LABORATORY TEST OF SNOW WETNESS INFLUENCE ON IMPULSE RADAR AMPLITUDE DAMPING

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ABSTRACT

The two-way travel time of impulse (ground penetrating) radar operated from helicopters and snowmobiles is used to determine snow water equivalent for annual snowpacks, glaciers and polar ices. Presence of liquid water in a snowpack creates uncertainties, which for a typical snowpack with 5% (by volume) liquid water can lead to overestimation of the snow water equivalent by about 20%. It would therefore be helpful if the radar could also be used to determine snow wetness. Damping of wave amplitude varies with electric conductivity and the dielectric constant. The dielectric constant can be calculated using the radar pulse travel time and knowledge about the depth, hence studying amplitude damping should reveal how conductivity depends on snow wetness. A laboratory test was set up to study the relationship between conductivity and wetness. Three sets of measurements, two with "old" and one with new-fallen snow, were made on initially dry one-meter thick snowpacks contained in a plywood box with cross-section area about $0.5m^2$. Water was stepwise added to the box in between radar measurements. A tentative relationship between conductivity and wetness was found, but further tests including studies of the effect of variations in snow salt concentrations and in experiment design are needed to assess the generality of the relationship.

INTRODUCTION

The snow water equivalent (SWE) is a measure of how much water is stored as snow. The Scandinavian hydropower industry needs more precise SWE estimates to achieve a more accurate prediction of the spring flood resulting in a more efficient energy production. Accurate SWE information is also essential for glaciologists, hydrologists, or a town planner who must ensure that reservoirs contain enough water to sustain a town for the year.

Ground penetrating radar (GPR) evaluates the two-way travel time of a radar pulse. The radar can be operated from a snowmobile or helicopter to cover large areas such as snowfields or glaciers. In Sweden, the use of GPR for SWE measurements was first studied by Ulriksen (Ulriksen, 1982). By 1986 Sweden was using GPR from helicopters traveling at 50km/hr to conduct *SWE* surveys, which has led to significant time gains as compared to traditional manual measurements (Marchand et al., 2003). Since the first study there have been many improvements in the quality of data collected from GPR measurements, mostly due to a better understanding of how factors like density, snow wetness, and melt-freeze cycles affect radar signals (Lundberg et al. 2005). However, if free water is stored in the snowpack, the uncertainties in GPR-determined SWE-values will increase. For a snowpack with a density around $300kg/m^3$ and 5% (by volume) liquid water SWE will be overestimated by approximately 20% (Lundberg and Thunehed, 2000). Measuring amplitude damping of a radar pulse can provide information about liquid in the snowpack, which can be used to reduce these uncertainties (errors) in SWE estimates. Conveniently, both the amplitude and two-way travel time can be obtained from the same radar data.

The overall aim with this study is therefore to establish the relationship between electric conductivity of a snowpack (obtained from amplitude damping) and snow wetness, in order to improve the accuracy of SWE measurements taken with ground penetrating radar.

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THEORY

As can be derived from Maxwell's equations, the amplitude A (-) of an electromagnetic wave can be expressed as a function of the initial amplitude A_0 (Wangsness 1979). The energy of a pulse is distributed over an area $f(x^2)$, which increases with the distance x (*m*) from the source. Amplitude is a measure of wave energy, therefore it will decrease with distance:

$$A = \frac{A_0}{f(x^2)} e^{-\alpha x}, \tag{2}$$

where the damping factor ($e^{-\alpha x}$) depends on the electrical properties of the medium.

When a radar pulse passes from air to a dielectric medium with a higher dielectric constant (ε) (such as a snowpack), the pulse is bent toward the normal according to Snell's Law (Nordling and Österman 2002). The refractive index (n) is related to the relative dielectric constant (ε_r) by $n = \sqrt{\mu \cdot \varepsilon_r}$. Here ε_r is specific to each medium and is related to the dielectric constant by $\varepsilon = \varepsilon_0 \cdot \varepsilon_r$ (*As/Vm*), where ε_0 is a constant equal and μ is the magnetic permeability (*Vs/Am*) that can be set equal to unity since snow is a paramagnetic material (Engström, 2000). If we assume the largest angle of incidence for the radar wave to be 90° to the normal (i.e. the source is located close to the snow surface) then the largest angle of refraction (Φ) through the snowpack will be $\Phi = \arcsin\left(\sqrt{\frac{1}{\varepsilon_{r\_snow}}}\right)$, where the relative dielectric constant of the snowpack is denoted by ε_{r_snow} (Figure 1). This is derived from Snell's Law.

