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Disaggregation Models of Seasonal Streamflow Forecasts

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ABSTRACT

Pacific Gas and Electric Company (PG&E) has developed a regression-based procedure for disaggregating a seasonal runoff forecast into monthly flow forecasts. These forecasts can be used to estimate the most likely or mean value for each month's runoff, or to estimate runoff for particular meteorological scenarios, given current conditions. Four basic seasonal-to-monthly disaggregation model families were studied: (1) linear, (2) polynomial, (3) exponential, and (4) logit. The logit model, which ensures that monthly forecasts are always non-zero and less than the forecasted seasonal volume, had larger or comparable R^2 than the polynomial and exponential models, so the latter two were dropped from further analysis. A program was developed which considers the linear and logit models with up to three explanatory variables: the total seasonal forecast, previous month's flow, and future monthly precipitation. It can select, using standard statistical criteria, a model reasonable for each site and season.

INTRODUCTION

PG&E generates seasonal streamflow forecasts for use in hydropower scheduling (Freeman, 1992). Substantial effort at PG&E and elsewhere has gone into the development of procedures for estimating the total seasonal runoff from previous flows, snowpack water content, and precipitation (Stedinger et al., 1989 and 1992). These seasonal runoff forecasts are subsequently disaggregated into monthly flow forecasts for use in PG&E's HYSS model (Ikura and Gross, 1984), which derives a monthly hydropower operations schedule. Similar applications are described in Johnson et al. (1991) and Staschus et al. (1989). The seasonal-to-monthly disaggregation procedure is a simple and quick alternative to more elaborate conceptual runoff models such as the National Weather Service River Forecasting System (Day, 1985), which requires extensive calibration for individual watersheds. This paper describes streamflow-forecast disaggregation models which can automate the disaggregation of seasonal forecasts to monthly values. Jacobs et al. (1993) describe the anticipated hydro-scheduling system these forecasts would support.

We considered four basic seasonal-to-monthly disaggregation model families. Each can be used to estimate the most likely or mean value for month t's runoff, given the total seasonal flow from month t through July. Thus, given an April 1 seasonal forecast, one of these models could be used to estimate April's runoff. The April-through-July forecast would then be decreased by the estimated April runoff yielding the May-through-July forecast. The May-through-July forecast would be the basis of the estimated May streamflow, and so forth. This is simpler than the all-at-once procedures described in Pei et al. (1987).

This process yields estimates of monthly flows in April through June from an April-July forecast made on April 1; July's estimated flow would then be found by difference. On May 1 a new forecast of May-July flows would normally be available, and that would be disaggregated into individual monthly flows starting with the May disaggregation model.

In the next section the four basic model families are introduced. The following sections summarize the ability of those models to describe the observed relationships in the longer streamflow data sets available for PG&E watersheds.

ALTERNATIVE SEASONAL-TO-MONTHLY DISAGGREGATION MODELS

Several families of models could be used to divide a seasonal streamflow forecast among the months within the forecast period. Let

- q_t = the monthly flow in month t,
- Q_t = the forecast of the seasonal flow from month t through July,
- p_t = the monthly precipitation in month t at a nearby station, and
- a_t, b_t, c_t and d_t = parameters of a disaggregation model.

Four families of disaggregation models were considered, each corresponding to a different functional relationship between the forecasted runoff Q_t, and the estimated runoff q_t for the month following the forecast date. The model families are (i) linear, (ii) polynomial, (iii) exponential, and (iv) logit. A proportional model is a special case of all four families. Each is described below.

Proportional Model

The simplest model one might consider would be that a fixed fraction, or proportion a_t, of the seasonal flow Q_t arrives in each month t:

$$q_t = a_t Q_t \tag{1}$$

The residual seasonal flow $(Q_t - q_t)$ is then the forecast Q_{t+1} which needs to be divided among the remaining months.

Linear Models

The proportional model in (1) will serve as our base case. However, more sophisticated models can be employed to account for the general delay in runoff with an increase in the seasonal runoff volume. One such model is:

$$q_t = a_t + b_t Q_t \tag{2}$$

It may be the case that a_t is not significantly different from zero, in which case the two-term linear model in (2) reduces to the simpler model in (1).

