# IMPROVING THE ACCURACY OF COVARIANCE ESTIMATES USED IN SPATIAL MODELING AND ESTIMATION OF SNOW WATER EQUIVALENT

Steven S. Carroll[1]

## ABSTRACT

With the increasing demand for water in the United States, it is essential that water resources be accurately monitored to insure that the available water supply is used optimally. Hence, the National Weather Service (NWS) maintains a set of conceptual, continuous, hydrologic simulation models to generate extended streamflow predictions, water supply outlooks, and flood forecasts for river systems throughout the U.S. The accuracy of these forecasts is very dependent on the accuracy of snow water equivalent estimates simulated by the models. Hence, periodically throughout the snow season, the simulated snow water equivalent estimates are updated using snow water equivalent estimates obtained from snow data collected in river basins around the country. The estimates are obtained using a geostatistical model and snow course, SNOTEL, and airborne snow data. In this research, we examine ways to improve the accuracy of spatial covariance estimates that are necessary to obtain snow water equivalent estimates used to update the snow simulation models. We illustrate our methodology using snow data collected in the North Fork Clearwater River basin in Idaho.

Key Words: Kriging, Spatial Prediction, Geostatistics

## INTRODUCTION

The demand for water in the United States continues to intensify making the need to forecast and manage available water supplies very critical. To forecast water resources, the National Weather Service maintains a set of conceptual, continuous, hydrologic simulation models used to generate extended streamflow predictions, water supply outlooks, and flood forecasts (Day, 1985; Hudlow, 1988). These forecasts are used in making major water management and disaster emergency services decisions for the United States by federal, state, and private agencies including the U.S. Army Corps of Engineers, the Natural Resources Conservation Service (NRCS) of the U.S. Department of Agriculture, and the Salt River Project in Arizona.

Much of the water available for irrigation, power generation, recreation, and industrial and other societal requirements, particularly in the Western U.S., comes from snow melt runoff. Hence, an integral part of the NWS hydrologic simulation models is a snow accumulation and ablation model that uses observed temperature and precipitation data to simulate snow cover conditions (Anderson, 1978; 1986). Obtaining valid simulated snow cover conditions to be incorporated into the hydrologic model is critical to making accurate streamflow and water supply forecasts. In an effort to obtain precise forecasts, ground-based and airborne snow data are periodically collected throughout the snow season by several federal and state agencies. These data, collected from over 2000 snow course and SNOTEL sites and over 1700 flight lines throughout the U.S., are incorporated into the snow model to update the simulated model states (Day, 1990; McManamon et al., 1993; Carroll et al., 1995).

Recently, to improve the accuracy of snow water equivalent estimates used to update the snow simulation models, the National Weather Service has developed a spatial statistical model that uses the ground-based and airborne data to estimate snow water equivalent in areas where no observed measurements are available (Day, 1990; Carroll et al., 1995). Carroll and Cressie (1996) developed a modification to the spatial estimation technique that enables hydrologists to model geographic attributes at sites where snow water equivalent is estimated. In this research, we will adapt the spatial model to account for differences in mean annual precipitation and differences in solar radiation intensity (Hetrick

[1] Associate Professor, Department Economics, Arizona State University, Tempe, AZ 85287-3806

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et al., 1993) at sites around a river basin and examine how these differences affect the accuracy of the snow water equivalent estimates.

An essential component of the spatial model is a covariance function that yields the covariance between the snow water equivalent at two sites. In the approach typically used in spatial modeling, the covariance is modeled as a function solely of the spatial co-ordinates of the two sites. Carroll and Cressie (1996) developed modified covariance models that enable one to incorporate other geographic attributes, such as, elevation, aspect, slope, and tree cover, into the covariance function. In their research, Carroll and Cressie showed that, by incorporating differences in elevation into the covariance model, the accuracy of snow water equivalent estimate may be substantially improved. In this research, we examine the effects of incorporating differences in mean seasonal precipitation and differences in total seasonal solar radiation intensity into the covariance function. Because mean seasonal precipitation and total seasonal solar radiation intensity affect the accumulation and ablation of snow water equivalent, we expect that, by incorporating these factors into the spatial covariance model, more realistic estimates of the covariances will result. Hence, more accurate snow water equivalent estimates may be obtained. More accurate snow water equivalent estimates are expected to ultimately result in better water resource forecasts. We shall illustrate our results on snow data collected in the North Fork Clearwater River basin in Idaho.

