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Improving Estimates of Snowpack Water Equivalent Using Double Sampling

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ABSTRACT

Equations are presented to estimate the average water equivalent of a snowpack and the variance of this estimate using double sampling. Double sampling refers to sampling the snowpack in two ways: measuring the depth at a number of points, and measuring the depth and the water equivalent at a smaller number of points. In a given sampling interval, double sampling can yield an estimate of snowpack water equivalent having a lower variance than is possible by measuring water equivalents only. The equations presented in this paper were tested during extensive snow surveys conducted on the North Slope of Alaska. The snowpack of the North Slope is similar to that of the Great Plains, thin and variable in both depth and density due to extensive redistribution of snow by wind. This variability makes estimation of the average snowpack water equivalent of an area difficult, and provides a rigorous test of the equations presented here.

Two two-hour snow surveys conducted by sampling water equivalent only, and by sampling both depth and water equivalent yielded identical estimates of water equivalent (48.1 mm). However, the double sampling estimate had a variance of only 80% of the variance of the estimate using water equivalent only (4.9 mm^2 vs 6.1 mm^2). Equations are also presented for calculating the optimal ratio of depth to water equivalent measurements. By considering time as the only cost of collecting the two measurements it is possible to calculate the ratio of depth to water equivalent measurements that will result in the best estimate of average snowpack water equivalent in a given sampling time. The optimal ratio of depths to water equivalent measurements varies between individual surveys, so times measured during snow surveys in a number of different locations and snow conditions on the North Slope were used to estimate an regional optimal ratio of depth to water equivalent measurements of about 14 to 1.

INTRODUCTION

The estimation of average snowpack water equivalent is necessary for the prediction of snowmelt runoff for purposes ranging from power generation and irrigation supply planning to flood prediction. The water equivalent of a snowpack is measured at a point using a standard corer specifically designed for snowpack measurement. A core is taken through the depth of the snowpack, depth of the snow is recorded, and the corer and snow are weighed. The weight of the snow can then be converted to water equivalent at the point sampled, commonly expressed as depth (centimeters or inches) of water. A number of cores are taken in an area to yield an estimate of the average snowpack water equivalent. While this method can yield a reliable estimate of average water equivalent, it has one major disadvantage; collecting and weighing cores is time consuming and therefore limits the number of samples that can be collected.

Another measure of the water equivalent of a snowpack is the depth of the snow. While depth is quick and easy to measure, it is an uncertain indicator of the water equivalent due to variability in the density of snow on the ground. Sampling the snowpack in two different ways, or double sampling, combines the accuracy of collecting and weighing cores with the large sample size possible with measuring snow depth, and produces improved estimates of average snowpack water equivalent when compared with collecting and weighing cores only.

Several methods of snowpack water equivalent estimation that utilize variations of double sampling have been designed and shown to have the potential to produce improved estimates of snowpack water equivalent (Dickinson and Whiteley 1971). Dickinson and Whiteley (1972) found that while snow depth in their study basin in Ontario was highly variable, density of the snowpack varied within a small range. They therefore concluded that the snowpack could be adequately sampled by collecting numerous depth samples and relatively few density samples. Steppuhn (1976) developed a method called centralized sampling which utilized a large number of depth measurements in conjunction with a smaller number of density samples. This method called for density samples to be taken at or near the mean (central) depth encountered in the depth survey, and proved to be an efficient shortcut for determining mean areal water equivalent in prairie snowpacks.

This paper presents equations developed to estimate average snowpack water equivalent, and to estimate the variance of the estimated water equivalent. Also presented are the equations used to predict the ratio of the two types of snowpack measurements that will produce the lowest variance estimate of snowpack water equivalent in a given time. Finally, the application of the equations is demonstrated using the results of snow surveys conducted in 1992 and 1993 on the North Slope of Alaska.

