A MONTE CARLO MODEL FOR SIMULATION OF RAIN-ON-SNOW EVENTS IN THE PACIFIC NORTHWEST

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ABSTRACT

Northwestern North America experiences blasts of heavy winter rainfall, when rain plus melting snow can cause high streamflows and erosion. Geographic variability and sporadic occurrence of *rain-on-snow* complicate study, but some questions can be addressed through modeling. A computer program combines probabilistic and deterministic elements to perform Monte Carlo simulation of ROS events over many "years", generating realizations of initial and weather conditions; within each event, snow accumulation/melt and percolation are tracked, and water available for runoff is the output. Frequency distributions are based on data from the Washington Cascades, some as functions of elevation and date, so the model can be applied to specific sites or generalized zones. One version calculates snow and percolation for measured weather conditions, to test algorithms and calibrate parameters. Validation focuses on Stampede Pass, a major weather station and snow measurement site. The model is evaluated by comparing statistics and frequency relations of the instrumental record and model realizations for precipitation, R+SM and WAR. In early model applications, the regional zone of greatest ROS enhancement to WAR seems to lie at ~750 m; future uses include assessment of ROS in forest clearings, and possible consequences of climatic change.

INTRODUCTION

Rain-on-snow (ROS) refers to the sum of rain plus rapidly melting snow occurring when warm storms invade terrain supporting an existing snowpack. Northwestern North America is particularly susceptible to occasional blasts of heavy rainfall onto snowpacks, as the great stochastic rain gods of the Pacific direct warm, moist maritime air masses toward the continent along the jet stream. Cyclonic-frontal systems typically arrive from late autumn to mid-winter, when pouring rain lasting several days can combine with melt of shallow early-season snow; but they can occur into late spring, when seasonal melt and runoff may be enhanced by rainfall. The likelihood of ROS in any particular place and time depends on a vast array of interacting environmental variables. Atmospheric pressure and circulation patterns control the weather, while larger-scale climatic forces control them in turn. These conspire at the synoptic scale to deliver storms to particular sites, and over the season to create snowpacks with certain characteristics that vary across the landscape, depending on latitude, elevation, physiography, vegetation, etc. This large variety of governing factors makes ROS phenomena profoundly probabilistic.

Rain-on-snow is common in the Pacific Northwest (PNW, considered N California to British Columbia and east to Idaho), thanks to the combination of frequent winter storms and terrain supporting seasonal snowpacks. Especially in the humid west, most major flooding and landsliding occur during ROS conditions. Documented recognition of these processes extends from the earliest episode mentioned in pioneer memoirs and newspapers, in Dec 1852–Jan 1853, followed by disastrous regional floods in 1861-62. The flood that drowned Vanport on Memorial Day 1948 was caused by spring ROS in the upper Columbia Basin; the Christmas 1964 and February 1996 events were triggered by low-elevation snowfall followed by very warm heavy rainfall across the PNW lasting several days. (Some floods are caused by torrential rain alone, such as those of Nov 1996 and Nov 2006.) Slightly lesser ROS disasters occurred in western Washington and/or Oregon in the winters of 1965, 1975-76, 1977, 1980, 1983, 1989-90, 1990-91, and 1996-97; minor events occur somewhere in the region almost every year.

Scientific appreciation of ROS grew through the 20th century, with research and commentary on the association of rain plus snowmelt by Robert Horton, Walter Parsons, James Church, George Clyde, R.W. Gerdel and other hydrologists, much of it presented in early meetings of the Western Interstate Snow Survey Conference and AGU Hydrology Section (Colbeck, 1987; Mergen, 1997). Early work was aided by the growing network of snow courses; related research focused on climate and storm weather, mountain transportation, water-supply/hydroelectric/-flood control projects, cold regions warfare, etc.; and especially the Cooperative Snow Investigations carried out by the Army Corps of Engineers and Weather Bureau in 1944–60 (USACE, 1956). From the 1960s through 1990s, much investigation concentrated on the hydraulic properties and processes of water movement through snow; Samuel Colbeck's (1972) adaptation of porous-medium flow equations has been followed by many theoretical and field

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studies having applications for ROS. Later bursts of interest were triggered by several of the larger ROS disasters. In forestry, wide-scale harvest in mountainous terrain stimulated studies related to the consequences of large clearings on snow accumulation and melt rates, especially before and during ROS, and thus on runoff and erosion: the work of Dennis Harr (1981, 1986; Christner and Harr, 1982) inspired several regional field studies, some reported at the ROS-themed 1983 Western Snow Conference. Recently, McCabe and others (2007) abstracted ROS data from 4318 weather stations across the continental western U.S. (1949-2003), in order to examine the broad spatial and temporal characteristics of the phenomena. They found that ROS occurs almost everywhere, most commonly in the PNW; most events happen in Oct-May, although they can fall in other months; it tends to occur more often in the north during La Niña and the south during El Niño winters; trends suggest that the frequency of ROS is decreasing at lower levels and increasing in higher mountains, as warming reduces available snow but delivers more rain.

RESEARCH QUESTIONS AND APPROACH

Rain-on-snow is an important hydrometeorologic process, especially in the Northwest. As a frequent source of major regional water input lasting several days, runoff generated by ROS commonly produces high streamflows and erosion, which can be damaging as floods and landslides in developed areas. Other effects of ROS on human works include heavy wet-snow loads that hinder transportation, trigger avalanches, and damage buildings. People can also affect the processes, through engineering works that modify runoff pathways; or land-use changes such as timber harvest, which seems to alter snow accumulation and melt rates in some places. Climate change, anthropogenic or not, is altering the spatial and temporal distribution of ROS. In regard to long-term and large-scale processes, ROS events (distinct from simple rainfall and snowmelt) may be significant contributors to the geomorphic work moving sediment and debris to shape the landscape. For these reasons, we are interested in knowing more about the geographic distribution, frequency, magnitude, behavior – the significance – of rain-on-snow.

Research on ROS has followed several pathways, including energy-balance and flow theory, instrumental weather/snow records, and field studies in the Northwest, Alaska, the Alps, etc. Yet many questions surrounding ROS are not easily studied by these means, chiefly because of the events' geographic variability and sporadic occurrence: ROS is a probabilistic phenomenon, due to the large variety of controlling factors. For example, in early Feb 1996, the region experienced a period of cold weather followed by several days of heavy precipitation and unseasonably warm temperatures. At Saddle Mountain in the Oregon Coast Range (990 m), about 52 cm of rain and complete melting of ~35 cm SWE of snow totaled 87 cm of input, apparently the highest combined total recorded (Taylor, 1997; informative case studies also in Marks and others, 1998; Colle and Mass, 2000; McCabe and others, 2007). But at Stampede Pass in the Washington Cascades (1175 m), much of the ~28 cm rain and ~8 cm melt was absorbed by a deep snowpack, with net outflow of ~20 cm. This ROS event had crucially varying effects not only at these sites, >200 km apart and in different mountain ranges, but at locations much closer in distance and terrain.