One can also include a third term which reflects whether current flow levels are relatively large or small, which would reflect baseflow and groundwater levels, whether the snowpack is beginning to melt, or recent winter rainfall contributing to current streamflow levels. The linear model in (2) then becomes:

$$q_t = a_t + b_t Q_t + c_t q_{t-1}. \tag{3}$$

Finally, a fourth term can be added to incorporate the impact of precipitation p_t in the current month:

$$q_t = a_t + b_t Q_t + c_t q_{t-1} + d_t p_t \tag{4}$$

Equation (4) is particularly attractive when p_t corresponds to a precipitation scenario used to generate Q_t.

Polynomial Models

It is also important to consider the hydrology and the physics of the situation. Certainly, if the seasonal forecast were zero, then all of the monthly flows should also be zero. Thus one might consider polynomial models of the form:

$$q_t = Q_t [a_t + b_t Q_t + c_t q_{t-1} + d_t p_t] \tag{5}$$

The terms $b_t Q_t$, $c_t q_{t-1}$ and $d_t p_t$ within the brackets might not be statistically significant; if all these terms were dropped, this polynomial model would reduce to the proportional model in equation (1).

Exponential Models

A problem with the linear and polynomial model families is that they could, in unusual situations and for particular combinations of the parameters, generate negative monthly flow forecasts. This is a common problem with such linear and polynomial models; it is also encountered in extremely dry years with linear regression models that estimate seasonal runoff volumes. A seasonal-to-monthly disaggregation model that does not suffer from that deficiency is the exponential model:

$$q_t = Q_t \exp[a_t + b_t Q_t + c_t q_{t-1} + d_t p_t] \tag{6}$$

Again, the second, third and fourth terms inside the brackets may not be necessary. Were all of them dropped, the exponential model would also reduce to the simple proportional model in equation (1).

Logit Models

The exponential model in equation (6) does not generate negative flow estimates (for $Q_t \geq 0$), but could in unusual circumstances generate monthly flows q_t which exceeded the specified seasonal total Q_t. A logit model that ensures that the estimated runoff ratio q_t/Q_t for a month is between zero and one, and hence that $0 \leq q_t \leq Q_t$, is

$$q_t = Q_t/\{1 + \exp[a_t+b_tQ_t+c_tq_{t-1}+d_tp_t]\} \tag{7}$$

Again, the second through fourth terms may not add significant forecasting power and could be omitted.

With this model, the ratio q_t/Q_t is estimated by

$$\{1 + \exp[x_t]\}^{-1} \qquad (8)$$

which is always between zero and one; in this case

$$x_t = [a_t + b_t Q_t + c_t q_{t-1} + d_t p_t]. \qquad (9)$$

When the estimated ratio q_t/Q_t is small, the $\exp[x_t]$ term in (8) is substantially larger than one. In such cases, the model in (7) will look and behave like the model in equation (6), and to a lesser extent like the model in equation (5). However, when the estimated value of q_t/Q_t approaches 50% or more, the logit model in equation (7) begins to look like

$$q_t = Q_t \{1 - \exp[x_t]\} \qquad (10)$$

so that q_t approaches Q_t for large negative x_t.

MODEL COMPARISON

In the initial research on disaggregation models we examined models that incorporate only information known at the time of the forecast, i.e. the previous flow q_{t-1} and the total seasonal forecast Q_t. These initial "screening tests" will be described first. On the basis of these tests some models were judged to be dominated by others and were dropped from further consideration; the remainder were implemented in general fitting and forecast programs currently in trial use at PG&E.

Recently PG&E has been developing a stochastic scheduling model that uses multiple inflow "scenarios" that start from current conditions. Since the seasonal inflows in these scenarios depend on anticipated seasonal precipitation, we investigated whether including monthly precipitation in the disaggregation models would significantly improve their explanatory power. That research is described in a subsequent section.

Screening Test

Standard linear and non-linear regression procedures were used to investigate which of these models best described the relationships between the month t through July runoff, Q_t, and the observed flow in month t, q_t. Linear regression is adequate to estimate the parameters of the linear and polynomial models, whereas nonlinear regression was used to obtain least-squares estimates of the exponential and logit models' parameters without transformation of the flows.