SAMPLING METHODS

The snow surveys analyzed in this paper were conducted using Adirondak snow tubes. These are fiberglass tubes, five feet (152 cm) in length, and 2 7/8 inches (7.3 cm) inside diameter (3 in. (7.6 cm) O.D.), and equipped with metal teeth on the lower end. The tubes are driven vertically through the snow, and the depth of the snow is recorded. The tube is then driven deeper into the uppermost layer of soil or vegetation and tipped sideways, retaining the snow and a small plug of the vegetation or soil under the snow. The collection of a small amount of soil or vegetation insures that the entire depth of the snow has been sampled and is especially important where well developed hoar frost is present, as is frequently the case in Arctic Alaska. The plug of vegetation or soil is then removed, and the tube and snow are weighed on a scale that has been zeroed with the empty tube, and calibrated to read directly in depth (inches) of water. Additional measurements of depth are also collected using the same procedure as above, but omitting removal of the snow and the soil plug. The use of the snow tube rather than another type of probe to collect additional depth measurements eliminates the possibility of discrepancies between measurements made with the tube and the probe skewing the survey results.

Sample points are regularly spaced in a pattern that does not coincide with any natural patterns in the land surface. For instance, points may be regularly spaced on a grid or along one or more straight lines or circles. The number of depths measured is dependent on the number of cores. The ratio of depths to cores is that ratio which is estimated to be the optimum ratio for achieving the best possible estimate of average snowpack water equivalent in the available time. Generally, after each core is collected, the chosen number of depths per core is sampled along the line or circle at a regular increments. The next core is then collected another increment along the transect and the process is continued until a predetermined time limit or number of samples is reached. Time limits or sample sizes are typically determined by time availability. A typical sample spacing used for the surveys reported here is 2 meters.

EQUATIONS FOR DOUBLE SAMPLING

The equations used for the estimation of water equivalent were developed from equations presented in Thompson's "Sampling" (1992), and their development is described in detail in the next section.

Double sampling yields two sets of data: a set of depths which includes all of the depths measured (note that not all depths are associated with a measured water equivalent), and a smaller set of water equivalents (each of which is associated with a depth measurement from the larger set of depths). Let depth be denoted x, and water equivalent be denoted w. The number of depths measured will be n_x, while the number of water equivalents will be n_w.

The average specific gravity of the samples, referred to hereafter as sample density $\overline{\rho}$, is obtained from the sample where both w and x values were measured (total of n_w sample points).

$$\bar{\rho} = \frac{\sum_{i=1}^{n_w} w_i}{\sum_{i=1}^{n_w} x_i}$$

Density as it is defined here is a dimensionless variable (depth water equivalent/ depth snow) which can be converted to a true density by multiplying by the density of water.

The density is used to estimate the average water equivalent $\hat{\mu}_w$ for all of the samples (total of n_x sample points).

$$\hat{\mu}_w = \bar{\rho}\bar{x}_{n_x}$$

Where $\bar{x}_{n_x}$ is the average depth of all n_x depth samples.

The estimated variance of $\hat{\mu}_w$ is given by:

$$v\hat{a}r\ (\hat{\mu}_w) = \frac{s^2}{n_x} + \frac{(n_x - n_w)}{n_x n_w (n_w - 1)} \sum_{i=1}^{n_w} (w_i - \bar{\rho} x_i)^2$$

Where s^2 is the sample variance of the water equivalent samples.

$$s^2 = \frac{1}{n_w - 1} \sum_{i=1}^{n_w} (w_i - \bar{w})^2$$

The optimal ratio of depth to density samples is the ratio which in a given time will yield the estimate with the lowest variance. The cost of sampling a snowpack is time. By measuring the time required for the two types of samples, the optimal ratio of the two types can be estimated using the following equation.

$$\frac{n_w}{n_x} = \sqrt{\frac{c_x}{c_w} \frac{s_\rho^2}{(s^2 - s_\rho^2)}}$$

$$s_\rho^2 = \frac{1}{n_w - 1} \sum_{i=1}^{n_w} (w_i - \bar{\rho} x_i)^2$$

Where c_x is the average time to collect one depth measurement, and c_w is the average time to collect one water equivalent value. s_ρ^2 is the variance of the water equivalent values about a line defined by the product of average density and depth.

