MODELING OF INTERANNUAL SNOW STORAGE FOR LONG-TERM SIMULATION

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ABSTRACT

Snow models in hydrologic engineering field have barely incorporated the long-term effect of the interannual snow storage such as glaciers because the time scale of glacier dynamics is much longer than those of river flow and seasonal snowmelt. It will be shown in the presentation that general energy and mass balance models for snow process may fail without consideration of the interannual snow storage effect. This study proposes an appropriate treatment for inland glaciers as systems in dynamic equilibrium that stay constant under a static climate condition. It is supposed that the snow/ice vertical movement from high elevation areas to valleys (lower elevation areas) by means of wind re-distribution, avalanches, and glaciation, may be considered as an equilibrator of the glacier system because it stimulates snow/ice ablation. The simplest implicit modeling of such a dynamic equilibrium snow system is introduced and discussed for the long-term snow simulation, and its demonstrative model application in Wyoming will be presented. (KEYWORDS: glaciers, snow storage, mass balance models, equilibrium, glaciology, energy balance)

INTRODUCTION

The dynamic equilibrium is a state in which the reaction rate is equal to the action rate while both actively take place. However, the equilibrium state is determined by an equilibrium constant, usually a simple function of the forward and reverse reaction rates. If the reaction rate is changed by an external forcing such as temperature, pressure, etc., the equilibrium level will shift gradually, and the system will show dynamic behavior. In other words, the glaciers gain in volume at the high elevation regions while they melt and sublimate in the lower elevation regions, but the overall size of the glaciers may stay the same on average in the long-term if the climate stays at the same state. The glaciers and interannual snow storage on mountains should be considered to be in a dynamic equilibrium. This study presents the treatments of dynamic-equilibrated glacier systems at regional scale by means of degree-day method (Ohara et al. 2014) and the energy balance method (RegSnow).

The shrinkage of glaciers has been recognized as one of the clearest evidences of global warming [e.g. IPCC, 2007; World Glaceir Monitoring Service (WGMS), 2008]. However, it is important to interpret our equilibrium assumption within the context of this change. The assumption implies that the glacier volume should stay the same under a static climate condition, and the glacier shrinkage means a shift of the equilibrium state due to the climate change. Without this equilibrium assumption, shrinkage of the glaciers cannot be an evidence of the climate change.

Glaciers are considered as an equilibrium system in glaciology field although hydrologists have only estimated the energy and mass balance of ice/snow bodies in their standard snow models. For example, glaciers are often characterized by “response time”, which is the steady-state sensitivity to sudden changes in mass balance [Huybrechts et al., 1989]. Similarly, Kruss [1984] characterized the glacier by the time lag in phases of a given sinusoidal climatic forcing and the corresponding sinusoidal front variation of the glacier, namely the reaction time. These indicators have been utilized to express the long-term character of the glacier at the local-scale in a very detailed glacier dynamic model. In this study, a glacier under an evolving climate condition will be treated as a dynamic equilibrium system, and its modeling will be performed accordingly.

PROBLEM IDENTIFICATION

Snow models for mass balance computation can be essentially described by two conservation equations: mass and energy balance equations. In practice, due to the limitation of the data availability, a simplified energy

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Paper presented Western Snow Conference 2014
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Snow models for mass balance computation can be essentially described by two conservation equations: mass and energy balance equations. In practice, due to the limitation of the data availability, a simplified energy conservation equation known as a degree-day approach is often utilized instead of a full energy balance framework. For either the simplified or the full energy-balance approaches, the general snowmelt model for the snow and ice mass balance computation can be expressed as follows:

$$\frac{dS(\mathbf{x},t)}{dt} = P(\boldsymbol{a},\mathbf{x},t) - M(\boldsymbol{a},\mathbf{x},t) - E(\boldsymbol{a},\mathbf{x},t) \tag{1}$$