This case highlights the limitations of the instrumental record in characterizing ROS across broad areas or long times. National Weather Service Coop stations observe hourly to daily precipitation, high and low temperatures, and snow depth; first-order facilities measure more parameters and at shorter intervals. Many stations have been operating since the early 20th century; hourly rain gauges were introduced in the 1940s. Yet most are located at lower elevations or on transportation routes through valleys and passes. McCabe and others (2007) milked a lot of information regarding ROS occurrence from NWS data, but they could not approach some of the questions about hydrologic significance or mid-scale geographic details. Alternately, Cooperative Snow Survey courses and SNOTEL sites are usually located in higher terrain; snow depth and SWE are measured on snow courses, normally at just a few times per winter; snow pillows produce nearly continuous records of SWE (and can be used to monitor outflow as well), and most SNOTEL sites have recently been fitted with sensors for temperature and snow depth. However, these are placed to generate representative measurements useful for runoff forecasting, and not to completely sample broad regions or elevation zones. The temporal quality of the record is limited by the retirement of many snow courses as the SNOTEL system spread beginning in the 1980s, while many of the latter have not been in operation very long. Likewise, although they have measured additional weather parameters and produced interesting results for periods of events to seasons, field studies typically do not embrace longer times or broader areas. Thus, although some observations span 60+ years, the instrumental evidence is too short and its sites too spatially limited to sample the wide variation of ROS events, even if combined with research studies (van Heeswijk and others, 1996).

These issues may be addressed through stochastic modeling, which can overcome limitations of the record over space and time. I have been developing a computer model combining probabilistic and deterministic elements involved in big storms in the Pacific Northwest, in order to explore those that involve ROS. Using Excel workbooks and Visual Basic for Applications (VBA) code, the program is built around Monte Carlo (MC) simulation, whereby random sampling techniques are used to obtain approximate solutions to physical problems. Selection of the various properties is controlled by their frequency distributions, the parameters of which (mean, variance, etc.) are determined from the observational record; MC simulation "expands" the record by generating combinations of factors, and thus outcomes (*realizations*) that may not have happened yet – but could – over time periods much longer than measurements have covered. The model simulates heavy precipitation events over hundreds to thousands of "years", for either a real place (e.g., a weather station) or a hypothetical site having a certain elevation and/or other characteristics. In each event, the model generates the starting date and time; initial snow depth and SWE; precipitation amount, duration, and internal distribution; central and range values for temperature and wind speed; and the hourly quantities (*Ph*, *Th*, *Wh*) through the event. Once a "storm" has begun, the deterministic parts of the model calculate hourly snow accumulation or melt (if any), depending minimally on heat energy correlated to *Ph*, *Th* and *Wh*. If liquid water (R+SM) is generated in a snowpack, percolation volume and rate are calculated, and the output is water available for runoff (WAR) at the ground surface.

In essence, I am building a computational apparatus in order to run virtual experiments. I want to extend and generalize the instrumental record to simulate the kind of big, cold-season storms that arrive in the PNW, and explore how many of them involve rain-on-snow, where they are most likely, and how hydrologically significant they are in various places. Initially, I am examining the frequency relationships between precipitation events for a limited number of weather stations, as measured and as simulated by the model, to estimate the likelihood and magnitude of ROS conditions, and the differences between simple precipitation and WAR. Next, I compare those quantities for elevation zones, based on the site measurements but generalized for a region. Initial results of these efforts are presented in this paper. The model can also be used to investigate various change scenarios.

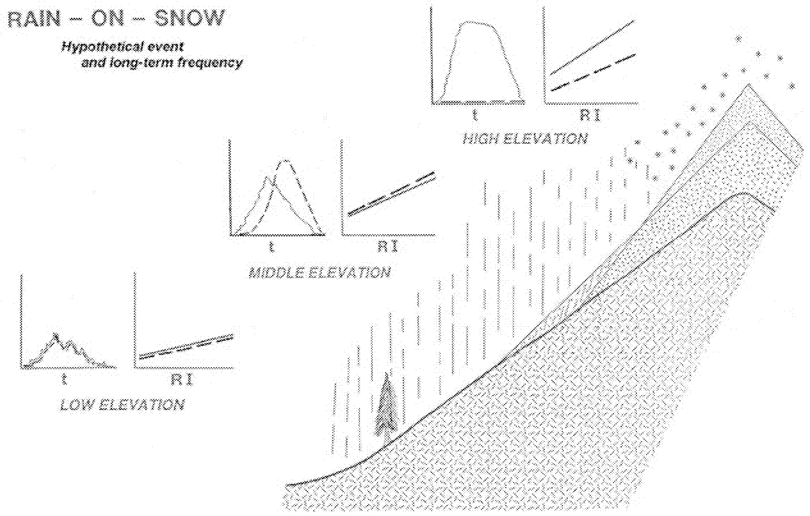

Figure 1. Elevation transect before/after a storm. In hyeto/hydrographs, t = time (hr) during the event; in frequency-magnitude graphs, RI = recurrence interval; in all, y-axis is amount of precipitation (solid) or WAR (dashed).

Some of the central issues around ROS frequency and significance involve the apparent variation with elevation. Some of these notions are illustrated in Figure 1, showing a hypothetical mountain range supporting a snowpack, before and after the arrival of a warm storm. At low elevations there is no snow, so the amount and timing of water to the ground is identical to rainfall, as shown in the hydrograph. At high elevations, the amount of precipitation may be orographically enhanced, but most falls as snow (the remainder absorbed in the pack), so WAR may be small or nil for this event. At an intermediate elevation, however, rain is supplemented by some amount of snowmelt (represented by the loss of the downhill edge of the snow wedge), and the volume of the WAR hydrograph is greater than that of the hyetograph. This middle area is the zone of significant ROS for this particular event, where $\mathrm{WAR} > P$. More important are the effects of these patterns over time: (1) If snow is rare at low elevations when rainstorms arrive, then $\mathrm{WAR} = P$ in most events, and the frequency-magnitude-duration (FMD) line for WAR will be close to that for gross precipitation. (2) If cold temperatures and deep snow normally suppress the production and transmission of liquid water at high elevations, then $\mathrm{WAR} < P$ in many events, and the FMD line for WAR should be lower than that for precipitation. (3) In middle elevations, if the occurrence of warm rains is common in areas that also support snowpacks, at least seasonally – i.e., if ROS occurs preferentially in a persistent zone – then $\mathrm{WAR} > P$ for some proportion of events, and the frequency line for WAR might be above that for precipitation.

For simulation and analysis, these concepts are translated into hypotheses and statistics. Although the outcomes of individual model events are interesting for some purposes (e.g., to calibrate/validate against event measurements, or synthesize a range of possible events for runoff or slope-stability applications), I focus on the long-term patterns of ROS in a limited region. My main questions involve the evaluation of FMD statistics at several sites and across elevation zones: to find the likelihood and consequence (hydrologic significance) of ROS at those places, and especially to determine if I can identify a peak ROS zone for my region. The hypotheses compare the statistical properties of the series of model realizations for precipitation, liquid R+SM water generated, and water delivered to the ground. In particular, for each 1000-yr run at a given site, the question "how often?" is answered by the proportion of events for which WAR > P (i.e., %ROS); the question "how much?" is addressed most straightforwardly by the means and variances of the series being compared, tested against null hypotheses (no significant difference) using appropriate statistical methods. Secondary comparisons, based on these parameters, include the trends of the various FMD lines; and the values of the integral of the product of frequency and magnitude ($\int$ F×M) for each series, which can be used to assess and compare the relative FMD trends over time.

The objectives of this project are: (1) create a computer model to simulate cool-season storms in the Northwest; (2) collect hydrometeorological data to inform the model; (3) calibrate and validate the model; (4) run the model. The first three parts are essentially complete, and I briefly describe some aspects of these elements in this paper. (Parts of this work have been reported in Brunengo, 1990; Wiberg, 1990; Wu and others, 1995.) Runs of the virtual experiments are in progress, and I offer some preliminary results here.