Development of equations

The equations for double sampling presented here were developed from equations for estimating the population total of a variable of interest w (in this case, water equivalent). The snowpack water equivalent is estimated using measurements of the water equivalent as well as measurements of an auxiliary variable x. The auxiliary variable is related to the variable of interest, and is cheaper or easier to measure (snow depth). Only snow depth (x_i) is measured over a sample of n_x units, while both water equivalent and snow depth (w_i and x_i) are measured over a smaller subset of n_w samples. Ratio estimation by double sampling may lead to improved estimates of the population mean or total where the w_i and x_i are highly correlated, with a linear relationship in which w_i=0 when x_i=0, (Thompson 1992).

Thompson (1992) gives equations to estimate the population total of the x-values $\hat{\tau}_x$, and the population total of the w-values $\hat{\tau}_w$.

$$\hat{\tau}_x = \frac{N}{n_x} \sum_{i=1}^{n_x} x_i \qquad \hat{\tau}_w = \bar{\rho}\hat{\tau}_x$$

Where N is the total number of potential sampling units in the population.

An estimator of the sample mean water equivalent, $\hat{\mu}_w$ in a population containing an infinite number of potential sampling units can be obtained from the above equations.

$$\hat{\mu}_w = \frac{\hat{\tau}_w}{N} = \bar{\rho}\frac{\hat{\tau}_x}{N} = \frac{\bar{\rho}}{n_x} \sum_{i=1}^{n_x} x_i = \bar{\rho}\bar{x}_{n_x}$$

The variance of $\hat{\tau}_w$ is given by:

$$v\hat{a}r\,(\hat{\tau}_w) = N(N - n_x)\frac{s^2}{n_x} + N^2 \frac{(n_x - n_w)}{n_x n_w (n_w - 1)} \sum_{i=1}^{n_w} (w_i - \bar{\rho} x_i)^2$$

Where s^2 is the sample variance of the w-values of the subsample.

An equation for the estimator of the variance of the mean water equivalent $var(\hat{\mu}_w)$ is obtained as follows.

$$\hat{var}(\hat{\mu}_w) = \frac{\hat{var}(\hat{\tau}_w)}{N^2} =$$

$$\left(\frac{N-n_x}{N}\right)\frac{s^2}{n_x} =$$

$$\frac{n_x-n_w}{n_x n_w(n_w-1)}\sum_{i=1}^{n_w}(w_i-\bar{\rho}x_i)^2$$

$$\text{as } N \Rightarrow \infty$$

$$\hat{var}(\hat{\mu}_w) = (1)\frac{s^2}{n_x} +$$

$$\frac{(n_x-n_w)}{n_x n_w(n_w-1)}\sum_{i=1}^{n_w}(w_i-\bar{\rho}x_i)^2$$

The equation for estimating the optimal ratio of sample types is presented in the previous section in the same form as is given by Thompson (1992).

FIELD TRIAL PROCEDURES AND RESULTS

Field trials were designed to address two questions about double sampling: Can double sampling yield better estimates of snowpack water equivalent than collecting and weighing cores only? What ratio of depth to water equivalent measurement will yield the best estimate in a given sampling time?

To determine if double sampling can indeed yield improved estimates of snowpack water equivalent, side by side snow surveys were conducted, some by double sampling and others by collecting and weighing cores only. Two sets of surveys were conducted. One involved two two-hour surveys of the snowpack on the Alaskan Arctic Coastal Plain near Prudhoe Bay, Alaska in April, 1992. These surveys were conducted over the same area on the same day. The surveys were conducted along four parallel rows of sample points, each approximately 50 meters long. The rows were spaced 7.5 meters apart. Sample points were spaced at approximate 1.5 meter intervals along the four lines. All of the sample points were located in a large area of uniform snowcover. The first and third rows of sample points were double sampled. Depth was measured at each sample point, and in addition, water equivalent was measured at every second point. The second and fourth rows were sampled by measuring depth and water equivalent at every second sample point, omitting measurement of depth at every sample point. Thus, two similar, side-by-side snow surveys were conducted, one by double sampling and the other by measuring water equivalent only. All sampling was conducted by a single scientist. The double sampling of two rows required 130 minutes, sampling the other two rows required 120 minutes.

