and Analysis

The data used here are measurements of snow depth and water equivalent from five weather stations and 20 snow courses in the Washington Cascades (fig. 3). Data from the weather stations are published in the *Climatological Data for Washington*; snow depth is reported as snow on the ground, and SWE was also reported for Stampede Pass. They thus provide records of snow depth for every day of the winter season. Most of the weather stations began observations of snow in 1949, and four continue to the present. Though the sites are in clearings, and so avoid the problems of canopy interception, in some cases they may be too open and windy for maximal accumulation. Four stations are in valley bottoms and in a relatively narrow elevation range, leaving slopes and ridges at higher elevations underrepresented.

The snow survey data were published by Soil Conservation Service in the *Water Supply Outlook for Washington*. The number of observations per season ranges from two to 15, generally on the 1st and 15th of November through June. (But measurements have been done a week before or after the nominal date; if the snowpack is light, trips are canceled, and shallow values may be underrepresented.) Most of the snow courses have 20-30 years of record, and several were instituted in the 1940s. Four snow pillows have been installed in the area; but observation at many courses was discontinued in the early 1980s, leaving only 12 sites active in 1988. Measurements made in May 1985 are the latest used here. At snow pillow sites, ground-truth was not determined on some occasions, and only the water-equivalent for the pillow was known. Although the courses were well dispersed geographically and by eleva-

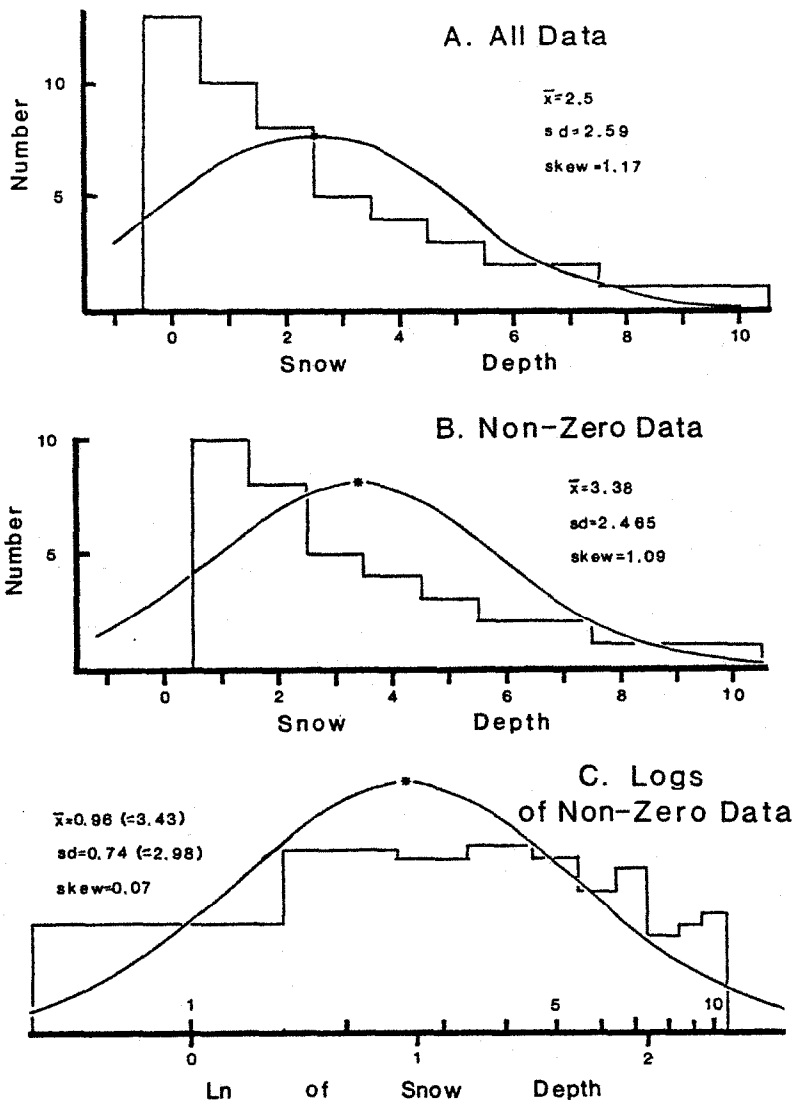


Figure 4. Histograms for hypothetical sample of snow measurements, showing mean (\bar{x}), standard deviation (sd), skew, and curve of the normal pdf. In C, the numbers in parentheses are values of x and sd corresponding to the parent distribution of a log-normal pdf having these parameters; compare with the original statistics in A.