DATA SOURCES

Study Region and Sites

Data to feed the model include a minimal array of observations: chiefly hourly precipitation, which drives the program, but also snow depth and SWE, and temperature and wind speed during storms. Measurement sites had to be sufficiently plentiful to allow generalization across elevations, but not be from so large an area that regional/-latitudinal/orographic differences in rainfall, snow depth, storm passage, etc. are so great that variances exceed trends. One limiting factor is the usual concentration of weather stations at lower elevations, and snow courses and pillows at higher altitudes; the middle elevations assumed crucial to ROS studies tend to be underrepresented by both. Fortunately, the west-central Cascades of Washington host a relative abundance of hydrometeorologic observation sites, thanks to the presence of the Seattle and Tacoma municipal watersheds, several major transportation corridors, flood-control projects and recreation areas. Figure 2 shows the region, between the Snoqualmie Valley and the north foothills of Mt Rainier. The humid, forested terrain of the Cascades generally slopes up from the Puget Lowlands (below ~200 m elevation) in the west, toward the drainage divide (at ~2000 m, the lowest part of the Washington Cascades); the idealized ramp is broken by a somewhat abrupt mountain front, and many major and minor valleys. The climate is humid-maritime, with cool wet winters and warm dry summers; average precipitation ranges from ~100-400 cm/yr, with snow accounting for practically none to >25% (or more, some years).

Within this data-rich environment the main resource is Stampede Pass (StpP), the only first-order NWS station on the Washington Cascade crest (1206 m). Except during a few periods (notably the 1990s), measurements have been extremely reliable; they include hourly precipitation, and temperature, wind, humidity, pressure, etc. many times per day. Daily snow amounts are read at the station, and a nearby snow course until 1982 (bi-monthly Dec-Jun) and the successor SNOTEL site since. Seven other Coop stations in the region have hourly precipitation records of good length and quality, ranging from Landsburg in the lowlands (163 m); to Snoqualmie Pass (920 m) on the crest, and White River Ranger Station (1075 m) near Mt Rainier. Stations at Palmer 3ESE (275 m), Mud Mountain Dam (400 m), Cedar Lake (475 m), and Greenwater (527 m) cover some intermediate elevations, but the ~500-1000 m band is poorly represented. Among the stations, simple correlations with elevation are complicated by local conditions (mountain front, Rainier's rain shadow) that affect episodic and long-term precipitation – some sites "act" higher or lower than their elevations. Coop stations also measure snow depth (not SWE); they typically report only daily temperature extremes and do not measure wind at all, limiting their usefulness for those factors.

The second major sources of model data are snow measurement sites of the NRCS cooperative snow survey and SNOTEL networks, providing probabilities and amounts of snow on the ground when model storms occur. For this project, 20 snow courses and eight snow pillows (some co-located) are used, most in the Cedar and Green basins. Courses provide snow depth and SWE around the first of the month (sometimes also mid-month), for two to (rarely) seven months in winter and spring. Snow pillows measure SWE, queried daily or more frequently. Data reduction and analysis are restricted to sites and measurement days having at least 10 yr record. As with the NWS stations, snow courses and SNOTEL sites vary in the degree to which they are representative of their nominal elevations (range 365-1762 m) in precipitation and temperature, both of which affect snow accumulation at a site.

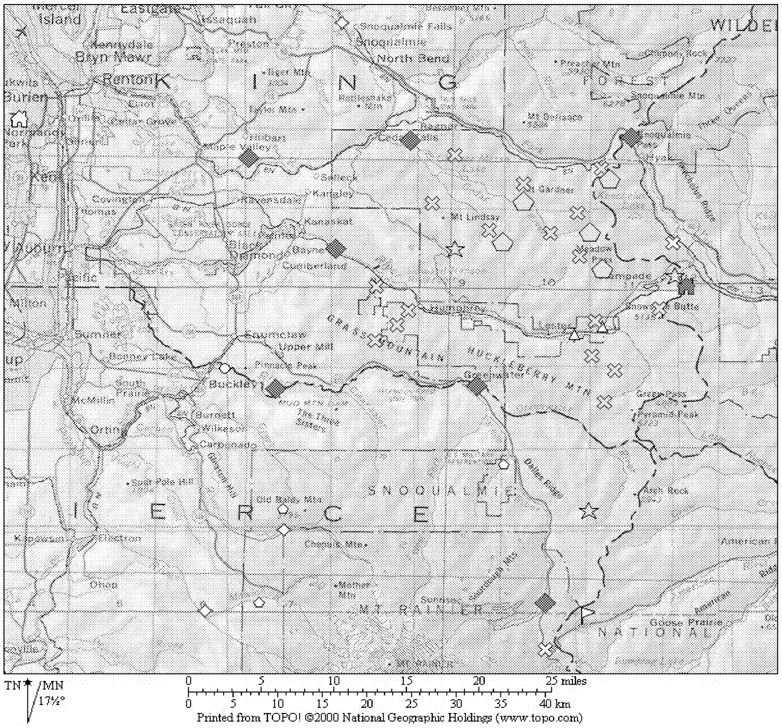

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Figure 2. Location of weather/climate observation sites in the central-western Cascades of Washington; Seattle is at map's NW corner (Sea-Tac Airport NWS station marked by hut symbol); N–S county lines (– – –) show the Cascades drainage divide. Stampede Pass hut marks area of NWS airways station, snow course and pillow. Darker and medium gray symbols represent sites used in analysis and modeling: diamond = NWS Coop station with hourly precipitation gauge; cross = snow course; pentagon = SNOTEL; star = long-term course and SNOTEL. Sites marked by small/white symbols (circle = Coop station, triangle = RAWS) not used, due to short/incomplete record, geographic inconsistency, tc.

Data: Records, Series, Statistical Analysis

Table 1 summarizes the parameters used in the model, their sources and frequency distributions. Statistics are based on up to 65 yr of record: the oldest measurements are from water year 1940, the most recent from WY 2005, though very few series span the entire period. Many of the parameters regarding event occurrence, duration and precipitation amount are based on hourly precipitation data from the eight weather stations. The analysis is organized around major storm events, which vary widely in duration: I define *long continuous storms* (LCS) as periods of steady precipitation as recorded at an hourly rain gauge, delimited by gaps of 6-8 hr. In order to examine and model a large sample of events (more than one per winter), I use the partial duration (PD) series to generate frequency distributions, comprising 170–295 events per station; although the annual maximum (AM) values are also used in characterizing and comparing series. All are based on water years (WY day 1 = 1 Oct).

Several kinds of mathematical models are applied to the parameters and data series. Some of these data are reduced to simple means and deviations based on normal or log-normal distributions. The precipitation input series, and the R+SM and WAR output realizations, are modeled using the exponential distribution, which is more appropriate for PD series than an extreme-value distribution (which are better for AM series; de Ploey and others, 1991). Some statistical parameters have been combined into curves and trend surfaces, as functions of factors such as date in the season and elevation. For example, data indicate that the amount and duration of event precipitation usually increase uphill, so the parameters (location/intercept and shape/slope) of those factors vary with effective elevation[1]; a lapse rate is applied to temperatures during storms. The likelihood and amount of snow on the ground depends on both elevation and date, so the parameters of a mixed model including the probability of no snow (P[0]) and the log-normal distribution of non-zero amounts are combined into 3-dimensional trend surfaces depending on date and altitude.[2] A few pairs of factors (e.g., precipitation amount and duration, snow depth and SWE) are so tightly linked that they are calculated using bivariate normal relations. Other elements, such as hourly precipitation and temperature, are based on a sampling of actual events (storm internal model as SIM codes; temperature frontal segments as T codes). Note that my treatment of these factors, and the model algorithms using them, are at various points on a spectrum that ranges from "extremely simple" to "extremely complex", based on compromises among relevance to the subject storms, data availability, and computational simplicity.