Another set of surveys was conducted in April 1993 in the basin of Imnaviat Creek, in the northern foothills of the Brooks Range in Northern Alaska. A transect of the Imnaviat Creek basin was sampled by the two snow survey methods. For the first 30 minutes of surveying, snowpack depth was measured at two meter intervals. At every fifth depth measurement, snowpack water equivalent was also measured, resulting in 10 meter spacing of water equivalent measurements. During the second 30 minutes of sampling, depth measurements not associated with a measurement of water equivalent were omitted, and only the depth and water equivalent at 10 meter intervals were measured. This pattern of alternating sampling schemes for 30 minute intervals was continued throughout the survey of the transect. This resulted in a total of seven individual, 30 minute surveys, laid out end to end, four completed by double sampling, three by the normal method of measuring water equivalent (and its associated depth) only. The results of the two sets of snow surveys are presented in Table 1.

To facilitate estimation of the optimal ratio of depth measurements to water equivalent measurements, a number of snow surveys were completed by first collecting and weighing all the cores for water equivalent, then measuring all depths. These two parts of the survey were timed to yield average times required for collection of each water equivalent and of each depth. This is necessary to estimate the optimal ratio of the two measurement types. The times and data sets for each survey were used to estimate the average time for collection of each data type and the optimal ratio for that survey. Optimal ratios averaged by year and location are presented in Table 2. Some of the surveys conducted on the Coastal Plain whose results are presented in Table 2 were conducted simultaneously rather than timed, providing the ratio of the times for collection of the two measurements types, rather than actual time for collection. For this reason the times for data collection on the Coastal

Table 1. Results of snow surveys conducted using the two different sampling schemes in similar snowpacks.

Sampling Scheme	Sampling Duration (min)	Number of Water Equiv't Measur'ts	Variance (cm^2)	Estimated Average Water Equiv't (cm)	90% Confid. Interval (cm)
Foothills Surveys - conducted along a valley transect (r=5/1)					
double	30	13	0.09	8.2	±0.53
normal	30	14	0.15	8.4	±0.66
double	30	9	0.06	9.2	±0.43
normal	30	16	0.36	13.1	±1.1
double	30	11	0.15	15.1	±0.7
normal	30	11	3.87	16.9	±3.6
double	30	5	2.26	21.1	±2.5
Coastal Plain Surveys - conducted side by side on level tundra (r=2/1)					
double	130	30	0.08	4.8	±0.5
normal	120	30	0.12	4.8	±0.6

Plain in Table 2 are normalized. The equations used to estimate the average water equivalent and sample variance for surveys involving only measured water equivalents are as follows.

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n_w} w_i$$

Where $\hat{\mu}$ is the average water equivalent and w_i is water equivalent at point i.

$$v\hat{a}r\ (\hat{\mu}) = \frac{1}{n_w - 1}\sum_{i=1}^{n_w} (w_i - \bar{w})^2$$

Where $v\hat{a}r\ (\hat{\mu})$ is the sample variance of the measured water equivalents, and w is the mean of the water equivalents.

DISCUSSION

All snow surveys presented here were conducted using the Adirondak snow tube and associated scales. The Adirondak sampler was chosen because it has a larger diameter than many other types of snow tubes and thus provides a larger sample, which is important in the shallow snowpacks of Arctic Alaska. However, there are a variety of similar snow surveying tubes of various diameters and lengths commonly in use, and the equations presented here should work for any type of similar snow sampling equipment. These equations are based on the assumption that data will be collected by random sampling. A systematic sampling scheme such as those commonly employed in snow surveying, where a variable is sampled at regularly spaced intervals along a line or grid, can result in an estimate of low variance if the various possible sample sets are partitioned in such a way that one set is similar to another (Thompson 1992). Thus, in snow surveying, estimates of low variance can be expected if grids or transects are laid out in such a way that the precise starting point of a grid does not effect the expected data from the grid. This is true where sampling patterns cover an area large enough to contain a representative collection of all parts of the snowcover, and where the location of sampling points does not coincide with any regular pattern of snow accumulation or density of the snowpack. Care was taken to assure that the snow surveys used for this paper were conducted in such a fashion, and thus it is assumed that the samples here are acceptable as random samples. The variance of the estimated average water equivalent in double sampling consists of two parts.