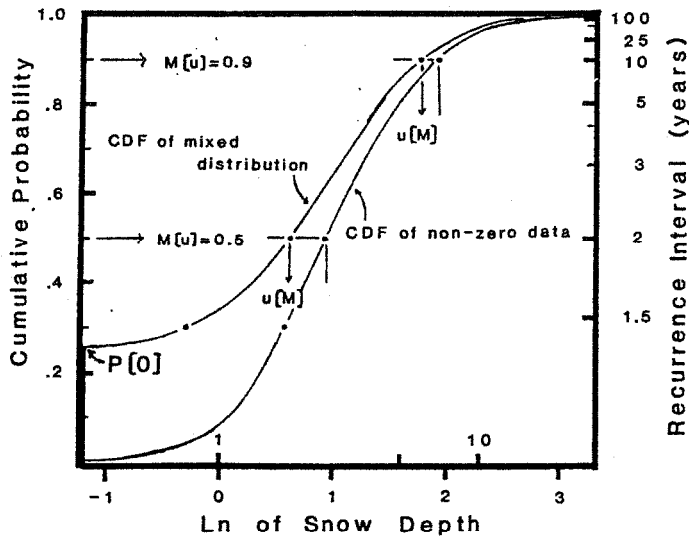


Figure 5. Diagram of the distribution function for a mixed-probability model of the hypothetical sample shown in figure 4. See text for explanation.

for $\bar{x}=0.96$
 $sd=0.74$
 $P[0]=0.26$

$$u[M] = 0.74 \times N^{-1} \left[\frac{M[u] - 0.26}{(1 - 0.26)} \right] + 0.96$$

Ex: for $M[u] = 0.9$,
 $u[M] = 1.8$
 $\Rightarrow d = \exp(1.8) = 6.05$

tion, the sites aren't perfect indicators of snow accumulation at a given elevation. The reports don't supply detailed information on vegetation, aspect, etc., so these can't be used for regression without inspecting the sites; in some cases the vegetation, and thus the environment for snow accumulation, has changed since a snow course was established.

To supplement the published information, I measured snow depth and water content at study sites in the Green River basin, where a pair snow courses in forest and an adjacent clear-cut, both at .730 m elevation, were visited from January 1982 to April 1984. Measurement at these sites were intermittent, and snow depth never exceeded 65 cm at observation times, but these data provide some insight into the effect of forest vegetation on snow accumulation. The observations reported by Beaudry and Golding [1982] and Berris and Harr [1987] are also being utilized for comparisons of forested and clearcut conditions. Initial calculations suggest that the ratio of snow in forests to that in clearings (F:C) seems to be sensitive to elevation: on average, F:C is approximately 20% at lower elevations (.750 m), .65% around 1000 m, and 100-260% (depending on time of year) at .1200 m.

Methods of Calculation, and Results

The series of weather-station measurements for each calendar day over the record period was considered a sample, and mean and standard deviation calculated. Then the parameters for groups of five days were averaged. (For samples with ≤ 3 non-zero members, the parameters were calculated with the data for the entire 5-d period.) For the snow-survey data, observations made on and about a certain date (1st or 15th of the month) were considered a sample, and the parameters indexed onto the nominal date. The probability of no snow was calculated as the ratio of null observations to total measurements for a given date.

In order to determine the patterns of frequency characteristics through the winter, second- and third-order polynomial equations, using date ($t = [\text{days since 1 Sep}]/100$) as the independent variable and statistical parameters (p) as the dependent variables, were fit by least-squares regression to the NWS data, and to those from the snow courses with at least four measurement dates. The equations have the form $p_t = A + Bt + Ct^2 + Dt^3$. Third-order polynomials were found to be more suitable, because they allow greater freedom in curve shape; those for the higher stations have a tendency to be asymmetrical, due to longer accumulation seasons and rapid spring melt. This can be seen in figure 6, which shows the

log-mean and P[0] curves for the weather stations; the Greenwater and Stampede Pass data show that the presence and depth of snow persists later into the season.