$$M = f(\boldsymbol{a},\mathbf{x},t) \tag{2}$$

where S is snow and ice storage [L], M is snowmelt amount [L/T], P is snow accumulation [L/T], E is evaporation and sublimation [L/T], $\boldsymbol{a}$ is atmospheric forcing variable vector, $\mathbf{x}$ is space, t is time, and f is an arbitrary function depending upon the snowmelt formulation. The first equation is the mass conservation equation and the second one is the snowmelt or energy conservation equation. If the physically-based energy-balance approach is taken, the snowmelt equation may be rewritten as,

$$M = L\left(Q_{sw}(\boldsymbol{a},\mathbf{x},t) + Q_{lw}(\boldsymbol{a},\mathbf{x},t) + Q_h(\boldsymbol{a},\mathbf{x},t) + Q_e(\boldsymbol{a},\mathbf{x},t) + Q_p(\boldsymbol{a},\mathbf{x},t) + Q_g(\boldsymbol{a},\mathbf{x},t)\right) \tag{3}$$

where L is the thermal quality of snowpack [L^3/J], Q_{sw} is the shortwave radiation energy flux [J/L^2/ T], Q_{lw} is the long-wave radiation energy flux [J/L^2/ T], Q_h is the sensible heat flux [J/L^2/ T], Q_e is the latent heat flux [J/L^2/ T], Q_p is the heat transfer from rain drops [J/L^2/ T], and Q_g is the ground heat flux [J/L^2/ T]. Alternatively, the snowmelt equation with the simplified approach, the degree-day formula, may be expressed as follows,

$$M = C_t(T_i - T_b) \tag{4}$$

where M is snowmelt production (mm/day), C_t is the degree-day factor (mm/(°C day)), T_i is the index air temperature (°C), and T_b is the base temperature (°C). It may be noted, more or less, that all existing snowmelt models can be described by the above general expressions (1) and (2) with some minor variations.
In order to examine the interannual variability of the snow storage under a static or stationary climate condition, the mass balance equation is integrated over a year. Under the static climate condition, the annual mean mass fluxes, snowfall, snowmelt, evaporation and sublimation over the snowpack become constants. Hence, the mass conservation equation can be reduced to,

$$\frac{dS(y,\mathbf{x})}{dy} = \overline{P(\boldsymbol{a},\mathbf{x},t) - M(\boldsymbol{a},\mathbf{x},t) - E(\boldsymbol{a},\mathbf{x},t)} = W(\mathbf{x}) \tag{5}$$

The bar in the equation (5) denotes the annual averaging operation. W is the annual mean mass balance over the snowpack and y is year. It is noted that W is constant over the years as the static climate condition is assumed in this exercise. The solution of this ordinary differential equation (ODE) with the initial condition $S(0,\mathbf{x})=S_o(\mathbf{x})$ can be obtained as

$$\mathrm{S}(\mathrm{y},\mathbf{x}) = \mathrm{W}(\mathbf{x})\mathrm{y} + \mathrm{S_o}(\mathbf{x}) \quad . \tag{6}$$

Since the snow storage S cannot be negative, the limit of the snow storage as the number of years y goes to infinity is,

$$\lim_{y\to\infty} S = \begin{cases} 0\,, W < 0 \\ S_o\,, W = 0 \\ \infty\,, W > 0 \end{cases} \tag{7}$$

If the annual mean mass balance is less than zero (snowmelt > snow accumulation), the snow cover develops only during winter and disappears during the following summer before the next winter arrives. This case obviously represents seasonal snow. When W is equal to zero, the snow storage will remain the same at the initial storage S_o. However, this case where the annual snowmelt production and the snow accumulation amount stay in perfect balance, is extremely rare. The third case in Equation (7) is more problematic and unrealistic. If the annual mean mass balance exceeds zero (snowmelt < snow accumulation), the snow storage goes to infinity asymptotically. For example, over a cold region using the above-mentioned standard snow models, the simulated snow/ice storage is likely to have an upward trend even under the stationary climate conditions. Therefore, the standard snow models, both the degree-day model and the full energy balance model, cannot handle the interannual snow/ice storage.