MODEL ELEMENTS AND OPERATION

The program has two forms. A simpler single-event (SE) configuration estimates snowpack changes and

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[1] Calculated as the average in a circle 2.5 km around each site; used to derive curves and trend surfaces.

[2] This model was outlined in Brunengo (1990), but numerical values reported in that paper have been superseded.

Table 1. Summary of model parameters, sources, statistical models and associations

Parameter	Source in Record	Series	Model, Distribution	Correlations & Functions	Notes
“Storm” Timing					
Number of events per year	hourly precip, all stations	LCS PD series	truncated normal (≥ 1/yr)		PD series comprise 170–295 events per sta
Event starting date (water year date: 1 Oct = 1)	hourly precip, all stations	LCS PD series	normal (adjusted around modal region)	→ snow depth & WE → central temperature	
Starting time			uniform		→ radiation melt in mid-day
Event duration	hourly precip, all stations	LCS PD series	log-normal	← elevation ← total precip → temperature code	bivariate normal with total precip
Precipitation					
Total amount	NWS stations: 7 Coop, 1 airways; hourly rain gauges (heated at StpP)	PD series on long continuous storms (also: PD & AM on 1- to 48-hr periods)	exponential (by regression) (also: extreme-value type 1)	← elevation → event duration	derive separate elev fcns for high- and low-precip stations
Hourly distribution	hourly precip, all stations	1000 LCS events (SIM codes)	4° polynomial on cum precip + random component	← total precip	event SIM codes chosen randomly
Snow					
Initial SWE	NRCS: 20 snow courses 8 SNOTEL sites NWS: StpP station	all avail daily data (≥10 yr record)	mixed log-normal with P[0]	← elevation ← event date (polynomial fcns)	P[0] estimated for depth & SWE together
Initial depth	NRCS: 20 snow courses NWS: 8 stations	all avail daily data (≥109 yr record)		← initial SWE	bivariate normal with SWE
Density				← SWE/depth	
Porosity (effective)				← density & irreducible saturation	
Permeability → hydraulic cond’ty				← density & grain diameter	Shimizu equation; adj for hydr K (cm/hr)
Temperature					
Central temperature	StpP station: ~ hourly temp readings; with other sta’s → storm lapse rate	126 LCS events	normal	← event date ← elevation (lapse)	
Temperature range	StpP station: ~ hourly temp		normal		
Hourly temperature		100 T codes (based on LCS events)	frontal + diurnal + random components on range	← duration	number & length of frontal segments bivariate normal with duration
Wind					
Central wind speed	StpP station: ~ hourly wind	126 LCS events	normal		
Wind speed range			normal		
Hourly wind speed			random component on range		no good model

percolation resulting from precipitation, temperature, etc. measured during an actual event, for comparison with observed outputs; SE is used to test algorithms and calibrate some parameters. The Monte Carlo (MC) version, simulating many hypothetical “centuries” of events, is the focus of most of the discussion below. The “full Monte” combines probabilistic (weather and snow conditions) and deterministic (snow accumulation/melt, percolation) elements to extend and generalize the existing record to longer time periods and larger areas. It is designed to be relatively simple and run quickly on personal computers, so several limitations should be noted. (1) It is based on *events*, and does not simulate conditions continuously through a winter. (2) It is a *point* model, applied to specific sites or generalized elevations, but it is not distributed below the ground surface or over the landscape. (3) Elements are simplified wherever possible, concentrating on the kinds of storms and snow conditions that typically occur in the region. (4) Storm precipitation is the controlling input, and the quantity against which the output realizations of *P*, R+SM and WAR are judged. Rather than creating storms with certain durations, temperature patterns, etc. and then letting those control the rainfall delivered – closer to the situation in nature – the model is driven by the frequency and patterns of storm precipitation, with some of the other parameters following from the selected amount.

MC model architecture is illustrated in Figure 3. The program operates within an Excel workbook, using code written in VBA language to transfer and manipulate information contained on several preformatted sheets. As noted, the parameter values and frequency distributions for precipitation, weather and snowpack properties are based on data from stations (especially Stampede Pass) in the Cascades (Table 1, Figure 2); some of these data have been combined into curves and trend surfaces, relating certain parameters to other variables such as elevation and seasonal date. Most of the variables used by the model are stored on the *parameters* sheet, where they are accessed to inform the entire run or any individual event. Variables based on recorded hourly precipitation and temperature patterns during storms are stored in the *SIM codes* and *temp codes* pages. Monte Carlo simulation requires a pool of random numbers (*R#*s) to provide the probabilities that are inverted to generate values of rainfall, duration, snow depth, etc. Because of the limitations of *R#* generators in Excel, and mainly to control the collection of numbers sampled and one source of extraneous variance, a set of 1.285 million *R#s* is stored on a separate sheet, enough to accommodate a 1000-yr run. (*R#s* occupy >60% of the ~28 Mb file size for the entire workbook, including code.)

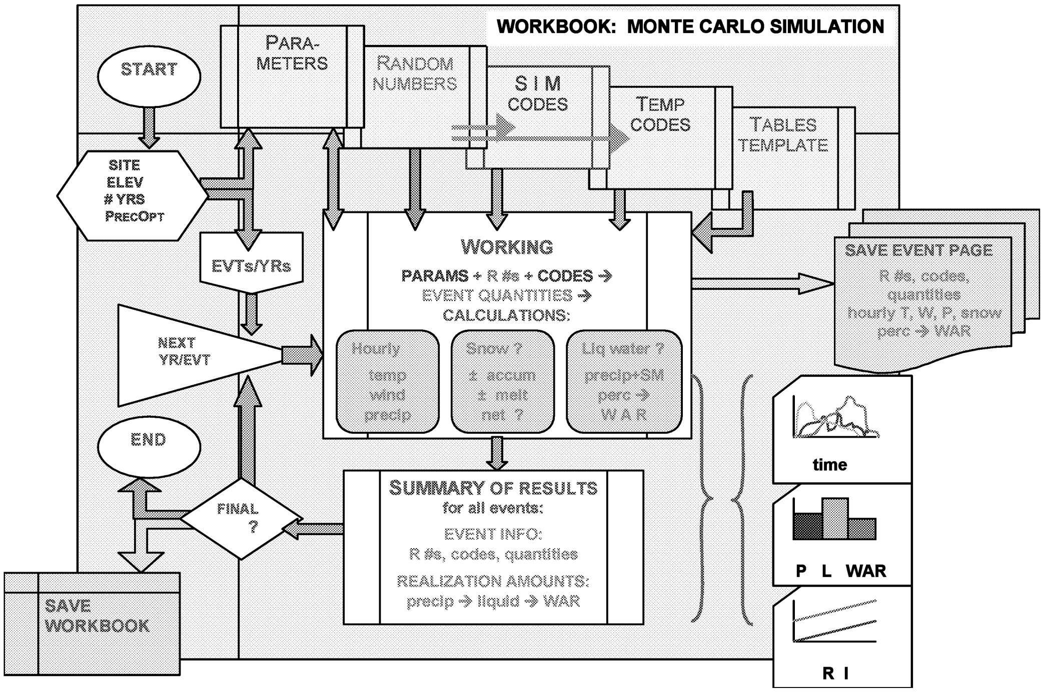

Figure 3. Flow chart of operations of the Monte Carlo model for simulation of winter-type storms in the PNW.

