ESTIMATING THE MEAN SQUARED PREDICTION ERROR OF THE AREAL SNOW WATER EQUIVALENT ESTIMATE FOR A RIVER BASIN

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ABSTRACT

With the increased demand for water in the United States, particularly in the West, it is essential that water resources be accurately monitored. Consequently, the National Weather Service maintains a set of conceptual, continuous, hydrologic simulation models used to generate extended streamflow predictions, water supply outlooks, and flood forecasts. A vital component of the hydrologic simulation models is a snow accumulation and ablation model that uses observed temperature and precipitation data to simulate snow cover conditions. The simulated model states are updated throughout the snow season using snow water equivalent estimates obtained from airborne and ground-based snow water equivalent data. The National Weather Service has developed a spatial geostatistical model to estimate the areal snow water equivalent in a river basin. The estimates, which are obtained for various river basins in the West, are used to update the snow model. To facilitate updating of the simulated snow water equivalent estimates generated by the snow model, it is necessary to have estimates of the mean squared prediction errors of the areal snow water equivalent estimates. In this research, we derive an expression for the mean squared prediction error of the areal snow water equivalent estimate for a river basin.

INTRODUCTION

Industrial, agricultural, and societal water requirements continue to increase making accurate forecasting of water supplies imperative. To forecast water resources, the National Weather Service maintains a set of conceptual, continuous, hydrologic simulation models used to generate extended streamflow predictions, water supply outlooks, and flood forecasts that are used to support major water management and disaster emergency services decisions for the United States. The forecasts are used by federal, state, and private agencies including the U.S. Army Corps of Engineers, the Soil Conservation Service (SCS), and the Salt River Project in Arizona.

An integral part of the hydrologic simulation models is a snow accumulation and ablation model that uses observed temperature and precipitation data to simulate snow cover conditions. Having accurate snow model states is critical to making accurate streamflow and water supply forecasts. In an effort to obtain precise forecasts, ground-based snow data are periodically collected throughout the snow season by the Soil Conservation Service and other federal and state agencies. These data, collected from snow-course and SCS SNOTEL sites, are incorporated into the snow model to update the simulated model states.

In addition to the ground data, the National Weather Service collects airborne snow water equivalent data to update the simulated snow water equivalent estimates in the snow model. The Office of Hydrology within the National Weather Service operates an airborne snow survey program which estimates snow water equivalent over more than 1700 flight lines in the United States and Canada. The airborne estimation technique uses the attenuation of natural terrestrial gamma radiation by the mass of the snow cover to make airborne estimates of snow water equivalent over a flight line that is typically 16 km long and 300 m wide covering an area of approximately 5 km^2. Consequently, each estimate is a mean areal measure integrated over the 5 km^2 area of the flight line. The gamma radiation flux near the

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ground originates primarily from the natural ^{40}K, ^{238}U, and ^{208}Tl radioisotopes in the soil. In a typical soil 96 percent of the gamma radiation is emitted from the upper 20 cm of soil (Zotimov, 1968). After a measure of the background (no snow cover) radiation and soil moisture is made over a specific flight line, a second measurement of these parameters is made over the flight line when snow is present. The attenuation of the radiation signal due to the snowpack is used to estimate the average areal amount of water in the snow cover (referred to as the snow water equivalent) over the flight line (Fritzsche, 1982).

Updating the National Weather Service snow model with accurate, reliable, real-time, ground-based and remotely sensed snow cover estimates is essential to effective water resource forecasting and management, particularly in the West. The use of streamflow simulation models in snow-covered areas is substantially improved when accurate input data are acquired on a real-time basis (Anderson, 1978). According to Castruccio et al., (1980), the benefit of a six percent improvement in streamflow predictions could be as high as $10 million for hydropower and $28 million for irrigation annually in the West. In one example, the 1985 flood in Fort Wayne, Indiana, the savings in flood costs (e.g. property damage costs and lost business revenue) attributed to the use of real-time airborne snow water equivalent estimates alone were estimated to be approximately $2.4 million (Carroll, 1986).