As with storm precipitation, elevation is not the only determinant of snow accumulation. Figure 6 demonstrates this: Palmer 3ESE, at 280 m but surrounded by mountains, gets more snow than Mud Mountain Dam, at 399 m but on a relatively open bench. However, the combination of snow-course and weather-station data provided a fair elevation transect, at least on major sampling dates (1st of the month, Dec - Jun). Therefore, polynomial equations were fit to the parameters for these dates, with elevation (e , 10^3 m or ft) as the independent variable, in the form $p_e = A + Be + Ce^2 + De^3$; again, 3^{rd} polynomials were necessary for asymmetric curves. The relation between elevation and the parameters is not perfect, but a good correlation is evident, especially for means and P[0]. The shapes of the curves can be used to demonstrate the geographic and temporal patterns of snow accumulation and melt (such as the shape of the snow wedge [U.S.A.C.E., 1956]). In particular, anomalies can be interpreted for insight into the microclimatic conditions of certain sites that apparently get significantly more or less snow than they would seem to be entitled.

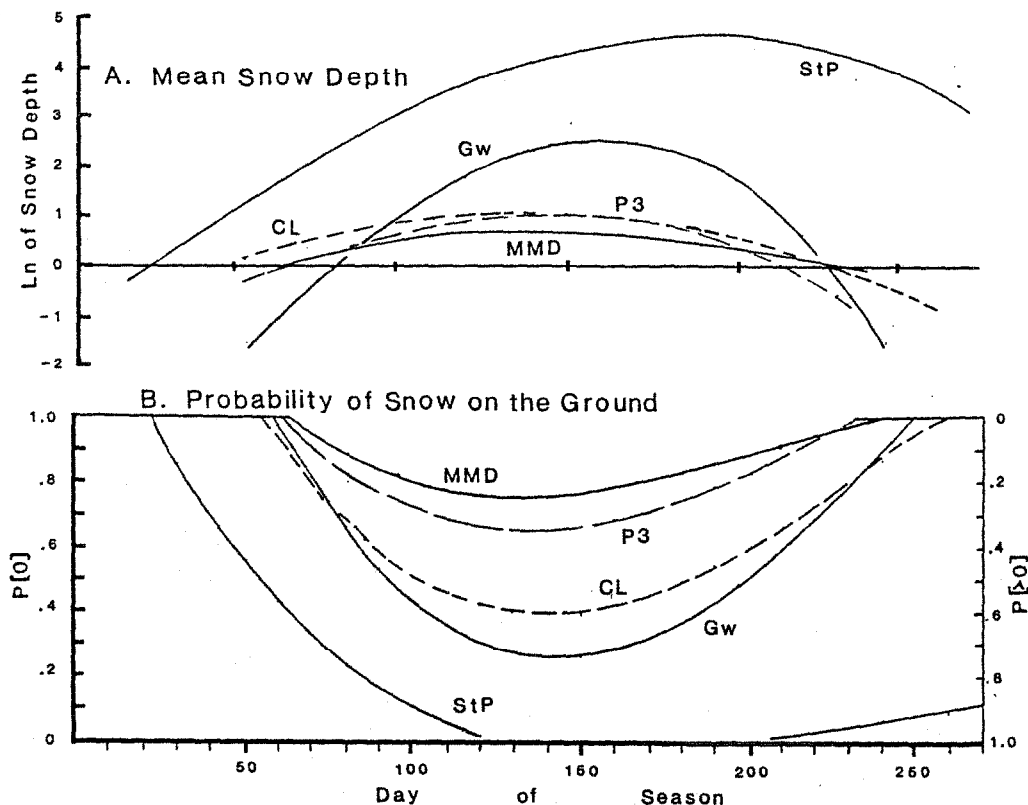


Figure 6. Curves derived from regression of logarithms of mean snow depth (in inches) and P[0] against date (1 Sep = 1), at Stampede Pass (StP), Greenwater (Gw), Palmer 3ESE (P3), Mud Mountain Dam (MMD), and Cedar Lake (CL) weather stations.