## ESTIMATING SNOW WATER EQUIVALENT

To obtain precise estimates of snow water equivalent, the National Weather Service has developed a spatial prediction model that incorporates both the ground-based and airborne data (Carroll et al., 1995). The snow water equivalent data obtained from snow course, SNOTEL, and airborne sites are first standardized to have mean zero and variance one. Standardization of the data is necessary for two reasons. First, due to orographic effects, precipitation in the West varies widely from site to site even if the sites are in close proximity to each other (Peck and Schaake, 1990). In order to obtain accurate estimates of snow water equivalent, it is imperative to account for the large-scale variation among the sites. Secondly, from historical data it is evident that the variance of the snow water equivalent is not constant from site to site. To use the spatial estimation techniques applied in this research, the variance of the observations must be equal. By standardizing the data, we account for both the large-scale variation and the nonconstant variance in the data. To standardize the observed data, the mean snow water equivalent for a specific site on a specific date is estimated using historical data or mean maps prepared by National Weather Service personnel. The mean maps are generated for specific dates throughout the snow season from a snow accumulation and ablation model that uses information about precipitation, temperature, and melt rate at the sites. The standard deviation is modeled as a function of the mean. Historical snow course data are used to estimate the parameters of this model.

When obtaining estimates of the snow water equivalent where no observations are collected, the standardized data are modeled using simple kriging (e.g., Journel and Huijbregts, 1978). Let $Y(\boldsymbol{s})$ represent the unstandardized snow water equivalent at location $\boldsymbol{s}$. For flight line $B_i$ and locations $\boldsymbol{s} \in B_i$, the area is represented as

$$|B_i| = \int_{B_i} d\boldsymbol{s} > 0, \tag{1}$$

and the aggregated unstandardized snow water equivalent for flight line $B_i$ is

$$Y(B_i) = \int_{B_i} Y(\boldsymbol{s}) d\boldsymbol{s} / \mid B_i \mid . \tag{2}$$

Hence, the standardized snow water equivalent for the flight line is

$$Z^*(B_i) = (Y(B_i) - \mu(B_i))/\sigma(B_i), \tag{3}$$

where $\mu(B_i)$ and $\sigma(B_i)$ are the mean and standard deviation respectively of the snow water equivalent for the flight line. If the data are ground-based (i.e., snow course or SNOTEL), we let $B_i = \{\boldsymbol{s}_i\}$ and

$Z^*(B_i) = Z(\boldsymbol{s}_i)$, where

$$Z(\boldsymbol{s}_i) = (Y(\boldsymbol{s}_i) - \mu(\boldsymbol{s}_i))/\sigma(\boldsymbol{s}_i) \tag{4}$$

and $\mu(\boldsymbol{s}_i)$ and $\sigma(\boldsymbol{s}_i)$ are the respective mean and standard deviation of the snow water equivalent at site $\boldsymbol{s}_i$. Using both the ground-based and airborne data, the best linear (i.e., simple kriging) predictor of $Z(\boldsymbol{s}_0)$ is

$$\hat{Z}(\boldsymbol{s}_0) = \sum_{i=1}^{n} \lambda_i \; Z^*(B_i), \tag{5}$$

where $n$ is the total number of the ground-based and airborne observations.

To obtain the coefficients $\{\lambda_i\}$, we minimize

$$\text{var}(Z(\boldsymbol{s}_0) - \sum_{i=1}^{n} \lambda_i \; Z^*(B_i)), \tag{6}$$

obtaining the simple kriging coefficients $\boldsymbol{\lambda} = \Sigma^{-1}\boldsymbol{c}$, where

$$\boldsymbol{\lambda}' = (\lambda_1, \lambda_2, \ldots, \lambda_n), \tag{7a}$$

$$\Sigma = \text{the } n \times n \text{ matrix whose } (i,j)\text{-th element is } \text{cov}(Z^*(B_i), Z^*(B_j)), \tag{7b}$$

$$\text{cov}(Z^*(B_i), Z^*(B_j)) = \int_{B_i}\int_{B_j} \sigma(\boldsymbol{s})\sigma(\boldsymbol{u})\text{cov}(Z(\boldsymbol{s}), Z(\boldsymbol{u}))d\boldsymbol{s}d\boldsymbol{u}/(\sigma(B_i)\sigma(B_j)|B_i||B_j|), \tag{7c}$$

$$\text{cov}(Z(\boldsymbol{s}_i), Z^*(B_j)) = \int_{B_j} \sigma(\boldsymbol{s})\text{cov}(Z(\boldsymbol{s}_i), Z(\boldsymbol{s}))d\boldsymbol{s}/(\sigma(B_j) \mid B_j \mid), \tag{7d}$$

and

$$\boldsymbol{c}' = (\text{cov}(Z(\boldsymbol{s}_0), Z^*(B_1)), \ldots, \text{cov}(Z(\boldsymbol{s}_0), Z^*(B_n)). \tag{7e}$$