EQUILIBRATOR OF GLACIER AND INTERANNUAL SNOW STORAGE

It is noted that this schematic of the snow-covered terrain represents single model unit or grid cell. The high elevation regions, such as Pamir and Himalaya, tend to have more snow/ice accumulation than melt due to orographic effect of precipitation and the low temperatures induced by the high altitude. If the annual snow accumulation was greater than the total of the annual snowmelt and sublimation rates, these mountains would be completely packed by snow and ice after some 100 years. However, in spite of substantial surplus in annual snow/ice balance, the Himalayan Mountains today have some rock surfaces near their summits although they have a very long history of existence.

The key process that is a missing component in the standard snow models is the vertical snow movement from higher to lower elevations by wind, avalanches, and glacial movements, which limits the snow water storage in the higher elevation sections of the mountains. The peaks and ridges are likely to receive more snowfall than the valleys because of the lower temperatures at the high elevations and the orographic effect of precipitation. Some portion of the newly fallen snow is immediately blown down to the valleys. For the rest of the snowfall, once the accumulated snow on the peaks and ridges becomes unstable, this snow occasionally slides down to the valleys as avalanches. The redistributed snow in the valleys melts much faster than the snow on the peaks and ridges. Furthermore, if the valleys are not sufficiently warm to melt the snow, the snow in the valleys turns into ice and gradually moves to lower elevations by gravity. These vertical snow movements by wind, by the avalanches, and the glacial movements relocate the snow, and render the long-term mass fluxes over the snow/ice cover even. Thus, this process may be considered as a mechanism to equilibrate the snow/ice water storage over the mountains.

DEVELOPMENT OF DYNAMIC EQUILIBRIUM MODEL

In Glaciology, the equilibrium line altitude (ELA) is a common index to characterize glaciers. ELA can be defined as the position where accumulation of snow is exactly balanced by ablation over a period of one year (e.g. Hoinkes, 1970). Figure 1 shows the schematic of snow and ice storage profiles and corresponding annual net mass balance curves. The ELA may be an ideal representative reference point for the standard mass balance snow computation because the snow accumulation and melt are balanced at that elevation.

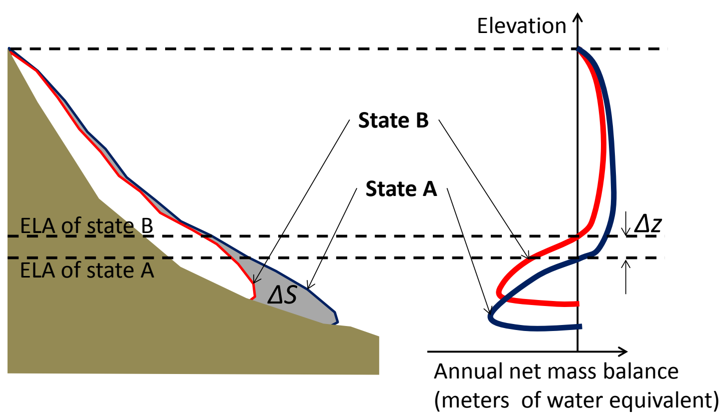

Figure 1. Schematic of snow and ice storage profiles and corresponding mass balance curves

Degree-day Model:

For example, the representative melt rate of the snow and ice storage may be computed by a degree-day formula using the reference air temperature at the ELA, assuming that snow and ice start melting at 0 °C and $T_b = 0$ in Equation (4), as follows:

$$M = C_t T_{ref} \tag{8}$$

where M is melt rate, C_t is a Degree-day coefficient, and T_{ref} is the reference air temperature at the ELA. Now, the dynamic ELA condition is considered as recently discussed by Benn et al. 2000 and others. Referring to Figure 2, the change in ELA is assumed to be proportional to the change in snow and ice storage in this study.

$$\Delta z = -k\Delta S \tag{9}$$

in which S is a total storage of snow and ice body such as glacier, z is the ELA, k is a coefficient of proportionality. Meanwhile, the air temperature difference associates with elevation as,

$$\Delta T_{ref} = -\mu \Delta z \tag{10}$$

where T_{ref} is air temperature at the ELA, and μ is lapse rete. Combining these two equations yields,

$$\Delta T_{ref} = \mu k \Delta S \tag{11}$$

Assuming the changes of the snow and atmosphere states are continuous, one can integrate this equation as:

$$\int dT_{ref} = \int \mu k dS$$

$$T_{ref} = \mu k S + C_1 \tag{12}$$

where C_1 is an integration coefficient.