Recently the National Weather Service has developed a spatial statistical model that uses the ground-based and airborne data to estimate the snow water equivalent in areas where no observed measurements are available (Day, 1990; McManamon, et al., 1993). The interpolated snow water equivalent estimates obtained for specific sites in a river basin are averaged to estimate the areal snow water equivalent in the basin. The estimates of the areal snow water equivalent are obtained for various river basins in the West and are used to update the snow model. To facilitate updating of the simulated snow water equivalent estimates generated by the snow model, it is necessary to have estimates of the mean squared prediction errors of the areal snow water equivalent estimates. In this research, we derive an expression for the mean squared prediction error of the areal snow water equivalent estimate for a river basin.

ESTIMATING SNOW WATER EQUIVALENT

To obtain accurate estimates of snow water equivalent, the National Weather Service has developed a spatial prediction model that incorporates both the ground-based and airborne data (Carroll et al., 1994). The snow water equivalent data obtained from snow-course, SNOTEL and airborne sites are first standardized to have mean zero and variance one. Standardization of the data is necessary for two reasons. First, due to orographic effects, precipitation in the West varies widely from site to site even if the sites are in close proximity to each other (Peck and Schaake, 1990). In order to obtain accurate estimates of snow water equivalent, it is imperative to account for the large scale variation among the sites. Secondly, it is evident from historical data that the variance of the snow water equivalent is not constant from site to site. To use the spatial estimation techniques applied in this research, the variance of the observations must be equal. By standardizing the data, we account for both the large scale variation and the nonconstant variance in the data. To standardize the observed data, the mean snow water equivalent for a specific site on a specific date is estimated using historical data or when historical data are not available using mean snow water equivalent maps. The mean snow water equivalent maps are generated on a weekly basis throughout the snow season using a physically based snow accumulation and ablation model and a geographic information system, which incorporate the effects of long term mean seasonal precipitation, temperature, elevation, aspect and forest cover at the sites. The standard deviation is modeled as a function of the mean. Historical snow-course data are used to estimate the parameters of the standard deviation-mean model.

When obtaining estimates of the snow water equivalent where no observations are collected, we use simple kriging on the standardized data (Cressie, 1991). Let $Y(\boldsymbol{s}_i)$ represent the unstandardized snow water equivalent at location $\boldsymbol{s}_i$. If the data are ground-based, the standardized snow water equivalent at site $\boldsymbol{s}_i$ is

$$Z(\boldsymbol{s}_i) = (Y(\boldsymbol{s}_i) - \mu(\boldsymbol{s}_i))/\sigma(\boldsymbol{s}_i)$$

where $\mu(\boldsymbol{s}_i)$ and $\sigma(\boldsymbol{s}_i)$ are the mean and standard deviation of the snow water equivalent at site $\boldsymbol{s}_i$.

For flight line B_i and locations $\boldsymbol{s} \in B_i$, the area of the flight line is represented as

$$|B_i| = \int_{B_i} d\boldsymbol{s} > 0,$$

and the aggregated unstandardized snow water equivalent for flight line B_i is

$$Y(B_i) = \int_{B_i} Y(\boldsymbol{s})d\boldsymbol{s} / \mid B_i \mid .$$

Hence, the standardized snow water equivalent for the flight line is

$$Z^*(B_i) = (Y(B_i) - \mu(B_i))/\sigma(B_i),$$

where $\mu(B_i)$ and $\sigma(B_i)$ are the mean and standard deviation respectively of the snow water equivalent for the flight line. The mean $\mu(B_i)$ is

$$\mu(B_i) = \int_{B_i} \mu(\boldsymbol{s})d\boldsymbol{s}/|B_i|$$

where $\mu(\boldsymbol{s})$ is the mean snow water equivalent at site $\boldsymbol{s}$.