If $B_i = \{\boldsymbol{s}_i\}$ and $B_j = \{\boldsymbol{s}_j\}$, then

$$\text{cov}(Z^*(B_i), Z^*(B_j)) = \text{cov}(Z(\boldsymbol{s}_i), Z(\boldsymbol{s}_j)). \tag{8}$$

In applications, the integrals above can be evaluated by numerical integration or some other approximation.

The kriging variance (or mean squared prediction error) is the minimized value of

$$\text{var}(Z(\boldsymbol{s}_0) - \sum_{i=1}^{n} \lambda_i \; Z^*(B_i)) \tag{9}$$

and is denoted by $\sigma_Z^2(\boldsymbol{s}_0)$. Upon substitution of $\boldsymbol{\lambda} = \Sigma^{-1}\boldsymbol{c}$, we obtain

$$\sigma_Z^2(\boldsymbol{s}_0) = 1 - \boldsymbol{c}'\Sigma^{-1}\boldsymbol{c}. \tag{10}$$

Having obtained $\hat{Z}(\boldsymbol{s}_0)$, we compute the unstandardized estimate of the snow water equivalent at location $\boldsymbol{s}_0$ as

$$\hat{Y}(\boldsymbol{s}_0) = \sigma(\boldsymbol{s}_0)\hat{Z}(\boldsymbol{s}_0) + \mu(\boldsymbol{s}_0) \tag{11}$$

with simple kriging variance

$$\sigma_Y^2(\boldsymbol{s}_0) = \sigma^2(\boldsymbol{s}_0)\sigma_Z^2(\boldsymbol{s}_0). \tag{12}$$

For further details on kriging, the interested reader can consult Journel and Huijbregts (1978) or Cressie (1991).

## THE SPATIAL COVARIANCE MODEL

Currently, to obtain the covariances necessary to solve for $\boldsymbol{\lambda}$, the National Weather Service uses historical data to estimate site-to-site covariances and then models the covariance between two sites as a function of distance. For all pairs of sites where substantial historical records exist, covariances are estimated from standardized historical data using $\hat{\text{cov}}(Z(\boldsymbol{s}_i), Z(\boldsymbol{s}_j)) = \sum_{t=1}^{T}((Z_t(\boldsymbol{s}_i) \cdot Z_t(\boldsymbol{s}_j))/(T-1)$, where $T$ is the total number of years of data used in the analysis. The covariance, $\text{cov}(Z(\boldsymbol{s}), Z(\boldsymbol{u}))$, between the (standardized) snow water equivalent at sites $\boldsymbol{s}$ and $\boldsymbol{u}$ is then modeled as an exponential function of distance. Specifically, we model the covariance (in this case the correlation because the data are standardized) as

$$\text{cov}(Z(\boldsymbol{s}), Z(\boldsymbol{u})) = \begin{cases} 1, & \boldsymbol{s} = \boldsymbol{u}, \\ A \exp\left(-B\|\boldsymbol{s}-\boldsymbol{u}\|\right), & \boldsymbol{s} \neq \boldsymbol{u}, \end{cases} \tag{13}$$

where $\|\boldsymbol{s}-\boldsymbol{u}\|$ is the distance in kilometers between sites $\boldsymbol{s}$ and $\boldsymbol{u}$, and $0 < A \leq 1$, $B > 0$ are covariance parameters to be estimated. The function (13) is used because it has been found to fit the data well and it is a positive-definite covariance model (e.g., Cressie, 1991, section 2.5).

For many river basins, we have found that the relationship between covariance and distance modeled by Eq. (13) is very strong. However, hydrologists at the National Weather Service believe that, by including in the covariance function additional geographic attributes that help to explain differing precipitation and snow melt rates at the sites, the accuracy of the covariance estimates will be improved.