In the initial screening test, the simple proportional model in equation (1) and each of the four families were considered, both with and without the $c_t q_{t-1}$ terms, but without including monthly precipitation p_t. The resulting models are listed in Table 1. Models were developed for the months of January through June for the 14 gage sites, listed in Table 2, which

Table 1. Models Considered in Regression Analyses

Abbreviation	*Functional Form*	*Family*
P[1,2]	$q_t = a_t Q_t$	Proportional
LQ[1,2]	$q_t = a_t + b_t Q_t$	Linear
LQq[1,2]	$q_t = a_t + b_t Q_t + c_t q_{t-1}$	Linear
LQp[2]	$q_t = a_t + b_t Q_t + c_t p_t$	Linear
LQqp[2]	$q_t = a_t + b_t Q_t + c_t q_{t-1} + d_t p_t$	Linear
PQ[1]	$q_t = Q_t [a_t + b_t Q_t]$	Polynomial
PQq[1]	$q_t = Q_t [a_t + b_t Q_t + c_t q_{t-1}]$	Polynomial
EQ[1]	$q_t = Q_t \exp[a_t + b_t Q_t]$	Exponential
EQq[1]	$q_t = Q_t \exp[a_t + b_t Q_t + c_t q_{t-1}]$	Exponential
NQ[1,2]	$q_t = Q_t / \{1 + \exp[a_t + b_t Q_t]\}$	Logit
NQq[1,2]	$q_t = Q_t / \{1 + \exp[a_t + b_t Q_t + c_t q_{t-1}]\}$	Logit
NQp[2]	$q_t = Q_t / \{1 + \exp[a_t + b_t Q_t + c_t p_t]\}$	Logit
NQqp[2]	$q_t = Q_t / \{1 + \exp[a_t + b_t Q_t + c_t q_{t-1} + d_t p_t]\}$	Logit

[1] Included in initial screening test
[2] Included in final model comparison

Table 2. Sites Considered in Regression Analyses

ID	*Name*	*Record*[1] *Screening*	*Final*
8105	Bucks Lake@Bucks Lake	1935-83	1935-92
8190	Lost Creek@Lost Creek Reservoir	1927-83	1940-92
8215	North Yuba@Bullards Bar	1940-83	1940-92
8230	Canyon Creek@Bowman Lake	1931-83	1931-92
8245	South Yuba@Lake Spaulding	1929-83	1929-92
8330	North Fork Mokelumne@Salt Springs Reservoir	1937-83	1937-92
8345	Middle Fork Stanislaus@Beardsley Reservoir	1920-83	1922-92
8350	South Fork Stanislaus@Lyons Reservoir	1943-83	1943-92
8440	Eel River@Lake Pillsbury	1922-83	1922-92
8090	North Fork Feather@Lake Almanor	1945-83	1945-92
8100	East Branch Feather@NF-51	1949-83	1949-92
8175	South Fork Feather@Little Grass Valley Res.	1948-83	1948-92
8265	Middle Fork American@French Meadows Res.	1951-83	1951-92
8280	Rubicon River@Hell Hole Reservoir	1947-83	1947-92

[1] Initial screening used only flow records and was performed in 1985-87
Final model comparison used contiguous flow and precipitation records, performed in 1993.

had 33 or more years of record as of 1983 and were of interest to PG&E.

Table 3 summarizes the average by month over all 14 sites of the R^2 values for each model considered. In all cases R^2 was calculated as

$$R^2 = 1 - \frac{\Sigma_y \left(q_{yt} - \widehat{q_{yt}} \right)^2}{\Sigma_y \left(q_{yt} - \mathrm{mean}(q_{yt}) \right)^2} \qquad (11)$$

where $\widehat{q_{yt}}$ is the particular model's estimate of month t's flow q_{yt} for each year y of record. For the proportional model at some sites, most notably in March and April, the computed R^2 was negative. In those cases the model $\widehat{q_{yt}} = a_t Q_{yt}$ was not as accurate as the constant model $\widehat{q_{yt}}$ = mean.

There was great variability among the values of R^2 for individual sites in the various months. The average values in Table 3 and displayed in Figure 1 illustrate the general trends. The R^2 values in Table 3 are not corrected for the number of parameters estimated. In many cases the models include parameters which were not significantly different from zero.