Because the flight lines are not straight, we use a finite approximation to the integral above to estimate $\mu(B_i)$. We first divide each flight line into smaller sections of approximately equal area along the path of flight. The number of sections, n_i, in the flight line (B_i) varies depending on the length of the flight line. Associated with each section, we determine the longitude and latitude of the center and estimate the average areal snow water equivalent for the section, $\mu(\boldsymbol{s}_k)$, from areal mean snow water equivalent maps using a weighted average of the means found in the section. The weights are proportional to the sizes of the corresponding areas covered by the various means in the section. We estimate $\mu(B_i)$ with

$$\hat{\mu}(B_i) = \sum_{k=1}^{n_i} \hat{\mu}(\boldsymbol{s}_k)/n_i$$

where $\hat{\mu}(\boldsymbol{s}_k)$ is our estimate of $\mu(\boldsymbol{s}_k)$. The variance, $\sigma^2(B_i)$, is

$$\sigma^2(B_i) = \int_{B_i} \int_{B_i} \sigma(\boldsymbol{s})\sigma(\boldsymbol{u})\text{Cov }(Z(\boldsymbol{s}), Z(\boldsymbol{u}))d\boldsymbol{s}d\boldsymbol{u}/|B_i|^2$$

and is estimated by

$$\hat{\sigma}^2(B_i) = \sum_{k=1}^{n_i} \sum_{l=1}^{n_i} \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\text{Cov }(Z(\boldsymbol{s}_k), Z(\boldsymbol{s}_l))/n_i^2.$$

If the data are ground-based, we let $B_i = \{\boldsymbol{s}_i\}$ and $Z^*(B_i) = Z(\boldsymbol{s}_i)$. Using both the ground-based and airborne data, the best linear predictor of $Z(\boldsymbol{s}_0)$ is

$$\hat{Z}(\boldsymbol{s}_0) = \sum_{i=1}^{n} \lambda_i \, Z^*(B_i)$$

where n is the total number of the ground-based and airborne observations.

To obtain the coefficients $\{\lambda_i\}$, we minimize

$$\text{Var }(Z(\boldsymbol{s}_0) - \sum_{i=1}^{n} \lambda_i \, Z^*(B_i)),$$

obtaining the simple kriging coefficients

$$\boldsymbol{\lambda} = \Sigma^{-1}\boldsymbol{c},$$

where

$$\boldsymbol{\lambda}' = (\lambda_1, \lambda_2, \ldots, \lambda_n),$$

$\Sigma =$ the $n \times n$ matrix whose (i, j)-th element is Cov $(Z^*(B_i), Z^*(B_j))$,

$$\text{Cov }(Z^*(B_i), Z^*(B_j)) = \int_{B_i}\int_{B_j} \sigma(\boldsymbol{s})\sigma(\boldsymbol{u})\text{Cov }(Z(\boldsymbol{s}), Z(\boldsymbol{u}))d\boldsymbol{s}d\boldsymbol{u}/(\sigma(B_i)\sigma(B_j)|B_i||B_j|),$$

$$\text{Cov }(Z(\boldsymbol{s}_i), Z^*(B_j)) = \int_{B_j} \sigma(\boldsymbol{s})\text{Cov }(Z(\boldsymbol{s}_i), Z(\boldsymbol{s}))d\boldsymbol{s}/(\sigma(B_j) \mid B_j \mid),$$

and

$$\boldsymbol{c}' = (\text{Cov }(Z(\boldsymbol{s}_0), Z^*(B_1)), \ldots, \text{Cov }(Z(\boldsymbol{s}_0), Z^*(B_n)).$$

If $B_i = \{\boldsymbol{s}_i\}$ and $B_j = \{\boldsymbol{s}_j\}$, then

$$\text{Cov }(Z^*(B_i), Z^*(B_j)) = \text{Cov }(Z(\boldsymbol{s}_i), Z(\boldsymbol{s}_j)).$$