Carroll and Cressie (1996) developed alternative models to Eq. (13) that incorporate geographic attributes at the sites, proved that these alternative covariance models are positive-definite, and used elevation, aspect, slope, and tree cover as examples of additional site characteristics that may be modeled.

In this research, we adapt the models of Carroll and Cressie (1996) to account for differences in mean seasonal precipitation and differences in total seasonal solar radiation intensity at sites where snow water equivalent is estimated.

## APPLICATION TO SNOW DATA COLLECTED IN IDAHO

To illustrate the effect of incorporating mean seasonal precipitation and total seasonal solar radiation into the covariance function, we provide an example using snow water equivalent data collected in the North Fork Clearwater River basin, a major drainage basin in the Columbia River system, located in northern Idaho. The data set used in the analysis consists of 35 years of snow water equivalent data (measured in millimeters). The data were collected on or near April 1 for the years 1961 through 1995 at nine SNOTEL and three snow-course sites in and around the basin. No airborne data were available. The locations of the twelve sites extend from 46.48° to 47.08° north latitude and from 114.58° to 116.34° west longitude. The elevations of the sites range from 938.78 to 1914.14 meters, and the October 1 to April 1 mean annual precipitation values range from 604 to 1322 mm. To model the effects of differing solar radiation intensities, daily integrated solar radiation indices were computed for each site on the first and fifteenth of the month from October 1 through April 1. The daily integrated solar radiation indices computed for a particular site were then summed to obtain the total seasonal solar radiation index for the site. The total integrated radiation indices ranged from 28.33 to 68.85 MJ/m$^2$.

To model differences in radiation intensity and precipitation at the sites, we apply the following positive-definite covariance models. The models that follow include a term to account for the differences in elevations at the sites. This term is included because Carroll and Cressie (1996) found, for the North Fork Clearwater River basin, that by modeling differences in elevation the accuracy of snow water equivalent estimates is substantially improved over the accuracy obtained when model (13) is used to estimate the covariances. Hence, for comparison, we first fit the following covariance model:

$$\text{cov}(Z(\boldsymbol{s}), Z(\boldsymbol{u})) = \begin{cases} 1, & \boldsymbol{s} = \boldsymbol{u}, \\ A \exp\left(-B\|\boldsymbol{s}-\boldsymbol{u}\| - CX_1\right), & \boldsymbol{s} \neq \boldsymbol{u}, \end{cases} \tag{14}$$

where $X_1$ is the absolute value of the difference in the elevations between sites $\boldsymbol{s}$ and $\boldsymbol{u}$.

We then account for differences in precipitation by modeling the covariance between two sites using

$$\text{cov}(Z(\boldsymbol{s}), Z(\boldsymbol{u})) = \begin{cases} 1, & \boldsymbol{s} = \boldsymbol{u}, \\ A \exp\left(-B\|\boldsymbol{s}-\boldsymbol{u}\| - CX_1 - DX_2\right), & \boldsymbol{s} \neq \boldsymbol{u}, \end{cases} \tag{15}$$

where $X_2$ is the absolute value of the difference in the mean seasonal precipitation values between sites $\boldsymbol{s}$ and $\boldsymbol{u}$.

Finally, to account for differences in radiation intensity at the sites we fit

$$\text{cov}(Z(\boldsymbol{s}), Z(\boldsymbol{u})) = \begin{cases} 1, & \boldsymbol{s} = \boldsymbol{u}, \\ A \exp\left(-B\|\boldsymbol{s}-\boldsymbol{u}\| - CX_1 - EX_3\right), & \boldsymbol{s} \neq \boldsymbol{u}, \end{cases} \tag{16}$$

where $X_3$ is the absolute value of the difference in the total seasonal radiation indices between sites $\boldsymbol{s}$ and $\boldsymbol{u}$. The parameters $0 < A \leq 1,\ B > 0,\ C > 0,\ D > 0,\ E > 0$ are all covariance parameters to be estimated.

The following example is provided for illustrative purposes; the parameter estimates and covariance models selected for these data are specific to the North Fork Clearwater River basin and should not necessarily be applied to other basins. However, the methodology used to derive the estimates and to select the models is appropriate for any region.

First, we estimate the parameters in models (14), (15), and (16) using a constrained (parameter constraints are shown above) weighted nonlinear least squares fit (using SAS (1989) PROC NLIN) of theoretical covariances to empirical covariances. The weights used are proportional to the inverse of the square of one minus the covariance function being fit, as recommended by Cressie (1985). The parameter estimates and the residual mean squared error (REMSE) for models (14), (15), and (16) are shown in Table 1.