After housekeeping, MC program operation begins with dialog boxes querying site name, elevation, length of run, and type of precipitation distribution (*PrecOpt*: exponential distribution, either with values from a particular site, EXP, or a supposed elevation, EXE). Based on the responses, parameters are collected from that sheet into code variables, and this initial information is copied onto the *summary* page and *tables* template. The first major action is calculation of the number of events for each run year: based on the average of ~4.4 LCS/yr for all stations, 1 to 12 storms/yr are modeled, ~4400 for a 1000-yr run.

The heart of the procedure begins with the first event of the first year. Based on the distribution parameters and marching through the array of stored *R#s*, the program generates for the event a starting date and hour; total precipitation, duration, and SIM; initial snow depth and SWE; central and range values and event code for temperature; and central and range values of wind speed. Variable snowpack hydraulic properties (effective porosity φ, permeability k) are calculated based on initial density (ρ = SWE/depth). Then, the core operations for an event occur on the *working* page, carried out in three blocks of procedures.

(1) Calculate and list hourly values of weather factors: *Th* from a combination of frontal (from *temp codes*), diurnal (sine wave), and random components acting on the range around the central value; *Wh* from a random component acting on the range around the central value; and *Ph* by reconstruction of hourly amounts from a 4th-order polynomial of cumulative precipitation in a sampled event (*SIM codes*), with a random component. The proportions of rain and snow are based on a linear ratio from –1.5 to +2.5°C.

(2) Accumulate or melt snow as appropriate for each hour: if there is a snowpack and/or snowfall that hour, potential melt volume is calculated based on USACE (1956) energy-balance melt equations simplified for ROS (Dunne and Leopold, 1978; Harr, 1981; Kattelmann, 1985):

$$Mp\ [\mathrm{cm}] = Th\ [\ 0.005917 + 0.002124\ Wh + 0.0125\ Ph] + 0.002 + rad$$

Most of the energy supplied for melting is from long-wave, conductive, sensible and latent heat, indexed to *T*,

W and liquid rainfall; small amounts of ground heat and short-wave radiation (*rad*, ~0.2 cm/d distributed in a triangular function around midday) are added. Potential melt is compared against any solid precipitation to find the net change in SWE: if positive, snow accumulates (depth calculated from density based on temperature); if negative, Mp is applied first to the hour's snowfall, then to the pack.

(3) For each hour with rain and/or melt on a snowpack, route the water to the ground. Procedures are based on Colbeck's (1972, 1974; Dunne and others, 1976; Colbeck and Anderson, 1982) adaptations of porous-medium flux using kinematic-wave methods.[3] The basic equation estimates the speed of a packet of water (celerity Vf) as proportional to the snow's hydraulic conductivity ($K = \rho\ g\ k/\mu$, as cm/h) and effective porosity (φ), an empirical coefficient (n ~3), and the R+SM input for the hour:[4]

$$Vf\,[\mathrm{cm/h}] = 3\ K^{1/3}\ (\mathrm{R{+}SM})^{2/3} / \varphi$$

The location of the flux is specified by $Vf \times$ time. Larger input fluxes are faster and can overtake slower ones, creating kinematic shocks; algorithms test for such interactions, and recalculate flux volumes and celerities by combining the slower and faster waves. In a next run-through, the output of water reaching the ground surface is summed for each hour (adjusted in some cases based on the maximum flux rate or snow water-holding capacity). In the last procedure, filters pass through the hourly P, R+SM, and WAR values to find the maxima for 1-, 6-, 12-, 24- and 48-h periods and for the total LCS duration (rainfall plus 6 hr).

All of the hourly and summary values for the event are printed on the *working* page, which is held in the workbook until 50 "years" are saved to disk. For individual events, such as in SE mode or when errors stop MC runs, standard graphs attached to the program are used to visually examine the various realizations. In any case, important event information is copied onto the *summary* page on the row representing that year and event. Then, the program moves on to the next event, selects new variables, sets up a new *working* page, and repeats these procedures. After the last event of the last year, the final set of event pages is saved in a file, and the entire workbook is renamed and saved in another. For a fast computer running at ~2.8+ GHz, a full Monte run of 1000 yr takes about an hour, ± ~15 min depending on whether the model site is more likely to have snow and more of it, thus requiring more calculation time for accumulation/melt and percolation.

PRELIMINARY RESULTS

Calibration and Validation (SE) Runs: Stampede Pass and DEMO Sites

Model realizations of individual events can be used for examples of process types, comparison with event records, or input for other models downstream; the summary information for long-term MC series are used for frequency analysis and hypothesis testing. But first, a few elements of the model had to be calibrated and validated. The most basic verification method is checking output realizations to ensure that the algorithms are doing what I intend, procedurally and mathematically, i.e. that hourly T, W, P, snow and percolation values are properly calculated. Beyond that, validation of a Monte Carlo model is difficult: it produces realizations of events that may not have happened yet, and might never, in exactly those combinations, over time periods longer than human observation may occur – so testing against measurements in nature may be limited or unfeasible. MC model validation assumes that if the various stochastic parts are each justified with respect to the instrumental records driving them, and if the major deterministic elements are physically appropriate (though simplified) and produce outcomes empirically comparable to observed results – particularly of the rates of liquid water reaching the ground, which integrate all other model elements – it should follow that the program as a whole will generate valid results.

Calibration and validation processes are hindered by the scarcity of fully usable data sets, comprising hourly T, W, P, melt/accumulation, snow depth and SWE, and WAR (preferably as lysimeter outflow, or change in SWE). I thought that the Stampede Pass weather station and SNOTEL together would provide these for several events, but found very few cases, since the station was restaffed (1997) and depth sensors installed at the snow pillow (WY 2003), in which all instruments were operating properly. A ROS event on 16-18 Jan 2005 provided some information, although precipitation started as freezing rain (indicating a difference between ground and air temperatures, violating some melt assumptions). Also, I was able to use Wetherbee's (1995) observations at DEMO sites in the Umpqua basin, southwest Oregon Cascades, during several ROS storms: events #1 (29 Nov–2 Dec 1994) and #4 (29 Jan–1 Feb 1995) were most useful (both 72 hr long, the other three being 24 hr or shorter).

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[3] The snowmelt numerical-analytical package (SNAP) of Albert and Krajeski (1998) is based on similar principles, but discrepancies between parallel equations have not yet been resolved; the results here are subject to revision.

[4] For computational simplicity, the snowpack hydraulic properties φ and k (calculated from initial ρ), irreducible saturation and n are assumed to remain constant during an event, although ρ can change.

Satisfied that the basic systems are working as intended, I used the SE version to compute realizations of accumulation/melt and water output from weather data observed over the several ROS events, and compared my results with the measurements. Relatively few of the variables in the model's deterministic components had to be calibrated. Most significant, perhaps, was the grain diameter (d) used in Shimizu's equation estimating snow permeability ($k = 0.077\, d^2 \exp[-7.8\, \rho]$), which controls percolation rate; a value of 0.1 cm seems generally best (despite coarsening during ROS). Other factors to be adjusted included the minimum and maximum limits for snowpack density, and the temperature range and proportions of snow versus rain near freezing; both affect accumulation or decay rates for given amounts of precipitation or melt. The ultimate results have been generally good, through to the integration of all functions in the quantities and patterns of water delivered to the ground. For example, Figure 4 is a comparison between model WAR output and lysimeter measurements from DEMO Event #4: the curves are very similar, indicating that the calculations for snowmelt and percolation are adequately simulating this event's processes and volumes. Unfortunately, substantially the same events are used for calibration and initial validation: this is poor procedure, but lacking additional observation sets I accept for now that the model is working acceptably; further testing can be performed when measurements have been collected for more events.