In applications, the covariances above are approximated by:

$$\hat{\text{Cov}}\ (Z^*(B_i), Z^*(B_j)) = \sum_{k=1}^{n_i}\sum_{l=1}^{n_j} \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\text{Cov }(Z(\boldsymbol{s}_k), Z(\boldsymbol{s}_l))/(\hat{\sigma}(B_i)\hat{\sigma}(B_j)\ n_i n_j),$$

and

$$\hat{\text{Cov}}\ (Z(\boldsymbol{s}_i), Z^*(B_j)) = \sum_{l=1}^{n_j} \sigma(\boldsymbol{s}_l)\text{Cov }(Z(\boldsymbol{s}_i), Z(\boldsymbol{s}_l))/(\hat{\sigma}(B_j)\ n_j).$$

To obtain the covariances necessary to solve for $\boldsymbol{\lambda}$ in the equations above, the National Weather Service uses historical data to estimate site-to-site covariances and then models the covariance between two sites as a function of distance.

The kriging variance (or mean squared prediction error) is the minimized value of

$$\text{Var }(Z(\boldsymbol{s}_0) - \textstyle\sum_{i=1}^{n} \lambda_i\ Z^*(B_i))$$

and is denoted by $\sigma^2_{Z_{sk}}(\boldsymbol{s}_0)$. Upon substitution of $\boldsymbol{\lambda} = \Sigma^{-1}\boldsymbol{c}$, we obtain

$$\sigma^2_{Z_{sk}}(\boldsymbol{s}_0) = 1 - \boldsymbol{c}'\Sigma^{-1}\boldsymbol{c}.$$

Having obtained $\hat{Z}(\boldsymbol{s}_0)$, we compute the unstandardized estimate of the snow water equivalent as

$$\hat{Y}(\boldsymbol{s}_0) = \sigma(\boldsymbol{s}_0)\hat{Z}(\boldsymbol{s}_0) + \mu(\boldsymbol{s}_0)$$

with simple kriging variance

$$\sigma^2_{Y_{sk}}(\boldsymbol{s}_0) = \sigma^2(\boldsymbol{s}_0)\sigma^2_{Z_{sk}}(\boldsymbol{s}_0).$$

MEAN SQUARED PREDICTION ERROR OF THE AREAL ESTIMATE

Let $Y(B) = \int_B Y(\boldsymbol{s})d\boldsymbol{s}/|B|$ represent the unstandardized snow water equivalent for the entire river basin, B, where

$$|B| = \int_B d\boldsymbol{s} > 0,$$

is the area of the basin. To estimate $Y(B)$, we use

$$\hat{Y}(B) = \sum_{k=1}^{m} \hat{Y}(\boldsymbol{s}_k)/m = \sum_{k=1}^{m}\ (\sigma(\boldsymbol{s}_k)\hat{Z}(\boldsymbol{s}_k) + \mu(\boldsymbol{s}_k))/m$$

where m is the number of gridded sites that are estimated in the basin. It can be shown that this estimator is unbiased and has minimum mean squared prediction error.

To compute the mean squared prediction error of $\hat{Y}(B)$, call it $\sigma^2_{Y_{sk}}(B)$, we write

$$\begin{aligned}\sigma^2_{Y_{sk}}(B) =& \text{E}(Y(B) - \hat{Y}(B))^2 = \text{Var}(Y(B) - \hat{Y}(B)) \\ =& \text{Var}(Y(B) - \textstyle\sum_{k=1}^{m} \hat{Y}(\boldsymbol{s}_k)/m) \\ =& \sigma^2(B) - (2/m)\text{Cov}(Y(B), \textstyle\sum_{k=1}^{m} \hat{Y}(\boldsymbol{s}_k)) + (1/m^2)\text{Var}(\textstyle\sum_{k=1}^{m} \hat{Y}(\boldsymbol{s}_k))\end{aligned}$$