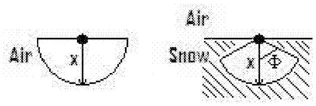

Figure 1: Difference in the shape of a radar pulse traveling through air (left) and passing from air to snow (right), where x is the traveled distance and Φ is the largest angle of refraction.

Therefore the following relation holds:

$$\frac{f(x^2)_{snow}}{f(x^2)_{air}} = 1 - \cos(\Phi). \tag{3}$$

This also explains why the amplitude of a radar pulse sent through snow can be larger than the amplitude of a reference pulse sent through air, as the same energy is distributed over a smaller area.

The exponent α in the damping factor $e^{-\alpha x}$ in equation (2) is (Jordan and Balmain 1968):

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}, \tag{4}$$

where ω is angular velocity (*rad/s*), σ is conductivity (*S/m*) and ε is the dielectric constant. For frequencies in the radar range we have $\sigma/\omega\varepsilon \ll 1$, so we can write the following approximation by using only the first two terms of the binominal expansion (Jordan and Balmain 1968):

$$\sqrt{1 + \frac{\sigma^2}{\omega^2\varepsilon^2}} \cong \left(1 + \frac{\sigma^2}{2\omega^2\varepsilon^2}\right). \tag{5}$$

Substituting (5) into equation (4) we obtain:

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}. \tag{6}$$

The damping factor can therefore be expressed as a function of electric conductivity and the dielectric constant, hence combining equation (2) with equation (6) and solving for σ gives:

$$\sigma = \frac{-2}{x} \cdot \sqrt{\frac{\varepsilon}{\mu}} \ln\left(\frac{A \cdot f(x^2)}{A_0}\right). \tag{7}$$

As snow is a paramagnetic material, we set μ equal to unity, and the dielectric constant can be obtained from the two-way travel time. This establishes the relationship between amplitude damping and electric conductivity. Once the relationship between conductivity and snow wetness has been experimentally established, the relationship between radar pulse amplitude damping and snow wetness will be obtained.

METHODS

A laboratory experiment was conducted to investigate how electric conductivity is related to snow wetness. In the experiment the initial amplitude A_0 was unknown, and amplitude damping was obtained by dividing the amplitude of a pulse sent through snow by the amplitude of a reference pulse sent through air. However, since the amplitude in the snow is enhanced due to concentration of the signal, equation (7) had to be modified to take equation (3) into account:

$$\sigma = \frac{-2}{x}\cdot\sqrt{\varepsilon}\ln\left(\frac{A_{snow}\cdot(1-\cos(\Phi))}{A_{air}}\right). \qquad (8)$$

Measurement

In the experiment a 99cm high plywood box with a 0.476m^2 opening at the top was filled with dry snow. The snow's temperature was kept close to 0˚C by storing it in a climate-controlled room for several days before the experiment. The snow was weighed and then added to the box. Its initial density was calculated from the weight of snow and the box's dimensions. A separate wooden structure housed radar antennas, one below and one above the box. This allowed reference measurements to be taken in-between stepwise addition of tap water chilled close to 0˚C. Both the box and the housing were built without metal parts in order to decrease disturbance in radar signals.

Three similar experiments were conducted, the first two with "old" snow, and the third with new-fallen snow. In each experiment a RAMAC ((Malå Geoscience AB, Sweden) ground penetrating radar with two 800Mhz antennae was used to take measurements. The first measurement was taken through dry snow. At each step about 4 and 1 liter of water (in the first and the other two experiments, respectively) was added to the box, followed by a radar measurement through snow and a reference radar measurement through air. The water was weighed, and sprinkled as evenly as possible over the snow using a watering pot. In the experiments the initial height of snow in the box was between 85 and 99cm with an initial density range of 207 -294 kg/m^3. The air temperature in the room during the experiments varied between +1 and -1.5˚C, with snow temperatures of -2 to -4˚C.

To minimize the effect of possible interference from reflections of the original radar pulse (for example, from the sides of the box) the value at the first significant minimum of amplitude was used for the calculations. In each experiment, samples of the snow and water were taken to test electric conductivity and total dissolved solids (the total amount of Ca^{2+}, Mg^{2+}, Na^{+}, K^{+}, Cl^{-}, SO_4^{2-}, HCO_3^{-}, and SiO_2 measured in g/l).

RESULTS

Based on experimental data plotted in figure 2, electric conductivity can be assumed to be linearly dependent on snow wetness $\sigma = m\cdot\theta + b$. Analyzing the combined data from all three experiments we obtain a tentative formula for electric conductivity as a function of snow wetness $\sigma = 0.498\cdot\theta + 0.0065$ (see Table 2).