When there is no interannual snow and ice such as the seasonal snow only case, the reference temperature can be expressed as a function of index temperature in the standard degree-day formula. It means that $T_{ref} = T_i - T_b$ when S = 0. Therefore, one can obtain

$$C_1 = T_i - T_b$$

Replacing the coefficient μk in (12) by C_g, the reference air temperature at the ELT can be written as follows:

$$T_{ref} = C_g S + T_i - T_b \tag{13}$$

Substitution of Eqn. (13) to Eqn. (8) yields,

$$M = C_t\left(C_g S + T_i - T_b\right) \tag{14}$$

Thus, the representative melt rate of the snow and ice storage can be computed by this modified degree-day formula considering the shift of equilibrium state. Comparing this equation to the standard degree-day formula, Equation (4), the reference temperature is adjusted by an additional term C_gS in order to incorporate the dynamic equilibrium nature of snow and ice storage. The newly introduced coefficient C_g may be called a “snow and ice movement parameter” in this study. This modification promotes the melt as the snow/ice storage volume increases, and renders the system to be in dynamic equilibrium.

Energy Balance Model:

With a similar manner, the dynamic equilibrium energy balance model can be developed by introducing the snow and ice movement parameter C_g. The dynamic equilibrium energy balance model is here named “RegSnow” model, which is designed to be coupled with a numerical weather model, such as Weather Research Forecasting (WRF) Model, to meet the atmospheric forcing data requirement of the snow model. Referring to the discussion above, there is a difference between the ELT and the mean elevation of a numerical weather model (WRF), as can be seen in the Figure 2. Among the atmospheric forcing variables, air temperature may be the most sensitive one to the elevation. Therefore, one can have,

$$T_{ref} = C_g S + T_{WRF} \tag{15}$$

where T_{WRF} is the air temperature output of WRF model at the mean elevation of the atmospheric computational cell, and S is the snow storage within a grid. The snow melt rate can be computed by the elevation adjusted energy fluxes as follows:

$$M = L\left(Q_{sw}\left(T_{ref}, \boldsymbol{a}', \mathbf{x}, t\right) + Q_{lw}\left(T_{ref}, \boldsymbol{a}', \mathbf{x}, t\right) + Q_h\left(T_{ref}, \boldsymbol{a}', \mathbf{x}, t\right) + Q_e\left(T_{ref}, \boldsymbol{a}', \mathbf{x}, t\right) + Q_p\left(T_{ref}, \boldsymbol{a}', \mathbf{x}, t\right) + Q_g\left(T_{ref}, \boldsymbol{a}', \mathbf{x}, t\right)\right) \quad (16)$$

This elevation adjusted snow melt equation yields more snowmelt as the snow storage is increased.