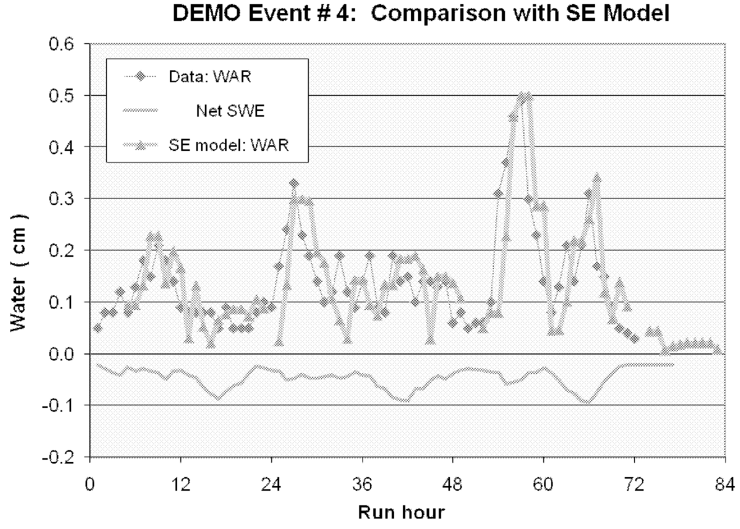


Figure 4. Comparison of model output (▲) and field measurements (♦) for ROS event of 29 Jan–1 Feb 1995 at DEMO sites 1 and 4, Umpqua National Forest, SW Oregon. SE model run used *T*, *W* and *P* observations to generate snowpack and WAR values. Negative net SWE indicates that the snowpack loses water (rain and melt) through the event. Data from Wetherbee (1995).

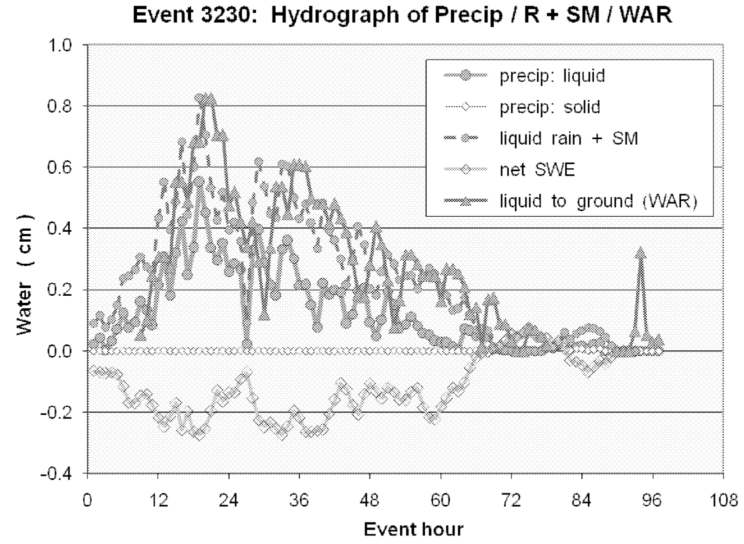
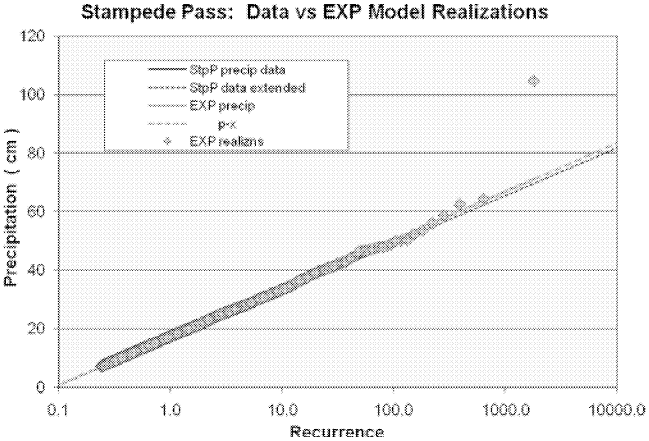


Figure 5. Hourly water input and output values generated for example event 3230 (MC–EXP version for StpP). Precipitation partitioned between rain and snow based on temperature (all R until last day); negative net SWE indicates that the snowpack loses water (R+SM) through most of the event; WAR closely tracks R+SM, except for last late flux to ground.

Monte Carlo (MC) Runs: Stampede Pass

In validation for the Monte Carlo model, I checked operations and the statistics of realization series against the governing series and parameters of the input distributions, particularly the Stampede Pass weather station, snow course and SNOTEL. Again, the MC model is evaluated initially based on its ability to function appropriately, but more critically on how well it replicates StpP's recorded rainfall amount, LCS duration, event temperature and wind, snow depth and SWE for days through the season, etc. Basic operation of the MC version is demonstrated with examples from an EXP run using StpP's effective elevation of 1065 m, over 1015 yr (generating 4135 "storms", analyzing 4101 in 1000 adjusted WY). Output from one ROS event is illustrative (Figure 5). In model terms, #3230 "occurs" in late March, with lots of snow on the ground and very warm temperatures (8-13°C) over three of four days, and so generates a lot of snowmelt. Although the 12+ cm of mostly rain would not qualify as a huge storm at StpP, the contribution of melt would be important, producing 23+ cm of WAR, almost double the precipitation. Note that this event is atypical for StpP: only ~10% of modeled events produce WAR > *P* at that elevation.

With regard to the aggregate series over hypothetical centuries, the Monte Carlo model reproduces the input distributions quite well. Figure 6 shows the correspondence between the exponential trend line for the partial-duration record of long continuous storms at Stampede Pass, and the realizations and trend from the MC simulation using the EXP precipitation model. Statistical tests on means and variances show that the output sample can be considered to belong to the same population as the governing parameters. This is despite the radical outlier generated in all the model outputs: once in a run, the minimum possible *R#* of the set (0.000001) is chosen to produce a very small exceedance probability and thus a very large precipitation amount; this event (lasting over a week) always "occurs" on bare ground in the summer (unrealistic but possible), so no snowmelt is generated.