**Table 1.** Parameter Estimates and REMSE

| Model | $A$ | $B$ | $C$ | $D$ | $E$ | REMSE |
|---|---|---|---|---|---|---|
| 14 | 0.98576 | 0.00888 | 0.00024 | - | - | 0.09780 |
| 15 | 0.98969 | 0.00069 | 0.00014 | 0.00017 | - | 0.08311 |
| 16 | 0.99786 | 0.00096 | 0.00022 | - | 0.00089 | 0.10027 |
| 18 | 0.99181 | 0.00065 | - | 0.00041 | - | 0.10997 |

Inspection of the parameter estimates, the associated standard deviations (not shown), and the residual mean squared errors from models (14) and (16), indicates that for this river basin solar radiation does not contribute substantially to the explanation of covariance. This conclusion is based partially on the fact that, even for large values of $X_3$ and for the estimate of $E$ in model (16), differences in the radiation indices affect the covariance estimates only very slightly. Moreover, the REMSE is larger for model (16) than it is for model (14). Thus, for the North Fork Clearwater River basin, we conclude that, in addition to elevation, only mean seasonal precipitation contributes to explaining the covariance. Hence, we compare models (14) and (15) more completely.

To select between models (14) and (15), we compare the accuracy of snow water equivalent estimates obtained when each of the two is used to model the covariances. Using each of the fitted covariance functions, we compare the closeness of the true values of snow water equivalent to the predicted values using a cross-validation technique suggested by Cressie (1991, p. 102). Having fit the covariance models using all 35 years of data, we cross-validate the models using data from each of 34 years. One year (1994) was not used for cross-validation because one site has an observation missing in that year. When we calculate the cross-validation statistic (shown below) for any particular year, we use only the data in that particular year in the computations.

Cross-validation in year $t$ is accomplished by first deleting datum $Z(\boldsymbol{s}_j)$ in year $t$ and predicting it with $\hat{Z}_{-j}(\boldsymbol{s}_j)$ [based on one of the covariance models (14) or (15) and the data in year $t$ without

$Z(\boldsymbol{s}_j)]$. Next, we compute the associated mean-squared prediction error $\sigma^2_{Z_{-j}}(\boldsymbol{s}_j)$. We then obtain predicted values of $Y(\boldsymbol{s}_j)$ through a transformation of $\hat{Z}_{-j}(\boldsymbol{s}_j)$ to $\hat{Y}_{-j}(\boldsymbol{s}_j)$ and obtain estimates of the mean-squared prediction error of $\hat{Y}_{-j}(\boldsymbol{s}_j)$ by transforming $\sigma^2_{Z_{-j}}(\boldsymbol{s}_j)$ to $\sigma^2_{Y_{-j}}(\boldsymbol{s}_j)$. These transformations are shown in Eqs. (11) and (12). In (11) and (12), we estimated $\mu(\boldsymbol{s}_j)$ and $\sigma(\boldsymbol{s}_j)$ using the mean and standard deviation of the 35 years (34 years at one site) of data at site $\boldsymbol{s}_j$.

Having obtained the predicted values $\hat{Y}_{-j}(\boldsymbol{s}_j)$ $(j = 1, \ldots, n)$, we compare the closeness of the true values to the predicted values using

$$CRV = [(1/n)\sum_{j=1}^{n}(Y(\boldsymbol{s}_j) - \hat{Y}_{-j}(\boldsymbol{s}_j))^2]^{1/2}. \tag{17}$$

In this illustration, there are $n = 12$ snow course and SNOTEL sites, and $CRV$ is computed twice for each of the 34 years, first using model (14) and then using model (15).

The quantity $CRV$ is a measure of goodness of prediction similar to the PRESS statistic often used in regression analysis (Draper and Smith, 1981). Small values of $CRV$ indicate that, in general, the predicted values are close to the true values.

For the models that we are considering, the values of the cross-validation statistics are shown in Table 2.

## DISCUSSION AND CONCLUSIONS

As our guide for selecting between the models, we focus on $CRV$. Inspection of Table 2 reveals that, based on this criterion, there is not a great deal of difference between model (14) and model (15). This result suggests that, by adding the absolute difference in mean seasonal precipitation to the elevation difference and the distance between the sites in the covariance function, the precision of the estimates of snow water equivalent is not substantially improved in this basin.