In order to combine the mathematical relations for date and elevation into trend surfaces for use in the Monte Carlo model, multiple regression was performed on the six parameters needed, with date and elevation as independent variables. Regression was performed using various combinations of polynomial orders (up to 3^{rd}) of the independent variables, and the forms of the equations were on the basis of higher coefficients of determination (r^2). The resulting equations:

$$\bar{x}_{t,e}(\ln d) = -3.846 + 4.148t - 1.661t^2 + 1.447e - 0.226e^2 + 0.557te \quad (.950)$$

$$s_{t,e}(\ln d) = 0.259 + 1.401t - 0.826t^2 + 0.141t^3 + 0.088e - 0.023e^2 - 0.014te \quad (.373)$$

$$P[0](d)_{t,e} = 2.396 - 2.115t + 1.083t^2 - 0.138t^3 - 0.435e + 0.055e^2 - 0.070te \quad (.941)$$

$$\bar{x}_{t,e}(\ln \text{SWE}) = -6.926 + 6.590t - 1.765t^2 + 1.409e - 0.154e^2 + 0.136te \quad (.960)$$

$$s_{t,e}(\ln \text{SWE}) = 0.131 + 0.7985t - 0.661t^2 + 0.200t^3 + 0.387e - 0.033e^2 - 0.1265te \quad (.471)$$

$$P[0](\text{SWE})_{t,e} = 1.608 - 1.462t + 0.919t^2 - 0.126t^3 - 0.258e + 0.049e^2 - 0.139te \quad (.892)$$

(r^2 values are shown in parentheses; functions were calculated for parameters in the original system of units, inches of depth and SWE, 10^3 ft elevation.) Figure 7 shows projections of the trend surfaces for depth, for a range of elevations. The mean and P[0] curves are quite reasonable. The low r^2 values for the standard deviations reflect the wide range of variances among the separate distributions, but they result in a fairly reasonable pattern.

As with the functions of date and elevation, there are anomalies around the trend-surfaces: some stations behave as if higher or lower than their actual elevations, because of site-specific climatic conditions (or oddities in the data series). Nevertheless, the equations provide a means of estimating snow amounts through the winter and across a range of elevations. Thus, for a given site elevation and date, the amount of snow on the ground corresponding to a randomly generated probability can be calculated from

$$u[M] = s_{t,e}(u) * N^{-1} \left[\frac{M[u] - P[0]_{t,e}}{P[>0]_{t,e}} \right] + \bar{x}_{t,e}(u)$$

with the mean, standard deviation, and P[0] all functions of date and elevation. Solution of this equation for $u[M]$, the log of the amount of snow corresponding to these parameters and the mixed-probability distribution, involves the use of a numerical-integration subroutine within the computer program (fig. 2). As was mentioned above, by coupling these calculations to an iterative program that can supply any number of randomly-generated probabilities, a record of snow depth or water content can be simulated.