An important issue is which of these nine models are the best candidates to use in an operational setting for disaggregating seasonal runoff forecasts. We observed that the performance of the exponential models (EQ and EQq) were almost identical with the corresponding logit models (NQ and NQq), both on average and at individual sites. In general, the performance of the polynomial models (PQ and PQq) was not quite as good as the corresponding logit models (NQ and NQq). In addition, the logit models honor the physical constraint that the runoff should be non-negative and less than the seasonal runoff volume forecast. Thus the logit models were judged as domi-

Table 3. Average R^2 Values from Screening Test Based on Data Through 1983

Model	*Jan.*	*Feb.*	*March*	*April*	*May*	*June*
P	0.29	0.34	0.27	0.01	0.73	0.962
LQ	0.31	0.36	0.36	0.37	0.84	0.971
PQ	0.30	0.36	0.33	0.37	0.84	0.977
EQ	0.31	0.36	0.35	0.43	0.87	0.977
NQ	0.31	0.36	0.35	0.43	0.87	0.976
LQq	0.35	0.38	0.38	0.40	0.86	0.975
PQq	0.34	0.38	0.35	0.44	0.87	0.979
EQq	0.34	0.38	0.37	0.47	0.88	0.979
NQq	0.34	0.38	0.37	0.46	0.88	0.978

nating the corresponding members of the exponential and polynomial families. On the other hand, while the linear models (LQ and LQq) performed worse on average than the logit models, there were clear instances in which they were superior.

For March and April forecasts, the proportional model (P) often had a negative R^2 value indicating that simply setting $\hat{q}_t$ to the mean would predict better than the proportional model. This makes sense: a deeper snowpack tends to ripen later, so streamflows early in the melt period may actually be less in a "wet year" than in an average or dry one. Prior to the melt, flows are more dependent on (warm) precipitation so wet years tend to have higher monthly flows; later in the melt period the correlation is positive as well.

PG&E's DFIT program considers six models as candidates for estimating q_t. These six include the simple constant model $\hat{q}_t$ = mean, which is considered to be a 1-parameter linear model, and the 2- and 3-parameter linear models (LQ and LQq). Also included are the two nonlinear logit models (NQ and NQq), and the proportional model (1), which can be thought of as the 1-parameter logit model: $\hat{q}_t = Q_t/\{1+\exp[a_t]\}$. These models have been used on a trial basis to guide streamflow forecasting at PG&E since 1992.

Including Monthly Precipitation

Under ordinary circumstances, variables whose values are unknown at forecast time would not be included in a forecasting model. However, future precipitation is often included in seasonal streamflow forecasts. In the case of streamflow disaggregation models, the inclusion of future precipitation can serve two useful purposes. First, it allows for efficient generation of "what if" inflow scenarios - e.g., what would streamflows be like if the rest of the spring was like the year 1977. Second, it allows the model to distinguish between snowmelt from the currently-measured snowpack and runoff from rain or snow that falls later in the spring. Thus a model that includes monthly precipitation may be more accurate in predicting the expected runoff from a heavy or light pack (assuming mean future precipitation) than one that does not.

The effect of including monthly precipitation p_t in disaggregation models was examined using records through 1992; see Table 1. Table 4 and Figure 2 report the results. The new linear models LQp and LQqp and non-linear logit models NQp and NQqp were compared to the linear and logit models currently in use (including the proportional model P).

A number of results are quite striking. First, including current monthly precipitation p_t in both linear and non-linear models increased the R^2 dramatically in January and February, and to a lesser extent in March and April. The effect of including previous flow q_{t-1} was less dramatic in these months, but often significant. In May and June, monthly precipitation was rarely of much value while previous flow had a somewhat greater impact. In these months the disaggregation models worked very well, with average R^2 for the logit models close to 90% in May and 98% in June. Thus the May and June runoff volumes can be estimated quite accurately given the volume of runoff to actually occur during the May-July or the June-July period.

The month at the start of the melt period, April, is anomalous in that neither previous flows nor current precipitation add much to forecast precision and the overall R^2 is less than 50% on average. Part of the reason is that precipitation generally tails off in April and May. Thus temperature must account for much of the unexplained variation in these months. In addition, in April a very heavy pack may yield small flows because it takes longer to ripen, while a very light pack may do the same because much of the

Table 4. Average R^2 Values from Final Comparison Based on Data Through 1992.