where

$$\sigma^2(B) = \int_B \int_B \sigma(\boldsymbol{s})\sigma(\boldsymbol{u})\text{Cov}\ (Z(\boldsymbol{s}), Z(\boldsymbol{u}))d\boldsymbol{s}d\boldsymbol{u}/|B|^2 \tag{1}$$

is the variance of $Y(B)$. Furthermore,

$$\begin{aligned}
&\text{Cov}(Y(B), \textstyle\sum_{k=1}^m \hat{Y}(\boldsymbol{s}_k))/m \\
&\quad = \textstyle\sum_{k=1}^m \text{Cov}(Y(B), \hat{Y}(\boldsymbol{s}_k))/m \\
&\quad = \textstyle\sum_{k=1}^m \text{Cov}(Y(B), \sigma(\boldsymbol{s}_k)\hat{Z}(\boldsymbol{s}_k) + \mu(\boldsymbol{s}_k))/m \\
&\quad = \textstyle\sum_{k=1}^m \text{Cov}(Y(B), \sigma(\boldsymbol{s}_k)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)Z^*(B_i))/m \\
&\quad = \textstyle\sum_{k=1}^m \sigma(\boldsymbol{s}_k)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)\text{Cov}(Y(B), Z^*(B_i))/m \\
&\quad = \textstyle\sum_{k=1}^m \sigma(\boldsymbol{s}_k)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)\text{Cov}(\int_B Y(\boldsymbol{s})d\boldsymbol{s}/|B|, Z^*(B_i))/m \\
&\quad = \textstyle\sum_{k=1}^m \sigma(\boldsymbol{s}_k)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)\int_B \sigma(\boldsymbol{s})\text{Cov}(Z(\boldsymbol{s}), Z^*(B_i))d\boldsymbol{s}/(|B|\cdot m),
\end{aligned} \tag{2}$$

where $\lambda_i(\boldsymbol{s}_k)$ is the simple kriging coefficient associated with $Z^*(B_i)$ when $Z(\boldsymbol{s}_k)$ is estimated, and

$$\begin{aligned}
&\text{Var}(\textstyle\sum_{k=1}^m \hat{Y}(\boldsymbol{s}_k))/m^2 \\
&\quad = \textstyle\sum_{k=1}^m \sum_{l=1}^m \text{Cov}(\hat{Y}(\boldsymbol{s}_k), \hat{Y}(\boldsymbol{s}_l))/m^2 \\
&\quad = \textstyle\sum_{k=1}^m \sum_{l=1}^m \text{Cov}(\sigma(\boldsymbol{s}_k)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)Z^*(B_i), \sigma(\boldsymbol{s}_l)\sum_{j=1}^n \lambda_j(\boldsymbol{s}_l)Z^*(B_j))/m^2 \\
&\quad = \textstyle\sum_{k=1}^m \sum_{l=1}^m \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\sum_{i=1}^n \sum_{j=1}^n \lambda_i(\boldsymbol{s}_k)\lambda_j(\boldsymbol{s}_l)\text{Cov}(Z^*(B_i), Z^*(B_j))/m^2 \\
&\quad = \textstyle\sum_{k=1}^m \sum_{l=1}^m \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)\sum_{j=1}^n \lambda_j(\boldsymbol{s}_l)\text{Cov}(Z^*(B_i), Z^*(B_j))/m^2 \\
&\quad = \textstyle\sum_{k=1}^m \sum_{l=1}^m \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)\text{Cov}(Z(\boldsymbol{s}_l), Z^*(B_i))/m^2
\end{aligned}$$