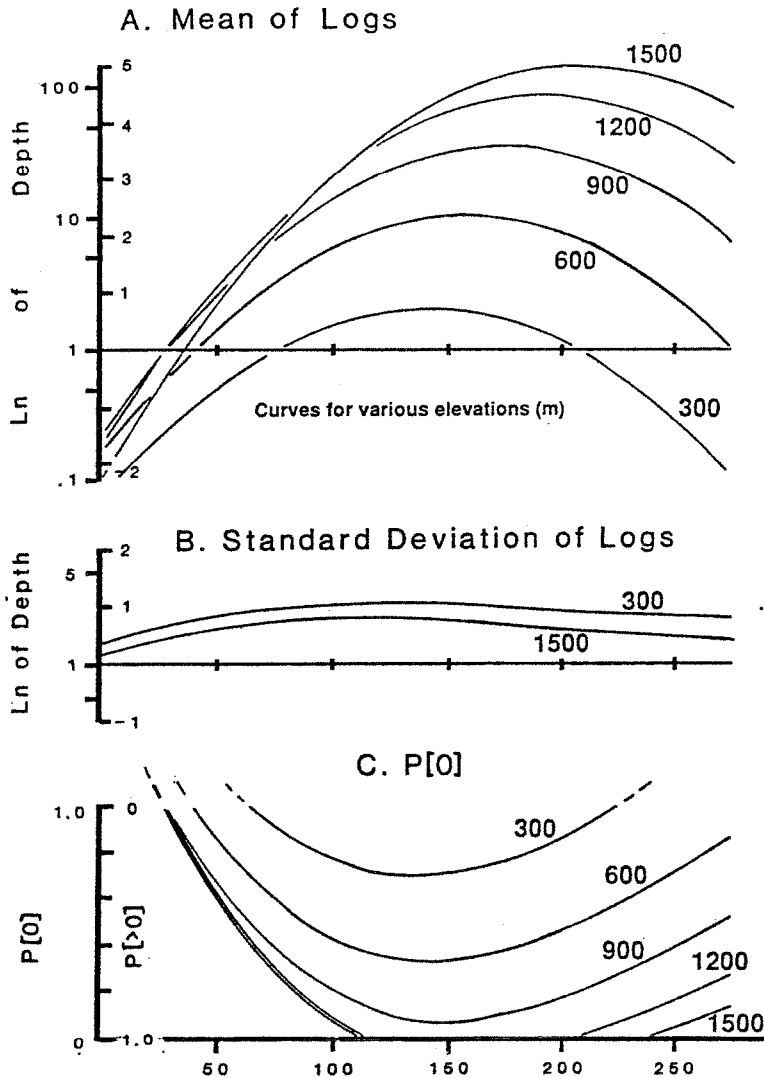


Figure 7. Projections (for selected elevations) of the trend surfaces determined by multiple regression of statistical parameters against date and elevation (ordinate scales in inches).

Tests of the model within Monte Carlo simulation have produced acceptable results. Slight adjustments must be made to the regression equations in the model, based on data from the snow courses and weather stations, to keep the functions physically reasonable. For example, the solution to the equations for P[0] should not fall outside the allowable range for a probability; so, assuming that there is a finite probability of snow on the ground any

day of the winter season, and likewise a finite probability that there will be no snow, $P[0]$ has been constrained to the range 0.0001 - 0.9999.

For illustrative purposes, the program was run for the case of a site at 3000 ft ($e = 3.0$), with snow depths simulated for 1 March ($t = 181/100$). The results of a "50-yr" simulation are summarized in table 1, along with the corresponding data and statistics for the same date from the snow courses close to the model elevation. This experiment produced a sample of snow depths that is consistent with the parameters of the log-normal distribution (generated from the regression equations), and mostly within the range of the parameters at the sites with similar elevations. This should not be surprising, since the data from those sites were used to create the regression equations. One apparent anomaly is that the model produced greater maximum realizations than the greatest depth recorded at any of the sites. This can be explained by noting that these values were produced in a simulated record of 50 yr, while the snow courses had records <29 yr long. (Without this peak event, the mean of the model series would be almost 5 in. lower, with corresponding reductions in deviation and skew coefficient, to quantities more like those for the observed records.) A 6-m snowpack at 900 m is not inconceivable: 592 cm accumulated at Stampede Pass (1206 m) in 1950.

Table 1. Sample Monte Carlo Run

Param	Model		Site		Data				
	Output	GM2	SFC	MtW	LCK	CM	MtG	TCK	
Elev (ft)	3000	2900	3000	3000	3100	3200	3300	3400	
Years	50	24	29	26	24	10	23	23	
Max	232	100	95	56	106	83	87	121	
All data									
Mean	50.3	41.0	46.5	19.3	57.1	43.6	37.3	51.9	
S.d.	48.1	31.4	31.2	19.5	29.6	31.9	29.3	34.1	
Skew	1.59	.51	-.01	.55	-.23	-.32	.14	.16	
$P[0]$.110	.120	.083	.103	.346	.042	.200	.087	
Logs									
Mean	3.55	3.72	3.46	3.70	3.18	3.91	3.85	3.41	3.82
S.d.	.90	.86	1.05	.90	.74	.73	.65	1.04	.79
Skew	-.08	-1.78	-1.88	-.98	-2.14	-1.22	-1.32	-1.54	
Parent distribution of log-normal sample									
Mean	52.3	59.1	55.1	60.5	31.6	65.6	58.0	52.0	62.7
S.d.	58.8	60.7	77.9	67.8	27.2	55.2	41.7	72.1	58.7

Note: Measurements in inches. Model output - numbers in left column were generated by regression equations; numbers on the right are statistics for this run.