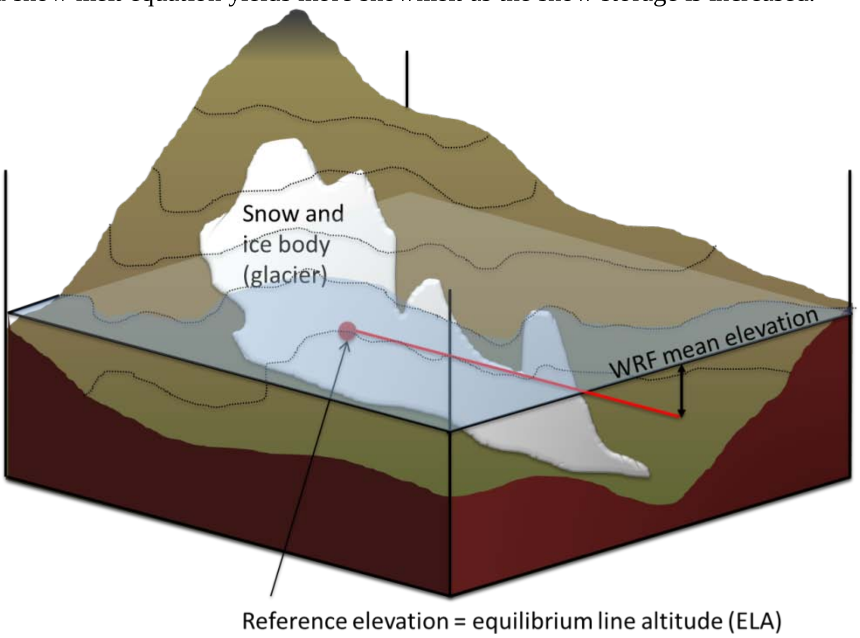

Figure 2. Schematic of glacier representation in RegSnow model

ANALYTICAL SOLUTIONS OF DYNAMIC EQUILIBRIUM MODEL

In this section, analytical solutions of the dynamic equilibrium Degree-day model are examined. The corresponding mass balance equation can be written as,

$$\frac{dS}{dt} = P - M - E = P - C_t(T_i - T_b) - E - C_t C_g S \quad (17)$$

In order to check the asymptotical behavior of the proposed system's equations, by assuming the stationary climate condition, the annual mean mass balance W' under the degree-day framework may be defined as,

$$W' = \overline{P - C_t(T_\iota - T_b) - E} \quad (18)$$

As such, the snow/ice mass balance equation can be reduced to,

$$\frac{dS}{dy} = W' - C_t C_g S \quad (19)$$

This ODE can be easily solved with the initial condition $S(0)=S_o$:

$$S(y) = \frac{W\prime}{C_t C_g} + \left(S_o - \frac{W\prime}{C_t C_g}\right) e^{-C_t C_g y} \quad (20)$$

The solution (20) is plotted in Figure 3. Because of the presence of the equilibrator, it can be seen that the snow/ice storage converges to a finite value depending on the annual mean mass balance. As the snow/ice water storage volume *S* does not become negative, the limit of S as $y \rightarrow \infty$ is

$$\lim_{y\to\infty} S = \begin{cases} 0\,, W' \leq 0 \\ \frac{W'}{C_t C_g}\,, W' > 0 \end{cases} \tag{21}$$

It is shown in Equation (21) that the system can unconditionally reach an equilibrium state even when the region receives excess snowfall.

The numerical examples in Figure 3 show the characteristics of the dynamic equilibrium model. The computed snow/ice water storage by the dynamic equilibrium model converges to a finite value due to the snow vertical movement formulation, while the standard model output diverts to infinity even under a static climate condition. In addition, the model parameters are estimated for the application to the Pamir Mountains, Central Asia (Ohara et al. 2014), and the analysis shows that the glacier system may take about 20 ~ 50 years to recover from the artificial initial snow/ice storage. This convergence time may be equivalent to the response time of the glacier, mentioned in the introduction section, and indeed, the value range of the analysis is almost the same as the value range of other studies (summarized in Table 1, De Smedt and Pattyn, [2003]).