Table 2. Estimates of the optimal number of depths per water equivalent measurement.

Location	Number of water equiv'ts, number of depths collected	Time to measure Water Equivalent (min)	Time to measure depth (min)	Optimal Ratio of Depths to water equivalent measurement
Foothills '92	50, 134	0.95	0.21	12
Foothills '93	40, 121	0.95	0.23	15
Coastal Plain '93	61, 392	1*	0.09*	15

* Times to collect data on the coastal plain are normalized.

The first part is the sample variance of the water equivalents adjusted for the larger sample size obtained by measuring additional depths. The second half of the equation estimates the variance of the measured water equivalents about a line representing the average density multiplied by depth. Thus, the closer measured water equivalents match the product of average density and depth at the points where water equivalent was measured, the lower the variance of the resulting estimate of average water equivalent. In all cases, a larger number of depth measurements will result in a lower variance. It should be noted that in the case where the number of depths is the same as the number of water equivalents, the equation reduces to the equation used when measuring water equivalent only.

The data in Table 1 demonstrate that double sampling will generally yield estimates of water equivalent having lower variance than will measuring water equivalent only. The first set of surveys shown was conducted at Imnaviat Creek, where high winter winds combine with hilly terrain to produce a snowpack with numerous windslabs and drifts. This variability of the snowpack is not consistent across the valley, as is evident from the data presented. The variability of the snowpack generally increases to the west across the valley (Surveys are numbered east to west in Table 1). Thus, comparisons are most appropriate between an individual survey and those adjacent to it. In every case, the surveys where double sampling was used produced lower variance estimates than adjacent surveys where double sampling was not used. The two surveys conducted on the Coastal Plain are in an area of much shallower snowcover and lower relief than the Imnaviat Creek surveys. This provides a different snowcover type as a test of the utility of double sampling. The data in Table 1 indicate that the two surveys presented produced the same estimate of average water equivalent, but that double sampling resulted in the lower variance. The large sample sizes of the two surveys may mean that the natural variance of the snowcover dominates the sampling error, and may thus explain the small difference between the variance of the two estimates of average water equivalent.

The optimal ratios of depth to water equivalent measurements for all three sets of snow surveys shown in Table 2 are similar. However, optimal ratios calculated from individual snow surveys varied greatly, ranging from 7 to 1 to 23 to 1. The optimal ratio is influenced by the ratio of the time required for the two types of measurement, by the variance of the water equivalent values, and by the variance of the water equivalent values about a line defined by the product of depth and average density. Thus optimal ratio can be influenced by a variety of weather and snow related factors. Cold or windy weather, wind slabs, or deep snow can alter the ratio of the time required for data collection by hampering walking, or by increasing the time and effort required for collecting or weighing samples. Uneven snow depth can increase the variance of the water equivalent values. The combination of newly fallen soft snow over denser, drifted snow will increase the variance of the water equivalent values about a line defined by average density and depth. For these reasons, the precise optimal ratio of measurements can vary both spatially and temporally. However, the purpose of calculating the optimal ratio is to provide a general guideline for efficient sampling during subsequent surveys in an area, and for this reason average optimal ratios are presented by year and geographic region in Table 2. To insure that a representative optimal ratio of depth to water equivalent measurements is obtained, it is advisable to time a number of snow surveys when attempting to estimate the optimal ratio in an area. It should be noted that it is not necessary to sample at the optimal ratio to utilize these equations.

CONCLUSIONS

The results of the snow surveys presented here demonstrate that double sampling can yield improved estimates of average snowpack water equivalent when compared with sampling water equivalent only. This, combined with its adaptability to sampling along a transect and overall simplicity, makes double sampling an attractive alternative for both existing snow courses and new snow surveys. By timing the collection of depth and water equivalent measurements, the optimal ratio of the two measurements types to be utilized in subsequent surveys can be calculated.

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