Discussion and Implications

Taken together, the multiple-regression trends can illustrate the general relations among date, elevation, and snow accumulation. Graphs such as the one in figure 8 show the average probability of having snow on the ground ($P[>0]$) and the mean SWE through the year, in this case for an elevation of 750 m. The family of such curves have shapes similar to those seen in the original data and distributions (figs. 5 and 6): the maximum snow likelihood increases and occurs later in the season at higher elevations; average SWE follows a similar pattern, with peak accumulation later in the winter than the maximum probability. By solving for $P[0] = 0.5$, the elevation of the average snowline for a given date can be estimated.

The function solutions can also be used to suggest boundaries for the transient snow zone. Therefore, an operational definition of the TSZ must almost necessarily be based on probabilities, as informed by empirical experience and at least partly controlled by the kinds of data available. The zone could be defined based on the probability of having snow for a certain length of time. If the TSZ is defined, for example, as the elevation band in which the probability of snow on the ground for at least 30 d is >50% but <75%, then the equations indicate that it lies at approximately 485-700 m in this part of the Cascades. It might be more appropriate to consider $P[>0]$ for periods when big storms are most likely: in mid-December, for instance, the 50-75% probability band is at 550-850 m. The SWE curves show how much water (on average) could contribute to infiltration or runoff; some criterion regarding a minimal amount of SWE available for melting could be added to refine the operational definition of the transient snow zone.

It must be kept in mind that these are trends and averages based on a short record of variable data, and that they merely suggest the time periods and elevation bands when snowpacks are likely. It remains for the full Monte Carlo simulations to indicate the zone in which

the combination of rain and snowmelt is most hydrologically effective. Work on the computer program is continuing; it has demonstrated its ability to simulate realistic series of most of the individual precipitation and weather components, but the merger of these elements in the snowmelt and percolation routines remain to be tested for realism and sensitivity.

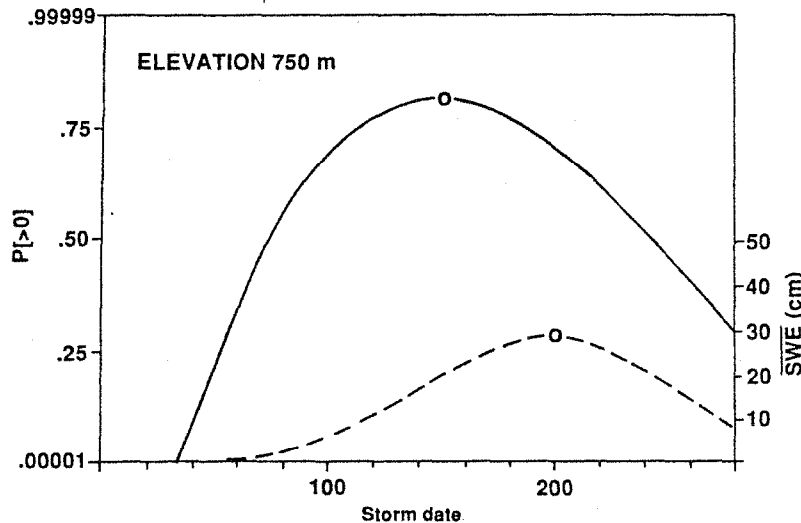


Figure 8. Graph of solutions for multiple-regression equations for snow-water content, showing likelihood of snow on the ground ($P[>0]$, solid) and average SWE (dashed), for sites at 750 m. Note that peak of snow quantity curve lags behind the peak of the likelihood curve.

However, based on the results of initial program runs, it seems safe to believe that Monte Carlo simulation can be used to generate series of F-I-D "data" and hydrographs from hypothetical R/SM events, and that examination of the statistical properties of these series of realizations can provide indirect but valid and effective means of studying their long-term characteristics. In particular, stochastic methods are means of investigating whether there are significant differences in the magnitude of hydrologic processes resulting from R/SM; how differences in geographic and topographic factors affect them; and whether there are important implications for forestry operations, due to the occurrence of this form of extreme hydrometeorological event in humid mountainous regions of the Pacific Northwest.

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