We estimate the variance and covariance given in Eqs. (1) and (2) by:

$$\hat{\sigma}^2(B) = \textstyle\sum_{k=1}^m \sum_{l=1}^m \ \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\text{Cov}\ (Z(\boldsymbol{s}_k), Z(\boldsymbol{s}_l))/m^2,$$

and

$$\begin{aligned}
&\hat{\text{Cov}}(Y(B), \textstyle\sum_{k=1}^m \hat{Y}(\boldsymbol{s}_k))/m \\
&\quad = \textstyle\sum_{k=1}^m \sigma(\boldsymbol{s}_k)\sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)\sum_{l=1}^m \sigma(\boldsymbol{s}_l)\text{Cov}(Z(\boldsymbol{s}_l), Z^*(B_i))/m^2.
\end{aligned}$$

Hence, the mean squared prediction error is estimated by

$$\begin{aligned}
&\hat{\sigma}^2_{Y_{sk}}(B) = \hat{\sigma}^2(B) - (1/m^2)\text{Var}(\textstyle\sum_{k=1}^m \hat{Y}(\boldsymbol{s}_k)) \\
&\quad = \textstyle\sum_{k=1}^m \sum_{l=1}^m \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\{\text{Cov}(Z(\boldsymbol{s}_k), Z(\boldsymbol{s}_l)) - \sum_{i=1}^n \lambda_i(\boldsymbol{s}_k)\text{Cov}(Z(\boldsymbol{s}_l), Z^*(B_i))\}/m^2 \\
&\quad = \textstyle\sum_{k=1}^m \sum_{l=1}^m \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\{\text{Cov}(Z(\boldsymbol{s}_k), Z(\boldsymbol{s}_l)) - \boldsymbol{c}'(\boldsymbol{s}_k)\Sigma^{-1}\boldsymbol{c}(\boldsymbol{s}_l)\}/m^2 \\
&\quad = \textstyle\sum_{k=1}^m \sigma^2_{Y_{sk}}(\boldsymbol{s}_k)/m^2 \\
&\qquad + 2\textstyle\sum\sum_{l<k} \sigma(\boldsymbol{s}_k)\sigma(\boldsymbol{s}_l)\{\text{Cov}(Z(\boldsymbol{s}_k), Z(\boldsymbol{s}_l)) - \boldsymbol{c}'(\boldsymbol{s}_k)\Sigma^{-1}\boldsymbol{c}(\boldsymbol{s}_l)\}/m^2
\end{aligned}$$

where $\boldsymbol{c}(\boldsymbol{s}_k)$ is the vector $\boldsymbol{c}$ associated with the estimation of $Z(\boldsymbol{s}_k)$.

DISCUSSION AND CONCLUSIONS

In operational applications, m, the number of individual sites that are estimated in a basin, can be quite large, and in these cases, the number of calculations required in the computation of the basin mean squared prediction error will be extremely large. For example in the Animas River basin in Southwest Colorado, m is approximately equal to 3000; hence, computation of over 4.5 million covariances is required. Consequently, when estimating $\sigma^2_{Y_{sk}}(B)$, one may want to use a larger grid size across the basin (and hence, fewer sites in the basin) to reduce the number of computations required to estimate the mean squared prediction error for a river basin.

The impact of obtaining accurate forecasts of water resources and understanding the errors associated with the forecasts can be immense. The simulation models are a major source of information for forecasting water availability for navigation, disaster emergency service requirements during flooding, and both the volume and timing of water supply for irrigation, power generation, and municipal water use.

The areal estimates of the snow water equivalent obtained from the spatial prediction model are used as real-time updates in the hydrologic simulation models maintained by the National Weather Service. Knowing the mean squared prediction errors of the basin snow water equivalent estimates is essential to appropriately update the simulated snow water equivalent estimates generated by the snow model. In this research, we have derived an expression for the mean squared prediction error associated with the basin estimate of snow water equivalent. Hence, when water resource decisions are made, the uncertainty in basin snow water equivalent estimates can be incorporated into the decision making process.

In future research, we will continue to refine the spatial prediction model to improve the accuracy of forecasts. Initially, we investigate how to include data of two different supports when the spatial covariance function is estimated. Moreover, we will explore the possibility of combining temporal and spatial information about snow water equivalent when estimates are generated.

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