Model	*Jan.*	*Feb.*	*March*	*April*	*May*	*June*
P	0.324	0.364	0.242	-0.100	0.781	0.966
LQ	0.344	0.376	0.312	0.366	0.860	0.973
NQ	0.338	0.381	0.319	0.422	0.891	0.979
LQq	0.388	0.390	0.405	0.414	0.880	0.976
NQq	0.374	0.398	0.406	0.458	0.905	0.981
LQp	0.560	0.520	0.401	0.407	0.867	0.974
NQp	0.624	0.606	0.443	0.473	0.905	0.980
LQqp	0.614	0.541	0.488	0.453	0.884	0.976
NQqp	0.682	0.626	0.518	0.507	0.914	0.982

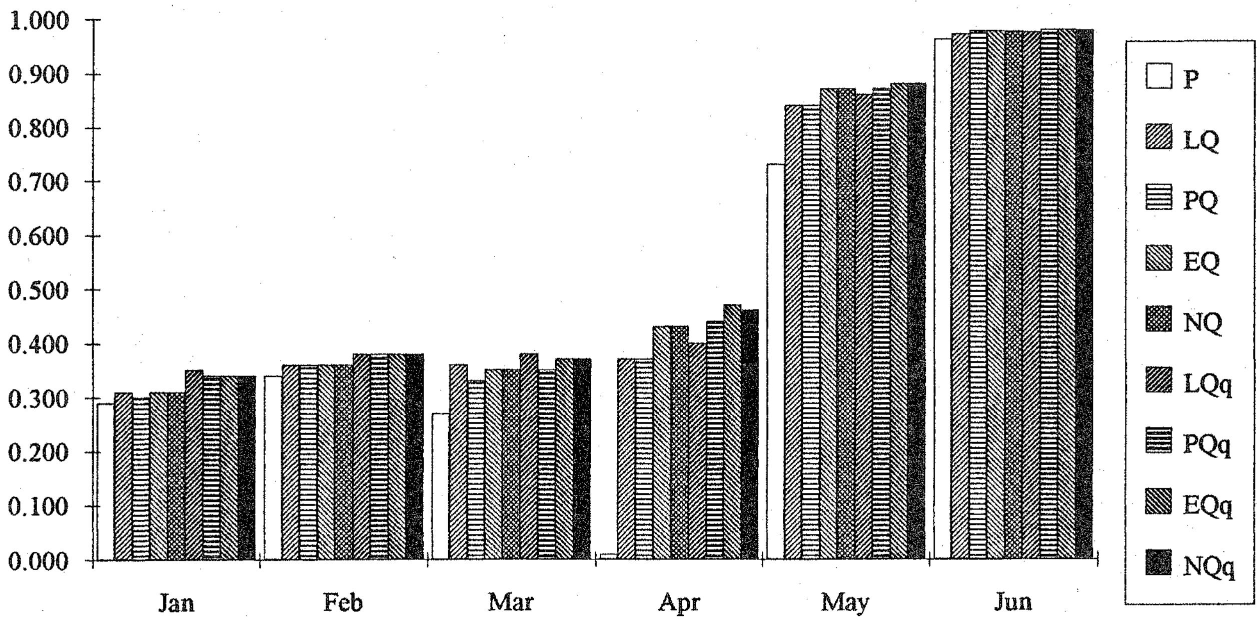

Figure 1. Average R-squared for Original Models Based on Data Through 1983

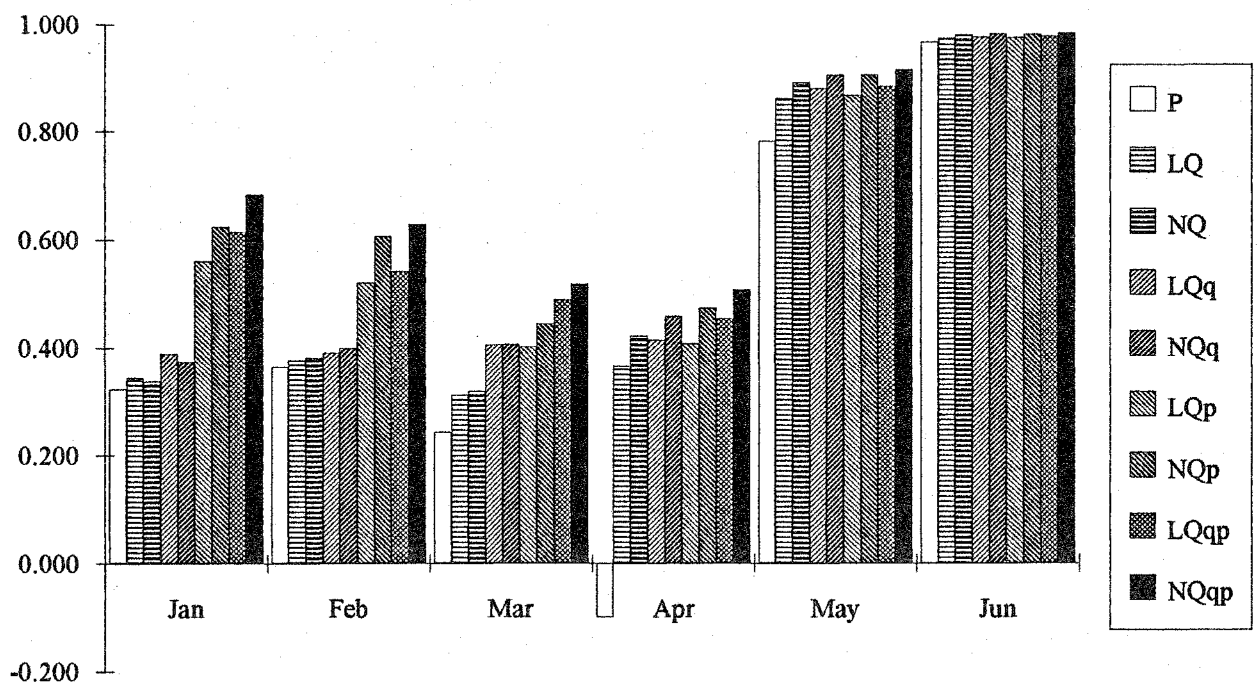

Figure 2. Average R-squared for Final Models Based on Data Through 1992

snow-covered area disappears quickly. While the logit model can mimic this behavior somewhat, a different form of non-linear model might perform better. Later in the melt in May and June, the rate of snowmelt is more directly related to the total remaining runoff Q_t because the pack has already ripened and the primary determinants of snowmelt would be temperature and snow-covered area, which is related to anticipated runoff volume. All of the disaggregation models perform well in May and June.

CONCLUSIONS

For the months of January through April, given a perfect seasonal runoff forecast and precipitation forecast at the beginning of a month, the actual flow in that month can be estimated by the better models with an average R^2 of 50-70%. Monthly precipitation has a particularly strong impact on the accuracy of the estimate in January and February. The results also illustrate the value of using a regression model. In many months the simple model $\hat{q}_t = a_t Q_t$ yields R^2 values substantially smaller than those achieved by the more general linear and logit models. Thus there is value in using a model with some sophistication.