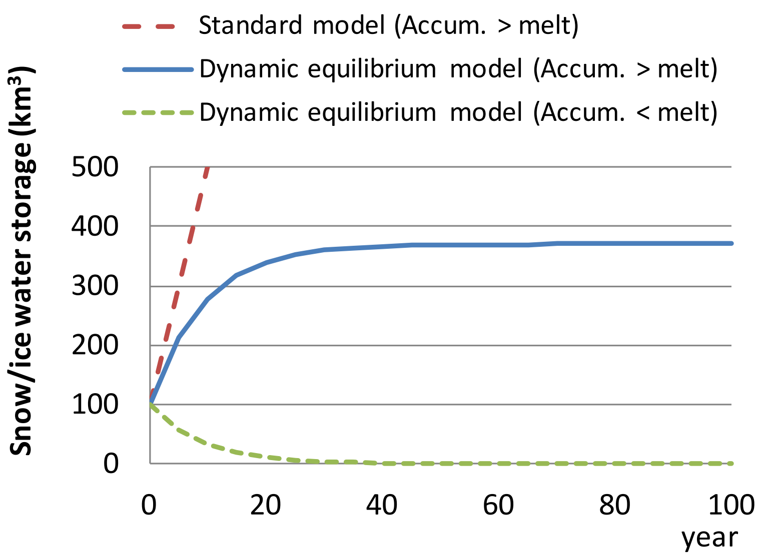

Figure 3. Asymptotic glacier volume under static climate conditions

PRELIMINARY RESULT OF ENERGY BALANCE DYNAMIC EQULIBRIUM MODEL

The preliminary model output of the RegSnow for Wyoming is shown in Figure 4. The model was applied to the entire state of Wyoming where some continental glaciers present, for 32-year-long historical period. The atmospheric forcing was reconstructed by dynamical downscaling of NCEP North American Regional Reanalysis: (NARR) data using the WRF model. Figure 4 shows that the computed total interannual snow storage in Wyoming reaches an equilibrium point around the 10-20 km^3. Note that the state-wide total glacier volume may be estimated to be 14.4 km^3 from the existing world glacier databases: World Glacier Inventory (WGI) (National Snow and Ice Data Center, 1999), MODIS snow product (Hall et al., 1995), GLIMS (Global Land Ice Measurements from Space, http://www.glims.org/). It can be seen the dynamic equilibrium model parameterization resists a false trend due to the interannual snow over the historical period. This confirms the fact that the dynamic equilibrium model parameterization is required for a multi-decadal simulation for regions with interannual snow storage.

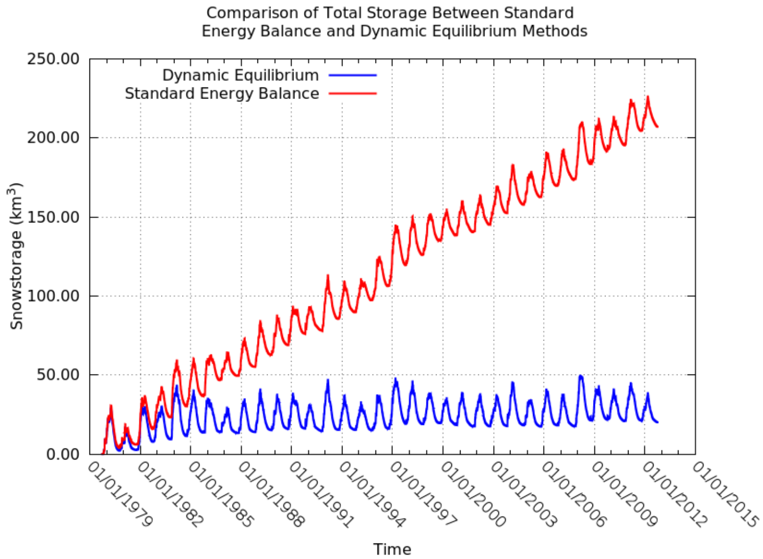

Figure 4. The preliminary model output of the RegSnow for Wyoming

CONCLUSIONS

The simplest way to check the suitability of a snow model for a long-term application in the presence of interannual snow storage and glacier is to run the model under a static climate condition over a century. If the model results show any everlasting trend in total snow mass throughout the simulation period, the model does not have an equilibrator in the system. Therefore, the model operator must use extra caution in the handling of the climate projections that are produced by such a model in order to distinguish between the internal model bias and the climate induced change. In fact, such an approach without an equilibrator often causes misleading projections of the future glacier conditions, for example, the unrealistic projection of Himalayan Glacier in IPCC AR4 [IPCC, 2007; Carrington, 2010] . In order to reduce such risk in the future glacier projections, this study proposes the treatment of the interannual snow storage and glaciers as dynamic equilibrium systems. This treatment provides great stability in the modeling results; thus, it was able to simulate the continuous spatially-distributed snow storage at the regional scale toward the future fairly easily.

The performance of the proposed dynamic equilibrium glacier model coupled with a numerical weather model was demonstrated for the entire state of Wyoming. The model simulation supported the importance of the dynamic equilibrium concept for the interannual snow modeling.

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