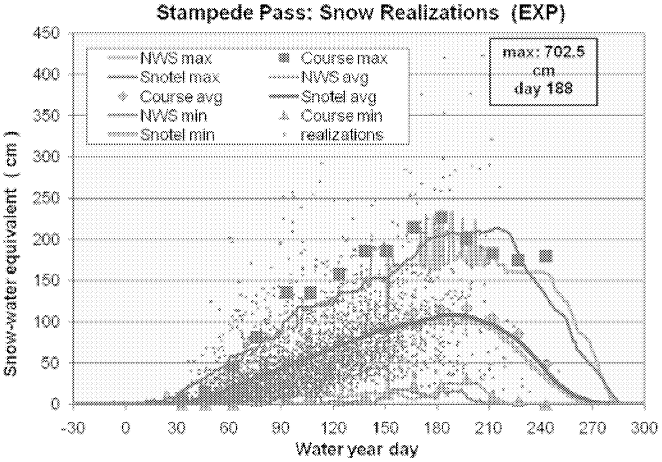
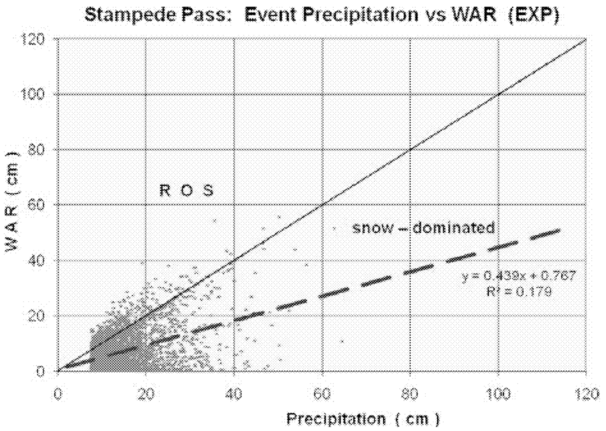


Figure 6 (L). Realizations of total precipitation (♦) from MC–EXP run for StpP, generating 4101 events over 1000 WY. Exponential trend line is very close to that of the frequency distribution sampled by the MC program, derived from 225 storms measured at StpP. The outlier is real. (Trend lines dashed beyond the frequency ranges of the data/realizations). Figure 7 (R). Realizations of initial snow volume from the same MC–EXP run for StpP (maximum value off scale). Lines and large markers indicate maximum/mean/minimum SWE recorded at StpP NWS station (almost daily), snow course (semi-monthly) and SNOTEL (daily). Data from NWS and NRCS publications and web sites, for varying available observation periods 1944-2005.

The MC model generates good realizations of the other stochastic factors as well. Figure 7 shows one of the more complex variables, initial snowpack volume, which is a function of elevation and the previously chosen initiation date (itself controlled by empirical storm arrivals). The concentrations of points reflect the preponderance in Oct–Mar, with fewer events in other months. The main cluster of initial SWE realizations falls just below the StpP average for a given day, with some points above the recorded maxima. This pattern is reasonable, considering that starting snowpack is modeled as log-normal, and can go beyond the record; samples for particular days closely match the governing distribution parameters.

We are most interested, though, in the ultimate model output of water to the ground, and its comparison with gross precipitation. Figure 8 graphs WAR against *P* for each event of the MC run for Stampede Pass site parameters. In almost 70% of the realizations in this sample, the amount of water transiting the pack is less than precipitation, as snowfall and/or a deep pack retard the generation and movement of liquid to the ground; in 11% of those, no water arrives at all. In ~9% of the events, WAR = *P*, usually indicating bare ground at beginning and end, perhaps with complete melt of any snowfall during the storm (this set includes the outlier at ~107 cm). For this run, 463 of 4101 events produce WAR in excess of the input precipitation: this ~11% ROS statistic represents the events that are at least marginally significant in terms of snowmelt enhancements to precipitation.



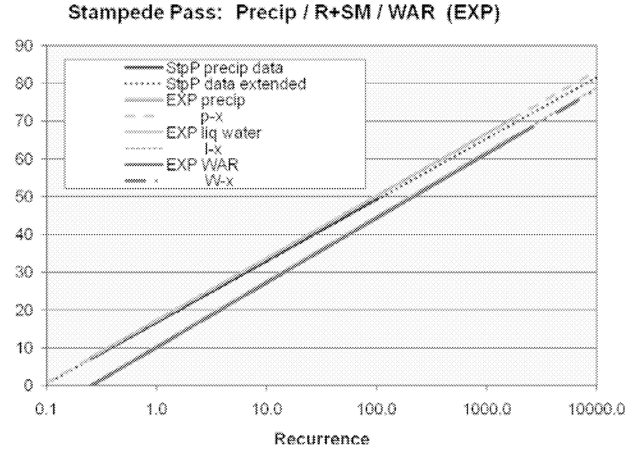


Figure 8 (L). Water available for runoff against total precipitation, from MC–EXP run for StpP (4101 events over 1000 WY). 1:1 line indicates WAR = *P*. Snowfall/pack dominate in lower right, where WAR < *P*; significant ROS in upper left, where WAR > *P*. Heavy dashed trend line suggests that WAR averages ~44% of precipitation in this run. Figure 9 (R). Exponential trends for realizations of water inputs and outputs from MC–EXP run for StpP (points not shown). As in Figure 6, the line for total event precipitation is very close to that of StpP data. Trends for liquid water and water to the ground (WAR) are almost collinear, though the distribution statistics are not quite identical.

Figure 9 shows the frequency-magnitude-duration curves for this Monte Carlo simulation. Again, the line for model realizations of total precipitation is very close to that of the empirical distributions from Stampede Pass. However, as can be surmised from the large number of model events in which WAR < *P*, the FMD lines for liquid water and water to the ground fall below those for gross precipitation. If the model is credible, this indicates that at this elevation, cooler temperatures, greater snowfall and deeper packs would hinder the delivery of WAR during a large proportion of big storms. The slight non-parallelism between the *P* and WAR lines may suggest some difference in the behavior of high- and low-frequency events, but any such inferences will require analysis of more runs. In addition, the near-congruence of the R+SM and WAR trends indicates that all or most of the liquid water generated in model events usually gets to the ground within the specified duration. This is fully reasonable for cases with little or no snow; but with deeper packs, it might suggest that modeled percolation rates are unrealistically fast.

Monte Carlo (MC) Runs: Elevation Zones

One of the main objectives of this project is to reveal where rain-on-snow is most probable and significant, at least in terms of elevation zones in the central-western Cascades. This is to be done by employing the Monte Carlo model to simulate sets of storm events at various supposed elevations, and comparing the statistics of the realization series. My conceptual view assumes that the mountain range approximates a simple ramp, with storm behavior varying just with elevation: this is not exactly true, either physiographically (mountain front, ridges and valleys) or climatically (decline in precipitation with moisture loss as a storm moves eastward). Nevertheless, in the MC–EXE version, the parameters of several important stochastic elements are calculated as direct or indirect functions of elevation, including precipitation amount, duration, initial snowpack, and temperature (Table 1). The samples in all runs are chosen using the same set of *R#s*, so the probabilities of corresponding quantities are identical.

The first test of the EXE program was by comparing EXP outputs for Stampede Pass–specific parameters with EXE realizations using StpP's effective elevation of 1065 m. The results (not shown) are similar but not identical, and not expected to be, as EXE outputs come from functions generalized among many observation sites. Next came trials for several elevation zones: Table 2 lists statistics from EXE runs for model sites at 500, 750, 1065 (in place of 1000) and 1500 m, for the series of precipitation, liquid water and WAR realizations. (The 2nd greatest value is noted because the highest in each series is always due to the outlier, so the #2s can be more informative.) In each elevation band, quantities tend to decline from *P* to R+SM to WAR, more sharply at higher levels. Note that minima for R+SM and WAR are all 0, events in which no liquid water reaches the ground; the proportion of no-WAR values increases from <1 to >33% with elevation (complicating statistical analysis and graphing of exponential series). Going "uphill", precipitation maxima and averages increase, as the governing distributions demand. However, the average R+SM and WAR outputs are similar at 500 and 1065 m, peak in between at 750 m, and fall off to 1500 m; the 2nd-ranked values are highest for 1065 m, and decline at 1500 m. The %ROS statistics tell a similar story, with ~1-2% of events having WAR > *P* at the lowest and highest levels modeled, reaching a zenith of ~11% at 750 m.