Still, the large residual or unexplained variation with the better models indicates that years with the same or nearly equal seasonal forecasts may experience very different runoff patterns. Thus attempts to use the runoff pattern from a particular historical year, which had a seasonal forecast roughly equal to the current year's forecast, may not yield the best or even particularly reasonable estimates of this year's runoff pattern, given the anticipated runoff volume Q_t, monthly precipitation p_t and antecedent streamflow levels q_{t-1}. There should be value in using disaggregation models which generate an average runoff pattern, based upon all of the available years of record, and the current hydrologic conditions.

The logit models yielded an average R^2 of 89-91% for May and 98% for June. In these two months all the multi-parameter models provide very accurate runoff predictions, given a perfect seasonal forecast. Meteorological variables have much less impact on the monthly pattern of runoff this late in the season.

Apart from the months of March and April, the models that include monthly precipitation had satisfyingly high R^2. Thus a monthly disaggregation model could be used as a simple alternative to the National Weather Service's Extended Streamflow Prediction procedure (Day, 1985) to generate scenarios of inflows given current conditions and assumptions about future weather. However, it should be noted that the accuracy of a monthly forecast based on assumed precipitation values will not approach the R^2 cited here unless the weather forecaster is gifted with perfect foresight. For the months of March and April, where R^2 was relatively low, the models might benefit from terms including monthly average temperature. Disaggregation models that include temperature are currently under development.

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