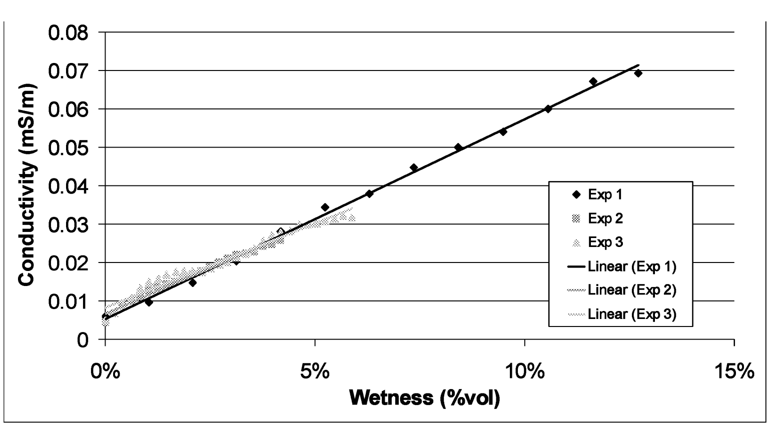

Figure 2: Electric conductivity vs. snow wetness (θ) for all three experiments, with linear regression trendlines.

Table 1: Variable from the experiments.

Experiment	**1**	**2**	**3**
Initial snowpack density (*kg/m*^{3})	290	294	207
Final snowpack density (*kg/m*^{3})	301	300	223
Initial snowpack height (*m*)	0.81	0.99	0.99
Final snowpack height (*m*)*	0.78	0.97	0.92
Final snow wetness (*%vol*)	12.0	4.1	6.1
Air temperature (*˚C*)	-1.5	1.0	0.0
Water temperature (*˚C*)	0.0	0.5	0.0
Initial snow temperature (*˚C*)	-4	-2	-2

*Linear decrease between measured initial and final snow heights assumed.

Table 2: Values from the linear regression

Experiment	**m (slope)**	**b (intercept)**	**R^2 ****
1	0.521	0.0053	0.9967
2	0.474	0.0067	0.9846
3	0.437	0.0083	0.9683
All data	0.498	0.0065	0.9888

** Coefficient of determination

Table 3: Test results from water and snow samples

	Snow		**Water**	
	exp 1 & 2	**exp 3**	**exp 1 & 2**	**exp 3**
Electric conductivity (*mS/cm*)	0.0083	0.0043	0.289	0.281
Total dissolved solids (*g/l*)	0.0053	0.0028	0.185	0.180

DISCUSSION

As demonstrated by the high R^2 values of the trendlines in Figure 2 (see Table 2), the experimental data strongly supports a linear dependency between conductivity and snow wetness. The slightly larger variance observed in the third experiment with lower-density new-fallen snow might be due to the chosen method of approximation for snowpack heights. Our simplification was that as water was added to the snow, the snowpack settled linearly. This should affect the new-fallen snow in the third experiment more since it had the lowest density resulting in the largest decrease in snow height, thus leading to a larger approximation error.

Several sources of error can be identified in the experimental setup, some of which can be eliminated in future work. For example, the salt content in the water added to the snow was much higher than the salt content in the snow itself (see Table 3). Higher salt content results in higher conductivity in the tap water leading to increased conductivity of the snowpack. Therefore, the slope of the trendline should be lower in field measurements. Water from melted snow, rather than tap water, should be used in future experiments. Additional experiments are needed to understand exactly how an increase in salt content affects the relationship between conductivity and wetness.

Another possible source of error is uneven water distribution, both horizontally and vertically. Water will not disperse evenly throughout the height of the snowpack, but since a radar wave travels the whole height of the snowpack, the error due to uneven vertical distribution is limited. However, horizontal variance is more serious. Dispensing water over the snowpack as evenly as possible using a watering pot can never produce a perfect spread. This can result in skewed values of water content used for calculations. This can be partly overcome in future experiments by performing measurements at multiple points of the surface (in our experiment the position of the antennae was fixed in relation to the box).

Interference from reflected radar waves may lead to incorrect readings of the amplitude. In our experiments this was taken care of by only using the first (significant) minimum of the amplitude in the calculations. Using a box with a larger cross-section area should also decrease the occurrence of interference.

CONCLUSION

This experiment produced promising results allowing future use of radar pulse amplitudes to measure snow wetness. The results suggest a linear relationship between conductivity and snow wetness. However, to assess the generality of the relationship additional experiments should be conducted to establish how snow density and salt content in the snow affect the relationship between electric conductivity and snow wetness.

Note also that in field measurements the radar pulse will travel through the snowpack, reflect from the ground, and travel back through the snowpack. Therefore, for this method to be applicable in real life, additional research is needed to establish how the reflection and the angle of incidence affect amplitude damping.

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