Because of the well documented positive relationship between elevation and precipitation (Daly et al., 1994), we fit one additional covariance model – model (18). In model (18), we includes the distance between the sites and the absolute difference in mean seasonal precipitation values at the sites, $X_2$. The parameter estimates are provided in Table 1. In our example, the correlation coefficient between elevation and precipitation is estimated to be .89. We compare the cross-validation statistics obtained from model (14) to those obtained from model (18) to determine if one of these factors produces a greater improvement in the accuracy of the covariance estimates. The cross-validation statistics for these two models (see Table 2) are not substantially different. Consequently, we conclude that the improvement in the accuracy of snow water equivalent estimates is nearly equal when we add either $X_1$ or $X_2$ to the covariance model in Eq. (13). Moreover, from our investigation of model (15), we conclude that if one of these two factors is in the covariance model, the addition of the second factor does not make a significant improvement.

The enormous economic implications of many water resource management decisions make it imperative that every effort be made to improve the accuracy of extended streamflow predictions and other water supply forecasts used in the decision making. In this research, we investigate how to improve the accuracy of the covariance estimates used to obtain estimates of snow water equivalent and illustrate how to modify the covariance model, which describes the spatial relationship between the snow water equivalent at two sites, to include mean seasonal precipitation and the total seasonal solar intensity at the sites.

For the North Fork Clearwater River basin data, we found that the improvements in the accuracy of the snow water equivalent estimates obtained by adding either elevation or precipitation are nearly equivalent. Moreover, given that one of these two factors is in the covariance model the addition of the second factor does not substantially improve the estimates. Additionally, there was no evidence that the inclusion of a measure of solar intensity at the that sites improves the prediction accuracy.

It is possible that further research may uncover additional geographic attributes that will be useful in modeling the covariances. We want to emphasize that, although the radiation index and precipitation did not contribute to improving the precision of snow water equivalent for the North Fork Clearwater River data, in other basins, these factors may be useful. Hence, all factors should be considered when models are fitted and selected in other basins.

TABLE 2. Cross-Validation Statistics (mm).

| Year | Model 14 | Model 15 | Model 18 |
|---|---|---|---|
| 1961 | 39.488 | 42.925 | 54.280 |
| 1962 | 56.876 | 59.095 | 68.733 |
| 1963 | 64.945 | 60.141 | 56.057 |
| 1964 | 79.559 | 73.341 | 72.354 |
| 1965 | 53.183 | 55.634 | 54.547 |
| 1966 | 89.044 | 89.249 | 91.817 |
| 1967 | 51.437 | 47.315 | 46.238 |
| 1968 | 65.601 | 66.359 | 66.186 |
| 1969 | 127.266 | 112.616 | 106.398 |
| 1970 | 115.770 | 104.752 | 94.982 |
| 1971 | 83.305 | 93.990 | 97.580 |
| 1972 | 93.857 | 96.655 | 87.568 |
| 1973 | 50.035 | 48.542 | 45.376 |
| 1974 | 120.720 | 121.385 | 118.183 |
| 1975 | 45.843 | 50.271 | 55.261 |
| 1976 | 67.436 | 73.801 | 85.154 |
| 1977 | 53.138 | 41.247 | 35.611 |
| 1978 | 73.367 | 76.350 | 71.983 |
| 1979 | 69.061 | 69.355 | 79.278 |
| 1980 | 45.782 | 51.526 | 58.083 |
| 1981 | 73.650 | 60.969 | 57.322 |
| 1982 | 52.579 | 54.198 | 60.012 |
| 1983 | 69.993 | 62.217 | 60.491 |
| 1984 | 54.468 | 56.330 | 58.099 |
| 1985 | 72.204 | 78.716 | 88.989 |
| 1986 | 47.269 | 36.997 | 33.282 |
| 1987 | 84.885 | 73.887 | 68.790 |
| 1988 | 37.679 | 42.603 | 47.966 |
| 1989 | 71.053 | 71.605 | 78.682 |
| 1990 | 64.156 | 65.336 | 65.924 |
| 1991 | 99.274 | 102.782 | 116.054 |
| 1992 | 63.411 | 72.458 | 89.326 |
| 1993 | 45.604 | 48.184 | 47.375 |
| 1995 | 47.277 | 51.148 | 58.407 |

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