The exponential trend lines corresponding to these realizations are illustrated in Figure 10. Points are not shown for clarity (and graphical file-transfer limits); most fall on the curves, but two kinds of exceptions should be noted: (1) several of the highest precipitation values are well above their lines, especially for 1500 m, reflecting the low-probability outlier and the orographic enhancement built into the algorithms; (2) WAR = 0 points fall along the x axis, again mainly for the high elevations at which the proportion of no-WAR events is greater. Both of these extremes tend to make the lines steeper, although their slopes are controlled chiefly by the thousands of points in the middle. In general, these lines demonstrate the same relations as the numbers in Table 2. The *P* and WAR lines for 500 m are very close (though not statistically identical), indicating no enhancement of precipitation with snowmelt. For the high-elevation simulations, the WAR curves fall well below those for gross precipitation, particularly so for 1500 m but also at ~1000 m; this supports the notion that ROS is rare in the higher mountains, and it is more likely for big storms to deliver snow and/or soak up liquid R+SM in deep snowpacks.

Table 2. Statistics of model realizations for elevation (M–C version EXE)

Parameters	Site elevation modeled				Notes
	500 m	750 m	1065 m	1500 m	
Precipitation					
Rank # 1 (# 2)	52.93 (38.90)	72.87 (53.15)	97.99 (71.10)	125.00 (95.89)	means and maxima of precip rise with elevation
min	5.65	6.50	7.90	8.70	
mean	9.42	11.70	14.56	18.52	
std dev	3.47	4.89	6.66	9.08	
Liquid rain + snowmelt					
Rank # 1 (# 2)	52.93 (46.54)	72.87 (44.27)	97.99 (62.86)	125.00 (58.76)	mean / maxima of liquid generated in snowpack peak at ~750 m
min	0	0	0	0	
mean	7.99	8.70	7.73	4.84	
std dev	3.88	5.75	7.31	7.42	
Water available for runoff					
Rank # 1 (# 2)	52.93 (46.52)	72.87 (44.27)	97.99 (62.57)	125.00 (58.76)	as with R+SM though slightly smaller, mean / maxima of WAR peak at ~750 m
min	0	0	0	0	
mean	7.98	8.67	7.65	4.62	
std dev	3.89	5.75	7.31	7.35	
% R O S					
WAR = 0	0.56	2.07	8.74	33.68	% ROS, expressed as % of events in which WAR > P, also peaks ~750 m
WAR < P	42.97	57.03	68.09	58.30	
WAR = P	54.40	29.66	14.42	6.95	
WAR > P	**2.07**	**11.23**	**8.76**	**1.07**	

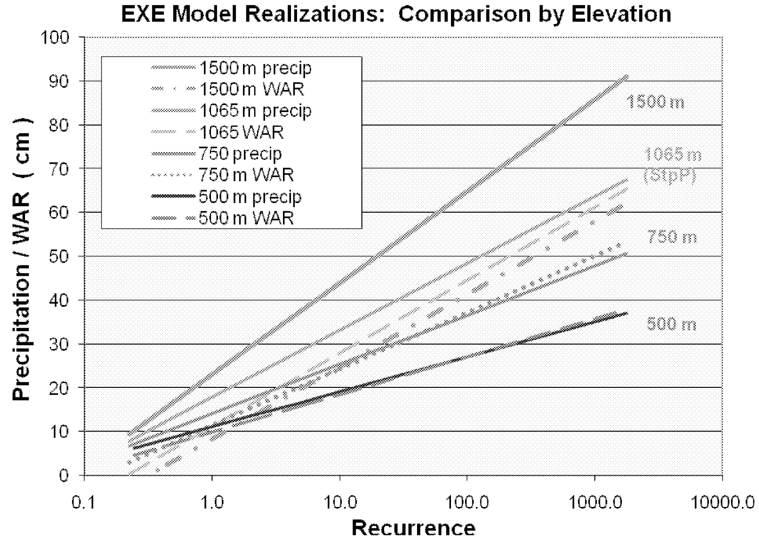


Figure 10. Exponential trends for realizations of water inputs and outputs from MC–EXE runs for various site elevations, generating 4487 events over 1000 WY (points not shown). *P* and WAR curves are closest for 500 and 750 m; WAR line falls away slightly for 1065 m, sharply for 1500 m.

On the other hand, the lines representing the 750 m simulation seem to show that the long-term frequency-magnitude relations of WAR at that elevation could be greater than those of precipitation, which would buttress the idea illustrated in Figure 1. This interpretation was offered in the oral presentation at the Western Snow Conference, but reanalysis shows that these lines present a graphical illusion. In fact, the realization points of the WAR series fall below the *P* points at higher recurrence periods; while 34% of the low-magnitude WAR points are less than the minimum *P* value, which seems enough to tilt the WAR curve upward to cross the *P* curve at the high end. Inspection of the series statistics in Table 2 confirms that the WAR series values are collectively lower than those for precipitation. Thus, these curves cannot be used as evidence for one of the particulars of the hypothetical enhancement of ROS in middle elevations, although the peaks in WAR amounts and %ROS still support the general idea.

SUMMARY & CONCLUSIONS

Rain-on-snow is an important hydrometeorological phenomenon, particularly in the Pacific Northwest: winter storms involving heavy rainfall and warm temperatures can be augmented by melting snow, causing high runoff, flooding and erosion. The sporadic nature and geographic variability of ROS events makes them hard to characterize from the instrumental record. However, Monte Carlo simulation is a useful tool for studying probabilistic phenomena such as the storms causing ROS. I developed a computer model combining MC sampling of stochastic factors including storm timing, precipitation amount, snowpack and weather conditions; along with deterministic components for estimating snow accumulation and melt, and movement of liquid water through a snowpack. The model generates realizations of storm events for ~1000 "years", based on data, frequency distributions and trends from weather and snow stations in the central-western Cascade Range of Washington. The model employs simplifications wherever possible, and is limited by its underlying assumptions and minimal data requirements. It seems to work appropriately: mathematical operations perform as intended, probabilistic elements reproduce the relevant input series, and deterministic components produce reasonable snow and percolation responses.

More to the point, the model generates interesting frequency-magnitude-duration distributions, for simulations of actual weather stations (particularly Stampede Pass) or imagined situations. Based on results so far, if the MC model can be accepted at least as reasonable, the realizations' statistics support the notion that the hydrologic significance of rain-on-snow peaks in a middle elevation band at ~750 m. This preliminary conclusion is founded on just one of the criteria listed initially, the differences in proportions of ROS events occurring in a modeled storm series. In contrast, the hypothesized relation of FMD curves for WAR for middle elevations is not completely upheld by these model runs. Nevertheless, I am encouraged that there does seem to be a preferred ROS zone in this region. And it has not escaped notice that 750 m (2460 ft) is remarkably similar to the elevations that Northwest hydrologists have been mentioning as a rain-on-snow zone – and siting their field studies – over many decades.

Future efforts with the model will include attempts to confirm the FMD relations with respect to data from other weather stations. To further explore the relation of ROS to elevation, I will perform runs at narrower intervals (probably 100 m); and analyze the realizations using the annual-maximum as well as partial-duration series. Eventually the model can be applied to scenarios involving possible changes of climate and land use. I must also address the feasibility of extrapolating the model to other regions without the laborious data-mining done for my study area.

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